Comparative Analysis of Spatial Covariance Matrix Estimation Methods in OFDM Communication Systems

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***Abstract*— The performance of the Orthogonal Frequency Division Multiplexing (OFDM) communication systems may be significantly improved by use of the multiple antenna techniques. One of the multiple antenna techniques applications is the co-channel interference mitigation by exploiting the Minimum Mean Square Error (MMSE) signal detection algorithm. The realization of the algorithm requires knowledge of interference plus additive noise spatial covariance matrix (SCM) estimate for every subcarrier of the OFDM symbol. In this paper the independent SCM estimation for every subcarrier of OFDM symbol is considered first and then the algorithm is proposed for the enhancement of such estimate by taking into account the frequency correlation of the propagation channel and additional knowledge of OFDM symbol structure. As a result, it is shown that exploiting of the a priori information about the channel impulse response delay spread and symbol structure may lead to the significant improvement of the estimation accuracy.**

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is known as an effective technique for communication in the frequency selective wireless Non Line-Of-Sight (NLOS) channel by using guard interval to mitigate intersymbol interference and FFT for channel equalization. OFDM is adopted in the most modern Wireless LAN and MAN standards such as IEEE 802.11 and IEEE 802.16, and OFDM-based communication devices are widely being spread now. The next step to wireless networks performance improvement is the deployment of the OFDM systems with multiple antennas on receive and (or) transmit sides for both base stations and mobiles. It is well known, that multiple antennas can be exploited for many purposes such as achieving diversity gain, array gain, spatial multiplexing and co-channel interference reduction.

Although many co-channel interference mitigation techniques are known [1], the linear Minimum Mean Square Error (MMSE) based processing techniques show the best tradeoff between the implementation complexity and system performance and such techniques will be first entering into the service. The practical use of the MMSE co-channel interference suppression algorithm [2], requires knowledge of the channel transfer function and the received signal (useful signal and background noise and interference) spatial covariance matrix (SCM) estimate, which should be obtained prior to the application of the MMSE algorithm. As the propagation channels are frequency selective the SCM should be estimated for every subcarrier of the OFDM symbol or at least for every group of several adjacent subcarriers.

Typically the OFDM signal structure includes data symbols which are preceded by known training symbols. The SCM matrices estimation should be done during the training stage as the MMSE algorithm should be already tuned when the first data symbols arrive.

Due to mutual statistical independence of the useful OFDM signal, the background noise, and the OFDM interference, SCM matrix of the received signal may be considered as a sum of the useful signal SCM and interference plus noise SCM. The SCM of the useful signal depends only on the channel transfer function of the useful signal and the channel transfer function may be estimated with high accuracy as the known training signals for its estimation are available. On the contrary, there are no available training signals for the OFDM interference. So it follows from these facts that the most accurate scheme for the received signal SCM estimation will include the estimation of the useful signal channel transfer function and the useful signal SCM, removing useful signal from the overall received signal and then estimating the SCM of the interference plus noise signal. Thus the problem arises to find the estimate of the interference plus noise signal SCM. This problem of finding the SCM estimate of the unknown OFDM signal and noise is considered in the present paper.

The classical solution for SCM estimation problem is the sample SCM estimate which is the Maximum Likelihood (ML) estimate in the case of Gaussian signals [3]. But it should be noted that as the OFDM system is considered, the number of available signal samples for each subcarrier is equal to the number of the available OFDM symbols. In practical case there are only a few OFDM symbols available for SCM estimation and a well-known problem of statistical signal processing with small number of observations arises [4]. So there is a need to improve the accuracy of the SCM estimates using some a priori information.

Such kind of information is available in all OFDM communication systems. It is known that in any well-designed OFDM communication system the length of a cyclic prefix (and correspondingly a typical length of the channel impulse response) is only a fraction of the symbol length. It leads to the fact that the SCMs for different subcarriers are correlated and consequently the more accurate estimates may be found than the independent sample SCMs estimates. Additionally, information about the structure of the OFDM symbol (positions of "zero" subcarriers) is also available in advance. It should be noted, that the similar ideas about exploiting of all available a priori information were recently used for the enhancing of the channel transfer function estimates (see, for example, [5]).

