Impact of Channel Estimation Errors on Various Spatial-Temporal Transmission Schemes

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***Abstract*— Herein, we investigate the impact of channel estimation errors on different spatial-temporal transmission schemes. The channel estimation errors are modeled by an zero mean complex Gaussian matrix. We then study the error floors of several transmission schemes, namely the Alamouti scheme, spatial multiplexing (SM) and the hybrid scheme. Our study reveals that for small estimation error, the Alamouti scheme has lower error floor comparing to the SM, while for 4 transmit antennas, the hybrid scheme is more robust than the pure SM. Moreover, simulation results show that the hybrid scheme with antenna shuffling can lower the error floor even further.**

I. Introduction

Recently, wireless broadband systems equipped with antenna arrays have attracted much attention both in academia and industry. They can provide diversity gain and/or spatial multiplexing (SM) gain comparing to the traditional systems using single transmit/receive antenna, hence increase the system capacity [1], [2] and/or lower the bit error rate (BER). Some popular spatial-temporal transmission schemes proposed before include orthogonal space-time block codes (OSTBCs) [3], [4], SM scheme [5], [6] and hybrid scheme [7], [8] that balances the benefit of diversity gain and SM gain. To further improve the performance, an algorithm has been proposed in [9] to optimally switch between the SM scheme and the Alamouti scheme.

Within the European Union (EU) 6th Research Framework, Project MEMBRANE (Multi-Element Multihop Backhaul Reconfigurable Antenna NEtwork) is one of the research projects that focus on the future wireless networks. The main target is to provide high speed transmission between the broadband wireless access networks and the wired internet, so called wireless backhaul networks.

To provide very high speed data transmission, multiple antennas are assumed to be deployed at each nodes within the wireless backhaul networks. This is a reasonable assumption since the nodes in backhaul networks are stationary and can provide enough spaces, battery power, and computation capability. Therefore, the spatial-temporal transmission schemes mentioned above are natural solutions to transmit the data with high data rate.

All schemes mentioned above require channel state information (CSI) at the receive side. In a real system, however, channel estimation errors always exist due to the channel noise and interpolation process. This leads to an error floor in the high SNR region that can not be neglected. In [10], the impact of channel estimation error has been studied together with the V-BLAST [6] structure. In this paper, we further study the impact of channel estimation error on the Alamouti scheme, the SM scheme, and the hybrid scheme.

This paper is organized as follows. Section II describes the system, channel, and error models used in the paper. The spatial-temporal transmission schemes are briefly described in Section III. In Section IV, the simulation results are presented. Finally, we conclude in Section V.

II. System Description

A. System Model

Assume there are N_t transmit antennas and N_r receive antennas. The input-output relationship can be expressed in the baseband as

$$\mathbf{y} = \sqrt{\frac{E_s}{N_t}}\mathbf{H}\mathbf{s} + \mathbf{n}, \tag{1}$$

where E_s is the total average energy transmitted over a symbol period, $\mathbf{H}$ is the N_r by N_t channel matrix, $\mathbf{s}$ is the transmitted symbol vector, $\mathbf{y}$ is the received signal vector, and $\mathbf{n}$ is the additive white Gaussian noise with variance σ^2. Note that we assume $\mathbf{H}$ is normalized so that its elements are complex Gaussian random variables $\mathcal{CN}(0,1)$, and the averaged symbol energy of $\mathbf{s}$ is unity.

B. Channel Model

To model the correlation between the elements of channel matrix $\mathbf{H}$, we use the well known Kronecker model [11], i.e. $\mathbf{R}_H = \mathbf{R}_{tx} \otimes \mathbf{R}_{rx}$, where $\mathbf{R}_{tx}$ and $\mathbf{R}_{rx}$ are the normalized transmit and receive channel correlation matrices respectively. The normalized channel matrix can then be written as

$$\mathbf{H} = (\mathbf{R}_{rx})^{1/2}\mathbf{G}[(\mathbf{R}_{tx})^{1/2}]^T, \tag{2}$$

where the stochastic matrix $\mathbf{G}$ contains independent and identically distributed (iid) $\mathcal{CN}(0,1)$ elements, $(\cdot)^T$ denotes transpose, $(\cdot)^{1/2}$ is defined such that $\mathbf{R}^{1/2}(\mathbf{R}^{1/2})^H = \mathbf{R}$, and $(\cdot)^H$ is the Hermitian transpose.

