

LIMITED DOWNLINK NETWORK COORDINATION IN CELLULAR NETWORKS

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ABSTRACT

We investigate the downlink throughput of cellular systems where groups of M antennas— either co-located or spatially distributed— transmit to a subset of a total population of $K > M$ users in a coherent, coordinated fashion in order to mitigate intercell interference. We consider two types of coordination: the capacity-achieving technique based on dirty paper coding (DPC), and a simpler technique based on zero-forcing (ZF) beamforming with per-antenna power constraints. During a given frame, a scheduler chooses the subset of the K users in order to maximize the weighted sum rate, where the weights are based on the proportional-fair scheduling algorithm. We consider the weighted average sum throughput among K users per cell in a multi-cell network where coordination is limited to a neighborhood of M antennas. Consequently, the performance of both systems is limited by interference from antennas that are outside of the M coordinated antennas. Compared to a 12-sector baseline which uses the same number of antennas per cell site, the throughput of ZF and DPC achieve respective gains of 1.5 and 1.75.

I INTRODUCTION

In conventional cellular networks, the throughput is limited by co-channel interference, either from within a cell (intra-cell interference) or from nearby cells (intercell interference). Downlink intracell interference can be eliminated by orthogonal channel allocation, for example through OFDMA. Intercell interference can be reduced in several ways, for example through frequency planning, soft handoff, or beamforming multiple antennas. While these techniques reduce the interference, intercell interference in theory can be completely eliminated by coherently coordinating the transmission of signals across base stations in the entire network. Each user's signal is transmitted simultaneously from multiple base station antennas (possibly spatially distributed), and the signals are weighted and pre-processed so that intercell interference is mitigated or completely eliminated [1].

Due to the complexity required to coordinate transmission over an entire network, we focus on downlink coordination over a limited subset of the network. We consider dirty paper coding (DPC), a capacity-achieving technique for the broadcast channel [2], and a simpler zero-forcing (ZF) beamforming algorithm which uses linear weighting and conventional single-user channel coding. Under ZF, multiple non-interfering beams are generated to serve multiple users simultaneously. Motivated by practical considerations that often require individual power amplifiers for each transmit antennas, we impose a per-antenna power constraint on the transmit powers. Under this constraint, power allocation for ZF was shown to be a con-

vex optimization, and its sum-rate throughput performance was shown to be optimal as the number of users increases asymptotically [3]. We assume a high-speed backhaul for transferring coherent channel knowledge and user data among the coordinating bases. The effects of delay or channel estimation error are not considered here but should be investigated in future work.

Previous work in this area has studied the performance of DPC and ZF under the condition of equal rate service for non-outage users during each frame [1]. This requirement is suitable for equal-rate circuit-based applications. In this paper, we consider the context of a scheduled packet-data system where the goal is to maximize the weighted sum rate of the users during each frame. Our methodology is an extension of the proportional fair scheduling (PFS) algorithm to a system where multiple users can be served during each frame.

The paper is organized as follows. In Section II we give the system model. In Section III we describe the system architecture, transmission technique, and system simulation methodology for the two precoding schemes, ZF and DPC. In Section IV we discuss the baseline sectorized system. In Section V we present the scheduling methodology used for all three transmission options, and in Section VI we discuss the simulation results.

II SYSTEM MODEL

We consider a cellular system of $N = 19$ hexagonal cells where a center cell is surrounded by two rings of cells. We evaluate the throughput performance of the central cell in the presence of interference caused by the 18 surrounding cells. For the precoding systems, let M be the number of coordinated antennas. For the sectorization cases, let $M = 1$ be the number of antennas per sector. The placement of the antennas is described later. Let K be the number of single-antenna users per cell, and let h_{km}^l be the channel response between the m th antenna ($m = 1, \dots, M$) of the l th cell ($l = 1, \dots, N$) and the k th user ($k = 1, \dots, K$) of the central cell:

$$h_{km}^l = \beta_{km}^l \sqrt{A(\theta_{km}^l) [d_{km}^l/d_0]^n \rho_{km}^l \Gamma} \quad (1)$$

where β_{km}^l is the Rayleigh fading, $\beta_{km}^l \sim \mathcal{NC}(0, 1)$, $A(\theta_{km}^l)$ is the antenna element response as a function of the direction from the m th antenna of the l th cell to the k th user in the central cell, d_{km}^l is the distance between the m th antenna of the l th cell and the k th user of the central cell, d_0 is the reference distance defined as the distance between the cell center and its vertex, n is the pathloss coefficient, and ρ_{km}^l is the lognormal shadowing between the m th antenna of the l th cell and the k th user.