The rest of the paper is organized as follows. Section II first describes the traditional sample covariance matrix estimate. Then the enhancement of the sample SCM estimate is proposed which takes into account the length of the channel impulse response and the OFDM symbol structure. After that, the moving average (MA) SCM estimation method, which is used for performance comparison, is shortly described. Section III presents the numerical simulations results for the three considered algorithms. The conclusion is given in Section IV.

II. Spatial Correlation Matrix Estimation Methods

Consider an OFDM system with multiple antennas receiving unknown (interfering) OFDM signal. The OFDM signal is assumed to consist of the set of orthogonal subcarriers modulated by QAM symbols. In the OFDM system the received frequency domain signal $y_{k,i}(n)$ for the i-th time moment, the k-th subcarrier and the n-th antenna can be written as:

$$y_{k,i}(n) = H_k(n)x_{k,i} + \xi_{k,i}(n)\,, \qquad (1)$$

where $H_k(n)$ is the channel frequency transfer coefficient for the k-th subcarrier and the n-th antenna, $x_{k,i}$ is the symbol transmitted at the k-th subcarrier at the time i, $\xi_{k,i}(n)$ is a frequency domain sample of the spatially uncorrelated additive white Gaussian noise. The index n changes from 1 to the number of receive antennas N_{RX}. The index k varies from 1 to the number of the active subcarriers N_{Sc}. The active subcarriers are the subcarriers where the signal is transmitted, i.e. pilot and data subcarriers. The structure of the OFDM signal (the FFT size N_{FFT} and the position of the data, pilot and guard subcarriers) is assumed to be known. The time index i spans from 1 to N_{Smp}. The channel functions $H_k(n)$ remain constant during the receive time. The channel transfer coefficients $H_k(n)$ and the transmitted symbols x_k are both unknown.

The considered problem is to obtain the estimates of the SCMs for all active subcarriers using the set of the received signal samples $y_{k,i}(n)$. As it was noted above, the straightforward method to find the SCMs estimates is to calculate such estimates independently for every subcarrier as sample covariance matrices $\hat{\mathbf{R}}_k$, which are given as follows:

$$\hat{\mathbf{R}}_k = \frac{1}{N_{Smp}} \sum_{i=1}^{N_{Smp}} \mathbf{y}_{k,i}\mathbf{y}_{k,i}^H \qquad (2)$$

where $\mathbf{y}_{k,i} = [y_{k,i}(1) \;\; \ldots \;\; y_{k,i}(N_{RX})]^T$ is a vector of the signals received from different antennas for the k-th subcarrier at time i. Superscripts ^{T} and ^{H} mean transpose and conjugate transpose respectively.

Such estimate is known to be a ML estimate [3] and thus it has some optimal properties in that sense, but no a priori information has been used yet. Therefore, as it was mentioned above, the accuracy of the SCM estimate may be enhanced by taking into account the a priori known information about frequency correlation of the channel transfer function and about the position of the active subcarriers within the OFDM symbol.

Let us consider the method of how such enhancement may be done assuming that time domain channel impulse response length is known and does not exceed some constant L. Note, that the set of the diagonal elements of SCM estimates for different subcarriers $\hat{\mathbf{S}}_{nn} = [\hat{\mathbf{R}}_1(n,n),\ldots,\hat{\mathbf{R}}_{N_{Sc}}(n,n)]$ is the estimate of the power spectral density for the signal from the n-th receive antenna. The set of the non-diagonal elements $\hat{\mathbf{S}}_{mn} = [\hat{\mathbf{R}}_1[m,n],\ldots,\hat{\mathbf{R}}_{N_{Sc}}[m,n]]$ is the estimate of the mutual power spectral density between the signals from the m-th and n-th antennas.