C. Channel Estimation Error Model

In real systems, the channel estimation procedure can never be perfect, and therefore the channel estimation errors always exist. The main sources of channel estimation errors include the noise in the estimation procedure, the interpolation process, the quantization, and the feedback delay. The first two sources have impact over the whole system, while the last two sources impact mainly at the transmitter. In this paper, we focus more on the channel estimation errors caused by the first two sources. Note that even in the very high SNR region, the channel estimation errors will still not disappear completely due to the interpolation error.

One popular channel estimation error model is to model the elements of the channel estimation error matrix using zero mean complex Gaussian variables [10], i.e.

$$\hat{\mathbf{H}} = \sqrt{1-\epsilon^2}\mathbf{H} + \epsilon\tilde{\mathbf{H}}, \tag{3}$$

where $\hat{\mathbf{H}}$ is the estimated channel matrix, $\tilde{\mathbf{H}}$ is the normalized zero mean complex Gaussian channel estimation error matrix, and the parameter $\epsilon \in [0,1]$ measures the accuracy of channel estimation. Note that $\epsilon = 0$ means no channel estimation error exists while $\epsilon = 1$ indicates a complete failure of channel estimation.

III. Transmission Schemes

A. Spatial Multiplexing Scheme

The SM scheme transmits data symbols over parallel spatial subchannels and achieves high data rate. This has been demonstrated in [5], [6], where the Bell Labs Layered Space-Time (BLAST) architectures were proposed along with a coding and decoding scheme. In this paper, we assume the transmitter has no CSI, therefore the power is equally allocated to each transmit antennas. Furthermore, we assume the receiver only knows the estimated CSI and study the impact of channel estimation error on the SM scheme using four different types of receivers. We briefly describe these receivers below.

1) ZF receiver: The zero-forcing (ZF) receiver belongs to the linear receiver, and can be expressed as [12]

$$\mathbf{G}_{ZF} = \sqrt{\frac{N_t}{E_s}}\mathbf{H}^{\dagger}, \tag{4}$$

where $(\cdot)^{\dagger}$ denotes the Moore-Penrose pseudo inverse.

2) MMSE receiver: Another type of linear receiver is the minimum mean square error (MMSE) receiver, which can be expressed as [12]

$$\mathbf{G}_{MMSE} = \sqrt{\frac{N_t}{E_s}}(\mathbf{H}^H\mathbf{H} + \frac{N_t}{\rho}\mathbf{I}_{N_t})^{-1}\mathbf{H}^H, \tag{5}$$

where ρ is the average SNR at the receive side.

3) V-BLAST receiver: In [6], the non-linear vertical BLAST (V-BLAST) receiver has been proposed. The main idea is to successively decode the symbols layer by layer. By using symbol cancelation, the interference from the decoded symbols is removed. This approach can be combined with either the ZF or MMSE receivers mentioned above. See [6] for more details on V-BLAST receiver.