The variable Γ is the reference SNR defined as the SNR measured at the cell vertex assuming a single antenna at the cell

center transmits at full power, accounting only for the pathloss. This parameter conveniently captures the effects of transmit power, cable losses, thermal noise power and other link-related parameters. For example, for a conventional macrocellular system with a 2 km base-to-base distance and with a 30 watt amplifier transmitting in 1MHz bandwidth, the reference SNR is $\Gamma = 18\text{dB}$ [4]. If instead we use a 3 watt amplifier and keep the other parameters the same, the reference SNR would be $\Gamma = 8\text{dB}$. Alternatively, we could have kept the transmit power the same (30 watts) and increased the bandwidth by a factor of 10 (to 10MHz) to obtain $\Gamma = 8\text{dB}$.

We model the element response as an inverted parabola that is parameterized by the 3 dB beamwidth θ_{3dB} and the sidelobe power A_s measured in dB:

$$A(\theta_{km}^l) \text{ dB} = -\min\{12(\theta_{km}^l/\theta_{3dB})^2, A_s\} \quad (2)$$

where $\theta \in [-\pi, \pi]$ is the direction of user k with respect to the broadside direction of the m th antenna of the l th cell. The different antenna array parameters considered in this work are summarized in Table 1, but we leave the discussion of these parameters for Sections A and A.

TX scheme	M	Antenna array	θ_{3dB}	A_s
3 sectors	1	none	$(70/180)\pi$	20 dB
12 sectors	1	none	$(17.5/180)\pi$	26 dB
ZF centr.	12	circular	$(90/180)\pi$	20 dB
ZF distr.	12	linear	$(70/180)\pi$	20 dB
DPC centr.	12	circular	$(90/180)\pi$	20 dB
DPC distr.	12	linear	$(70/180)\pi$	20 dB

Table 1: Antenna parameters.

We assume that antennas are coordinated in clusters of M antennas, and each cluster served K single-antenna users. Letting $l = 1$ be the index of the target cluster, the discrete-time complex baseband received signal by the k th user served by this this cluster is

$$y_k = \mathbf{h}_k^1 \mathbf{x}^1 + \sum_{l=2}^N \mathbf{h}_k^l \mathbf{x}^l + n_k, \quad k = 1, \dots, K \quad (3)$$

where $\mathbf{h}_k^1 = [h_{k1}^1, \dots, h_{kM}^1]$ is the channel vector between the antenna array of this cluster and its k th user, $\mathbf{h}_k^l = [h_{k1}^l, \dots, h_{kM}^l]$ is the channel vector from the l th interfering cluster, $\mathbf{x}^l \in \mathbb{C}^{M \times 1}$ is the signal vector transmitted by the l th cluster, $n_k \sim \mathcal{CN}(0, 1)$ is the complex additive white Gaussian noise at the k th user, M is the number of antennas per coordinating cluster and K is the number of users served per cluster. We assume that each cluster perfectly knows the channel state information of only its users. On a given symbol period, each base l serves a subset of users $S^l \subseteq \{1, \dots, K\}$. Under a per-antenna average power constraint, the transmitted signal must satisfy

$$\mathbb{E}[|x_m^l|^2] \leq P_m^l, \quad m = 1, \dots, M \quad (4)$$

where P_m^l is the power constraint for the m th antenna of the l th base station.

III PRECODING WITH FULL CSI AT THE BASE STATION

In this section we consider two precoding schemes that require full channel state information (CSI) at the transmitter side. The focus is on the zero-forcing (ZF) beamformer with a per-antenna power constraint. The well-known dirty paper coding (DPC) scheme with a sum power constraint is also considered as benchmark.

A System architecture

For both ZF and DPC, coordinated transmission occurs over a cluster of M antennas. As shown below in Figure ??, the antennas for a given cluster are either co-located or spatially distributed depending the type of coordination.