If all the OFDM symbol subcarriers are active (i.e. $N_{Sc} = N_{FFT}$) then the IFFTs of $\hat{\mathbf{S}}_{nn}$ and $\hat{\mathbf{S}}_{mn}$ are estimates of the cyclic autocorrelation $\hat{\mathbf{r}}_{nn}$ and cross correlation $\hat{\mathbf{r}}_{mn}$ sequences respectively. These transforms can be written in matrix-vector form as:

$$\hat{\mathbf{r}}_{mn} = \mathbf{F}^{-1}\hat{\mathbf{S}}_{mn} \qquad m,n = \overline{1, N_{RX}} \qquad (3)$$

where F is a FFT transform matrix which is equal to:

$$\mathbf{F} = \begin{pmatrix} 1 & 1 & \ldots & 1 \\ 1 & e^{-2\pi j/N_{FFT}} & \ldots & e^{-2\pi j(N_{FFT}-1)/N_{FFT}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-2\pi j(N_{FFT}-1)/N_{FFT}} & \ldots & e^{-2\pi j(N_{FFT}-1)(N_{FFT}-1)/N_{FFT}} \end{pmatrix} \qquad (4)$$

Now consider true cyclic correlation sequence $\mathbf{r}_{mn} = [r_{mn}(1), r_{mn}(2), \ldots, r_{mn}(N_{FFT})]^T$ of the signal. The received signal is a sum of the unknown OFDM signals from one or several transmitters, which have propagated through communication channels, and an additive white Gaussian noise. As the OFDM signal and the noise are statistically independent then the correlation sequence can be rewritten as a sum of the correlation sequences of the OFDM signal and the noise respectively:

$$\mathbf{r}_{mn} = \mathbf{r}_{mn}^{Noise} + \mathbf{r}_{mn}^{OFDM} \tag{5}$$

The bakground noise is considered to be white, so its correlation sequence has only one non-zero sample:

$$\mathbf{r}_{mn}^{Noise} = [r_{mn}^{Noise}(1), 0, \ldots, 0] \tag{6}$$

The initially transmitted OFDM signal (signal at the output of transmitter, which has not yet propagated through channel) is known to be almost uncorrelated in the time domain. In accordance with the made assumption the length of the channel impulse response does not exceed some known constant L. In that case the correlation time for the received OFDM signal is not greater than L samples. So the correlation sequence has zero samples for $L < k < N_{FFT} - L + 1$:

$$\begin{aligned} \mathbf{r}_{mn}^{OFDM} &= [r_{mn}^{OFDM}(1), \ \ldots, r_{mn}^{OFDM}(L), 0, \ldots \\ &\ldots, 0, r_{mn}^{OFDM}(N_{FFT} - L + 1), \ldots, r_{mn}^{OFDM}(N_{FFT})]^T \end{aligned} \tag{7}$$

Thus the total correlation sequence will also have zero samples for $L < k < N_{FFT} - L + 1$. This fact is exploited for improving accuracy of the correlation sequence estimate $\hat{\mathbf{r}}_{mn}$ and then of the power spectral density estimates and the SCM estimates. To do so a new estimate of the correlation sequence $\hat{\mathbf{r}}_{mn}^{(L)}$ is introduced as:

$$\hat{r}_{mn}^{(L)}(k) = \begin{cases} \hat{r}_{mn}(k), & k \le L, k \ge N_{FFT} - L + 1 \\ 0, & L < k < N_{FFT} - L + 1 \end{cases} \tag{8}$$

The above operation can be written in a matrix-vector form as:

$$\begin{gathered} \hat{\mathbf{r}}_{mn}^{(L)} = \mathbf{E}\hat{\mathbf{r}}_{mn}, \quad \mathbf{E} = diag(e_k), \\ e_k = \begin{cases} 1, & k \le L, k \ge N_{FFT} - L + 1 \\ 0, & L < k < N_{FFT} - L + 1 \end{cases} \qquad k = \overline{1, N_{FFT}}. \end{gathered} \tag{10}$$

The FFT of $\hat{\mathbf{r}}_{mn}^{(L)}$ gives an improved estimate of the power spectral density $\hat{\mathbf{S}}_{mn}^{(L)}$:

$$\hat{\mathbf{S}}_{mn}^{(L)} = \mathbf{F}\hat{\mathbf{r}}_{mn}^{(L)} = \mathbf{F}\mathbf{E}\mathbf{F}^{-1}\hat{\mathbf{S}}_{nm} \tag{11}$$

Thus the equation (11) proposes the algorithm for the enhancement of the sample covariance matrix estimates by taking into account the information about the channel delay spread. However the algorithm was derived under the assumption that all the subcarriers of the OFDM symbol are active. For the practical OFDM systems also there is additional a priori information about the OFDM symbol structure because such systems use some small fraction of "zero" (not active) subcarriers in the middle and on the borders of the bandwidth for the spectral shaping purposes. For example, the IEEE 802.11a standard establishes the use of 52 out of 64 subcarriers.