4) Impact of channel estimation error: The impact of channel estimation error on the ZF receiver has been investigated in [10]. Using the perturbation theory, when the Frobenius norm $|\hat{\mathbf{H}}^{\dagger} - \mathbf{H}^{\dagger}|_F$ is small, the perturbation of the channel matrix $\mathbf{H}$ is approximated as an noise term $\tilde{\mathbf{n}}$ to the unperturbed system, i.e. [10]

$$\tilde{\mathbf{n}} = \mathbf{n} - \frac{\epsilon\sqrt{\mathrm{E}_s}}{\sqrt{N_t(1-\epsilon^2)}}\tilde{\mathbf{H}}\mathbf{s}. \tag{6}$$

The covariance of the approximated noise term can be derived as [10]

$$\hat{\mathbf{R}} = \sigma^2\mathbf{I}_{N_r} + \frac{\epsilon^2 E_s}{N_t(1-\epsilon^2)}\mathrm{E}[\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H], \tag{7}$$

where $\mathrm{E}(\cdot)$ calculates the expected value of a random variable.

B. Alamouti Scheme

The Alamouti transmission scheme [3] is designed to exploit the transmit diversity for any system with 2 transmit antennas. It is a special case of OSTBCs [4] which can exploit diversity for any number of transmit antennas. Note that the Alamouti code is the only full rate code available among the OSTBCs. In this paper, we focus on studying a 2×2 system using Alamouti scheme.

Assume the channel remains constant for two consecutive symbol periods, the input-output relationship for the Alamouti scheme can be written as

$$\mathbf{Y} = \sqrt{\frac{E_s}{2}}\mathbf{H}\begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} + \mathbf{N}, \tag{8}$$

where $(\cdot)^*$ denotes complex conjugate, $\mathbf{Y}$ is the received signal matrix, and $\mathbf{N}$ is the noise matrix.

The above expression can be rewritten using the effective channel matrix as

$$\mathbf{y}_{eff} = \sqrt{\frac{E_s}{2}}\mathbf{H}_{eff}\mathbf{s} + \mathbf{n}_{eff}, \tag{9}$$

where $\mathbf{y}_{eff} = [\mathbf{Y}(:,1)^T, \mathbf{Y}(:,2)^T]^T$, $\mathbf{s} = [s_1, s_2]^T$ and $\mathbf{n}_{eff} = [\mathbf{N}(:,1)^T, \mathbf{N}(:,2)^T]^T$. The effective channel matrix $\mathbf{H}_{eff}$ is

$$\mathbf{H}_{eff} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \\ h_{1,2}^* & -h_{1,1}^* \\ h_{2,2}^* & -h_{2,1}^* \end{bmatrix}. \tag{10}$$

Similar to the SM scheme, using the perturbation theory and the equivalent channel transfer function (9), the covariance of the approximated noise term can be derived as

$$\hat{\mathbf{R}}_{Ala} = \sigma^2\mathbf{I}_4 + \frac{\epsilon^2 E_s}{2(1-\epsilon^2)}\mathrm{E}[\tilde{\mathbf{H}}_{eff}\tilde{\mathbf{H}}_{eff}^H], \tag{11}$$

where $\tilde{\mathbf{H}}_{eff}$ has similar structure as $\mathbf{H}_{eff}$.

C. Hybrid Transmission Scheme

For systems equipped with 4 transmit antennas, instead of transmitting using either the SM scheme or OSTBC scheme mentioned above, a hybrid transmission scheme has been presented [7], [8] in order to obtain a good balance between the SM gain and the diversity gain. The main idea is to group two antennas together so that there are two sets of antenna pair. The SM scheme is used between two sets of antenna pairs. While within each antenna pair, the Alamouti scheme is deployed. This hybrid scheme has also been studied in the 3GPP meeting [13].

1) Without antenna shuffling: Assume the channels are stationary for two consecutive symbol periods. In the first symbol period, the transmitted symbol vector is $\mathbf{s} = [s_1, s_2, s_3, s_4]^T$. The transmitter transmits $\tilde{\mathbf{s}} = [s_2^*, -s_1^*, s_4^*, -s_3^*]^T$ in the second symbol period. Using the effective channel matrix $\mathbf{H}_{hyb}$, the input-output relationship can be written as [7]

$$\begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2^* \end{bmatrix} = \sqrt{\frac{E_s}{4}}\mathbf{H}_{hyb}\mathbf{s} + \begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2^* \end{bmatrix}, \tag{12}$$

where the effective channel matrix $\mathbf{H}_{hyb}$ is [7]

$$\mathbf{H}_{hyb} = \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 & \mathbf{h}_4 \\ \mathbf{h}_2^* & -\mathbf{h}_1^* & \mathbf{h}_4^* & -\mathbf{h}_3^* \end{bmatrix}, \tag{13}$$