- *Centralized coordination.* The cluster consists of a circular array of $M = 12$ co-located antennas. The grey area in Figure ?? indicates that coordination occurs among each set of M antennas. The antennas are located at the "half-hour" points of an analog clock with sufficient element spacing so that the independent Rayleigh fading assumption is justified. We model the element response using (2), with $\theta_{3dB} = \pi/2$ and $A_s = 20$ dB, where each antenna's boresight direction corresponds to its radial direction with respect to the array's center. Users experience interference from other cells; therefore users at the cell edge will experience the most interference. The CSI is shared among M co-located antennas.
- *Distributed coordination.* The $M = 12$ coordinated antennas are spatially distributed in 3 groups of 4 antennas, where each group is associated with a different cell site. In Figure ??, the three groups on the corners of the grey area are coordinated. Each group is a linear array consisting of directional elements pointing towards the center of the grey area. Note that the three groups that share a common base are each associated with a different coordination group. If we were to label each coordination cluster with a different hexagon, these hexagons would tile the cellular network. Note that in discussing simulations for the distributed architecture, the notion of hexagonal cells is replaced by the hexagonal coordination regions.

Each element has parameters with $\theta_{3dB} = \frac{70}{180}\pi$ and $A_s = 20$ dB. The distributed architecture is similar to the conventional 3-sector architecture, and the element parameters as the same for the two cases. Because of non-ideal sidelobes, users near the sector border will experience interference from adjacent sectors. The implementation would be more difficult than for the centralized case since CSI must be shared among spatially distributed antenna groups.

B Transmission schemes

1) ZF

We now discuss the ZF transmission technique for a given cluster of M coordinated antennas. Assuming a linear precoder

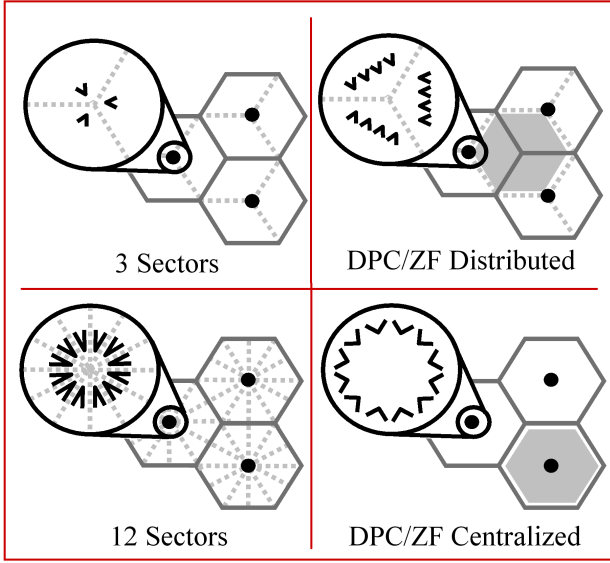


Figure 1: Antenna architectures for sectorized and precoding systems. Shaded regions indicate antennas that are coordinated.

(beamformer) is used, the transmitted signal for the central base station can be written as

$$\mathbf{x} = \mathbf{G}(\mathcal{S})\mathbf{u}, \quad (5)$$

where $\mathbf{G}(\mathcal{S}) \in \mathbb{C}^{M \times |\mathcal{S}|}$ is the beamforming matrix for the users in set \mathcal{S} and $\mathbf{u} = [u_1, \dots, u_{|\mathcal{S}|}]^T$ is the vector of information bearing signals. Note that in defining \mathbf{u} , the user indices have been rearranged so the users chosen for service correspond to the first $|\mathcal{S}|$ indices.