For the case when not all subcarriers of the OFDM symbol are known, i.e. $N_{Sc} < N_{FFT}$, for the SCM estimate enhancement the matrix F should be replaced with a new matrix $\mathbf{F}_U$ defined below. In general case the FFT matrix F can be considered as a set of row vectors:

$$\mathbf{F} = \begin{pmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \vdots \\ \mathbf{f}_{N_{FFT}} \end{pmatrix} \tag{12}$$

$$\mathbf{f}_i = \begin{pmatrix} 1 & e^{-2\pi j(i-1)/N_{FFT}} & \ldots & e^{-2\pi j(i-1)(N_{FFT}-1)/N_{FFT}} \end{pmatrix}$$

The new matrix $\mathbf{F}_U$ is $N_{Sc} \times N_{FFT}$ matrix obtained from F by removing $(N_{FFT} - N_{Sc})$ row vectors $\mathbf{f}_j$ corresponding to non-active subcarriers. In this case the inverse FFT matrix $\mathbf{F}^{-1}$ should be substituted by pseudoinverse matrix $\mathbf{F}_U^{\#}$ of size $N_{FFT} \times N_{Sc}$.

Thus the general algorithm for the improved estimate of the power spectral density is:

$$\hat{\mathbf{S}}_{mn}^{(L)} = \mathbf{F}_U\mathbf{E}\mathbf{F}_U^{\#}\hat{\mathbf{S}}_{nm} = \mathbf{P}\hat{\mathbf{S}}_{nm}, \quad \mathbf{P} = \mathbf{F}_U\mathbf{E}\mathbf{F}_U^{\#} \tag{13}$$

After that improved estimates of SCMs are found as:

$$\hat{\mathbf{R}}_i^{(L)} = \begin{pmatrix} \hat{\mathbf{S}}_{11}^{(L)}[i] & \ldots & \hat{\mathbf{S}}_{1N_{RX}}^{(L)}[i] \\ \vdots & \ddots & \vdots \\ \hat{\mathbf{S}}_{N_{RX}1}^{(L)}[i] & \ldots & \hat{\mathbf{S}}_{N_{RX}N_{RX}}^{(L)}[i] \end{pmatrix} \tag{14}$$

As a result, the proposed above method exploits the a priori information about both the channel delay spread and the OFDM symbol structure for enhancing the accuracy of sample SCMs estimates.

It can be easy seen from equation (13) that in general case the proposed SCM estimation algorithm uses different smoothing coefficients for different subcarriers of OFDM symbol for the accuracy enhancement. It is interesting to compare the proposed algorithm with variables weights (VW) and the moving average (MA) algorithm with the constant weights which enhances sample SCM estimate by averaging over the adjacent active subcarriers with a rectangular window of bandwidth w.

Performance comparison of all SCM estimation algorithms, described above is given in the next section.

III. Simulation Results

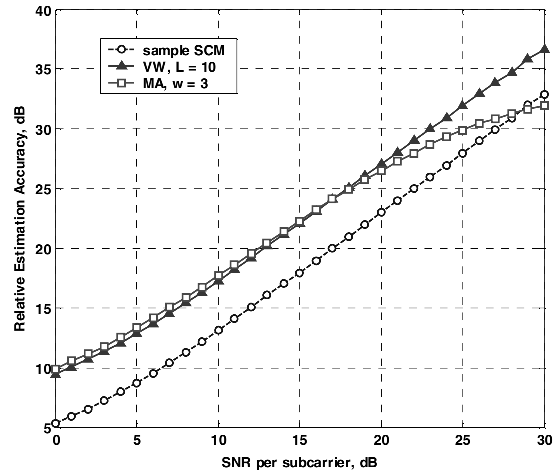
To investigate the performance of the SCM estimation algorithms, numerical simulations have been carried out. The OFDM communication system with 2 receive antennas was considered. The parameters of the system were taken in accordance with IEEE 802.11a standard where the channel bandwidth is 20 MHz, the 64-point FFT is used with the 52 active subcarriers, the subcarrier spacing is 312.5 KHz, the sample interval is 50 ns, the guard interval is 16 samples (800 ns). The subcarriers were modulated with the QPSK modulation. Only 4 OFDM symbols were used for the SCMs estimation. The Rayleigh channel model with 50 ns exponential delay profile (τ_{RMS} = 50 ns) was used for the simulations. This channel model is known to be the most widely used for modeling of the indoor WLAN signals propagation.