$\mathbf{r}_i$ is the received vector at symbol period i $(i = 1, 2)$, $\mathbf{n}_i$ is the noise vector at symbol period i, and $\mathbf{h}_k$ is the channel vector associated with antenna k $(k = 1, 2, 3, 4)$.

At the receive side, a linear ZF/MMSE or V-BLAST receiver can be used to decode the received signals [7]. Using the perturbation theory and (12), the covariance of the approximated noise term can be written as

$$\hat{\mathbf{R}}_{hyb} = \sigma^2\mathbf{I}_{2N_r} + \frac{\epsilon^2 E_s}{4(1-\epsilon^2)}\mathrm{E}[\tilde{\mathbf{H}}_{hyb}\tilde{\mathbf{H}}_{hyb}^H], \tag{14}$$

where $\tilde{\mathbf{H}}_{hyb}$ has similar structure as $\mathbf{H}_{hyb}$.

2) With antenna shuffling: The hybrid scheme described above does not select specific antennas to form the antenna pairs. Hence it does not provide optimal balance between the SM gain and diversity gain. In [8], [13], the antenna shuffling based on long term channel statistics has been studied to further improved the system performance. For each instantaneous channel realization, one possible approach is to find the optimal antenna pairs based on the capacity of the effective channel matrix, i.e.

$$[\mathrm{M}_1, \mathrm{M}_2] = \arg\max_{\mathrm{M}_1,\mathrm{M}_2} \log_2 \det(\mathbf{I}_4 + \frac{\rho}{4}\mathbf{H}_{hyb}^H\mathbf{H}_{hyb}), \tag{15}$$

where M_1 and M_2 are two sets of antenna pairs.

In real system when the exact number of capacity is not a concern, (15) can be rewritten as below to reduce the computation complexity, i.e.

$$[\mathrm{M}_1, \mathrm{M}_2] = \arg\max_{\mathrm{M}_1,\mathrm{M}_2} \det(\mathbf{I}_4 + \frac{\rho}{4}\mathbf{H}_{hyb}^H\mathbf{H}_{hyb}). \tag{16}$$

(15) and (16) are equivalent since the $\log_2(\cdot)$ is an increasing function.

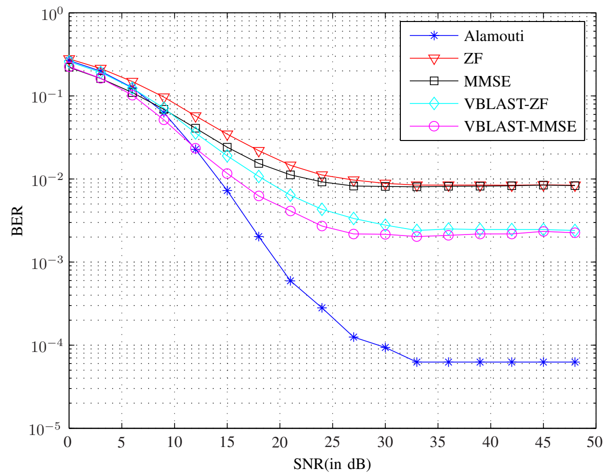

Fig. 1. Comparison of the Alamouti and SM schemes for 2×2 system with channel estimation errors $\epsilon = 0.1$, $\rho_{tx} = \rho_{rx} = 0$.

Note that in this case, a small feedback overhead is required since the receiver needs to find the best antenna pairs among 6 candidates and feedback this information to the transmitter.

IV. Simulation Results

In this section, we use the Monte-Carlo simulations to show the error floor associated with various transmission schemes (described in Section III) when there exist small channel estimation errors ($\epsilon = 0.1$). Note that throughout this section, we assume the channel estimation error matrix $\tilde{\mathbf{H}}$ has iid zero mean complex Gaussian element.

A. Two Transmit Antennas

For 2×2 systems, the Alamouti scheme and the SM scheme are simulated. In the simulations, 1000 channel realizations are generated. We further assume the channels are stationary for 8 successive symbol periods, i.e. each channel realization has been used 8 times for transmission. To keep the same data rate, we use 4-QAM modulation for the SM scheme and 16-QAM modulation for the Alamouti scheme.

Fig. 1 shows the result when no channel correlation is assumed at both the transmitter and the receiver. Similar to the results reported in [10], the error floor for the V-BLAST receivers are lower than the error floor for the linear ZF and MMSE receivers. For small channel estimation error, the error floor for the Alamouti scheme is much lower than the SM scheme. Furthermore, we observe that the Alamouti scheme reaches the error floor slightly later (i.e. at higher SNR) than the SM scheme.

In Fig. 2, we set the correlations $\rho_{tx} = \rho_{rx} = 0.7$ at both the transmitter and the receiver. We obtain similar results as shown in the no correlation case. However, the BER associated with the error floor becomes larger due to the loss of diversity and SM gain.

B. Four Transmit Antennas

We compare the SM scheme with the hybrid scheme for 4×4 systems. 2000 complex Gaussian channel realizations

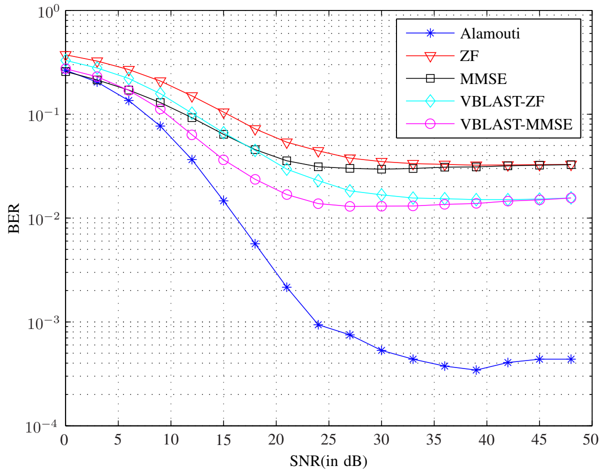

Fig. 2. Comparison of the Alamouti and SM schemes for 2×2 system with channel estimation errors $\epsilon = 0.1$, $\rho_{tx} = \rho_{rx} = 0.7$.

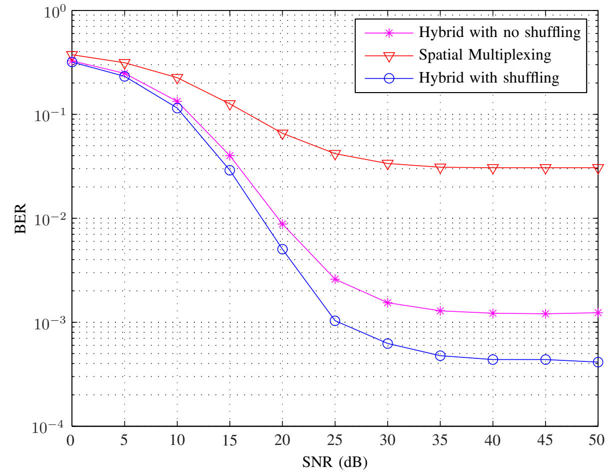

Fig. 4. Comparison of the SM and hybrid schemes for 4×4 system with channel estimation errors $\epsilon = 0.1$, $\rho_{tx} = \rho_{rx} = 0.7$.