Fig. 1 shows the performance of the sample SCM estimation algorithm (as a reference), proposed VW algorithm and MA algorithm for different assumed channel impulse response lengths *L* (in samples) and averaging window bandwidth *w* (in subcarrier spacings). On this figure the relative estimation accuracy is plotted as function of the Signal-to-Noise Ratio (SNR) which is the ratio of the received OFDM interference power to the power of the background additive white Gaussian noise. It can be seen from Fig. 1, that for the considered case of according selection of channel length parameter *L* for VW algorithm and window bandwidth *w* for MA algorithm ($L \cdot w \approx N_{FFT}/2$) the both algorithms have the same accuracy and significantly outperform sample SCM estimate for low SNR region (less than 15 dB) due to exploiting additional a priori information. For the higher SNR (more than 15 dB) the proposed VW algorithm has 2-3 dB accuracy gain over MA algorithm.

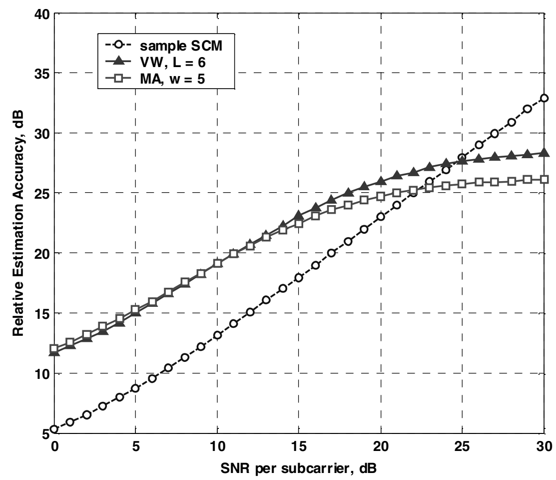
Fig. 2 plots the relative estimation accuracy (for different SNRs) of the proposed VW algorithm for the same channel delay spread (τ_{RMS} = 50 ns) in the dependence on the assumed channel impulse response length *L* exploiting in the VW algorithm. So, as it follows from Fig.2, the a priori information about the SNR can give additional significant performance gain by optimal choosing parameter *L* of the VW algorithm.

IV. Conclusion

In this paper the novel spatial covariance matrix (SCM) estimation algorithm with variable weights (VW) has been proposed for application to OFDM communication system in the case of small number of available OFDM symbols. The VW algorithm uses the a priori information about the channel delay spread and the OFDM symbol structure for the estimates enhancement. The characteristics of the VW algorithm were compared by numerical simulations with sample SCM estimation algorithm (ML algorithm) and moving average (MA) SCM estimation algorithm. It has been shown that the proposed VW algorithm and MA algorithms significantly outperform sample SCM estimation



(a)



(b)

Figure 1. Relative estimation accuracy of the sample SCM algorithm, the proposed VW algorithm and the moving average (MA) algorithm

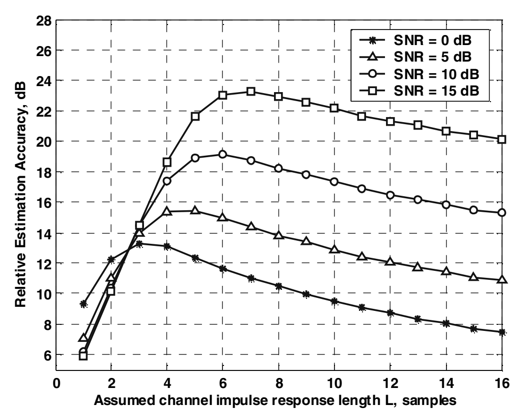


Figure 2. Relative estimation accuracy of the proposed algorithm in the dependence on the assumed channel impulse response length for different SNRs

algorithm for low SNR and that the proposed VW algorithm outperforms both sample and MA algorithms for high SNR region. It has been also revealed that an additional accuracy gain for the proposed VW algorithm may be obtained by using the information about the current SNR for the proper algorithm parameters optimization.

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