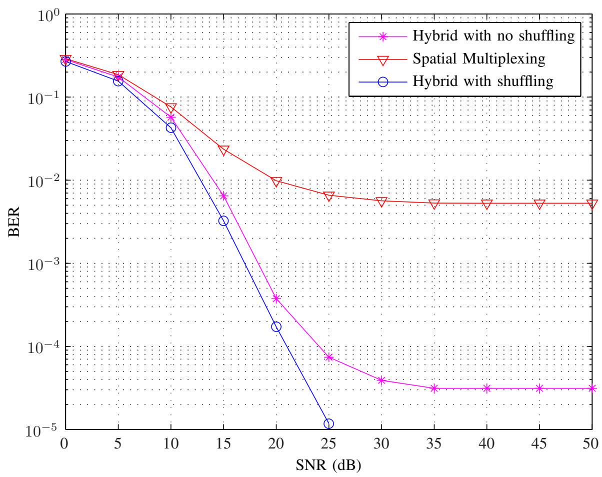

Fig. 3. Comparison of the SM and hybrid schemes for 4×4 system with channel estimation errors $\epsilon = 0.1$, $\rho_{tx} = \rho_{rx} = 0$.

are generated, and the channel is assumed to be stationary for 16 successive symbol periods. The V-BLAST ZF receiver has been used in both schemes. Here the modulation schemes used for the SM and hybrid schemes are 4-QAM and 16-QAM respectively.

Fig. 3 shows the results of using the SM scheme and the hybrid scheme. No correlation is assumed at both the transmitter and the receiver. Again, the error floor for the hybrid scheme is much lower than the SM scheme. This is due to the fact that in the hybrid scheme, the Alamouti scheme is combined with the SM scheme. Since the Alamouti scheme achieves lower error floor than the SM scheme when the channel estimation error is small, it is clear that the hybrid scheme also has lower error floor comparing to the pure SM scheme.

To study the performance of the SM and hybrid schemes in the correlated channels, we use the following correlation matrix structure at both the transmit and receive sides, i.e.

$$\mathbf{R}_l = \begin{bmatrix} 1 & \rho_l^* & \rho_l^{2*} & \rho_l^{3*} \\ \rho_l & 1 & \rho_l^* & \rho_l^{2*} \\ \rho_l^2 & \rho_l & 1 & \rho_l^* \\ \rho_l^3 & \rho_l^2 & \rho_l & 1 \end{bmatrix}, \tag{17}$$

where $l \in (tx, rx)$, and ρ_l is the correlation between the neighboring antenna elements at either the transmitter or the receiver.

Fig. 4 shows the performance for the channel with correlations $\rho_{tx} = \rho_{rx} = 0.7$ at both the transmitter and the receiver. We observed similar results as plotted in Fig. 3, although the BER associated with the error floor increases for both the SM and hybrid schemes.

C. With/Without Antenna Shuffling

We further study the error floor for the hybrid schemes with and without antenna shuffling. Figs. 3 and 4 clearly show that the hybrid scheme with antenna shuffling (15) performs better than the scheme without antenna shuffling. In Fig. 3, the hybrid scheme without antenna shuffling achieves the error floor at around 35dB with BER 3×10^{-5}, while the error floor for the hybrid scheme with antenna shuffling is not observed due to the limited number of runs in our simulation. In Fig. 4, it is shown that the error floor for the scheme with antenna shuffling (associated with BER 4×10^{-4}) is lower than the error floor for the scheme without antenna shuffling (associated with BER 10^{-3}).

V. Conclusions

In this paper, we have investigated the impact of channel estimation errors on various transmission schemes using complex Gaussian error model. For 2 transmit antennas, our simulations have shown that for small channel estimation errors, the Alamouti scheme is more robust than the SM scheme in the sense that its associated error floor of BER curve is much lower than that of the SM scheme. In the case of 4 transmit antennas, it has been shown that the hybrid scheme outperforms the SM scheme. With antenna shuffling, the performance of the hybrid scheme can be further improved.

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