If we use ZF beamforming, the beamforming matrix is chosen such that $\mathbf{H}(\mathcal{S})\mathbf{G}(\mathcal{S}) = \mathbf{I}_{|\mathcal{S}|}$, where $\mathbf{H}(\mathcal{S}) \in \mathbb{C}^{|\mathcal{S}| \times M}$ is obtained by extracting the appropriate rows of $\mathbf{H} = [\mathbf{h}_1^T \dots \mathbf{h}_K^T]^T$, and $\mathbf{I}_{|\mathcal{S}|}$ is the $|\mathcal{S}|$ -by- $|\mathcal{S}|$ identity matrix. Assuming $|\mathcal{S}| \leq M$, the ZF matrix can be chosen to be the pseudo-inverse of $\mathbf{H}(\mathcal{S})$,

$$\mathbf{G}(\mathcal{S}) = \mathbf{H}^H(\mathcal{S})[\mathbf{H}(\mathcal{S})\mathbf{H}^H(\mathcal{S})]^{-1}. \quad (6)$$

Letting α_k be the quality of service (QoS) weight for the k th user, for a given set of users \mathcal{S} ($|\mathcal{S}| \leq M$), the problem of finding the optimum user power allocations which maximize the weighted sum rate under ZF beamforming subject to per-antenna power constraints can be formalized as:

$$R(\mathcal{S}) = \max_{v_k, k=1, \dots, |\mathcal{S}|} \sum_{k=1}^{|\mathcal{S}|} \alpha_k \log \left(1 + \frac{v_k}{\sigma_k^2} \right) \quad (7)$$

$$\text{subject to} \quad \begin{cases} v_k \geq 0, & k = 1, \dots, |\mathcal{S}| \\ \sum_{k=1}^{|\mathcal{S}|} |g_{mk}|^2 v_k \leq P_m, & m = 1, \dots, M \end{cases}$$

where g_{mk} is the (m, k) th element of $\mathbf{G}(\mathcal{S})$, σ_k^2 is the thermal noise plus interference variance and where the power constraint follows from (5) and (4). The QoS weights are generated by a scheduler described in Section V. We emphasize that the term

σ_k^2 accounts for the intercell interference and that this interference is considered Gaussian. We describe how this interference is generated in the following subsection. Problem (7) is a convex problem which in general can be solved by using an interior point method [5]. In general, in considering service to the complete population of K users, we need to consider all possible sets of users with cardinality $|\mathcal{S}| \leq \min(M, K)$. Therefore the overall optimization is a generalization of (7):

$$\max_{\substack{\mathcal{S} \subseteq \{1, \dots, K\} \\ |\mathcal{S}| \leq \min(M, K)}} R(\mathcal{S}) \quad (8)$$

Solving (8) optimally requires a brute force search over all possible sets \mathcal{S} of 1, 2, \dots , M users. The number of sets to consider is $\sum_{j=1}^{\min(M, K)} \frac{K!}{(K-j)!j!}$, so the complexity of this search becomes unacceptably high for large K . We therefore use a suboptimum but efficient greedy algorithm for large K described in [6] (see also [7]) for selecting a subset of users $\mathcal{S} \subseteq \{1, \dots, K\}$ to serve.

2) DPC

As with ZF, we are interested in deriving the expression for the maximum weighted sum rate achievable by K users in the center cell in the presence of intercell interference σ_k^2 received by the k th user. Defining

$$\mathbf{\Sigma} = \text{diag} [1/\sigma_1^2, \dots, 1/\sigma_K^2] \quad (9)$$

and $\mathbf{H}_{eq} = \mathbf{\Sigma}\mathbf{H}$, for each given channel matrix \mathbf{H}_{eq}^{-1} , the maximum throughput is achieved by Dirty-Paper Coding (DPC) [8]:

$$C(\mathbf{H}_{eq}, P) = \max_{\mathbf{v}} \log \det (\mathbf{I} + \mathbf{H}_{eq}^H \text{diag}(\mathbf{v}) \mathbf{H}_{eq}) \quad (10)$$

where maximization is over $\mathbf{v} \in \mathbb{R}_+^K$ such that $\sum_i v_i \leq P$, $P = \sum P_m$ and P_m is the power constraint on the m th antenna. The convex maximization in (10) can be solved by the simple iterative algorithm of [9]. Let $\mathcal{R}^{DPC}(P)$ be the rate region achieved by DPC. To determine the optimum throughput when a scheduler is used, it is necessary to determine the point \mathbf{R}^* on the boundary of the rate region that achieves the maximum weighted rate sum for a given set of QoS weights

$$\mathbf{R}^* = \arg \max_{\mathbf{R}} \boldsymbol{\alpha} \mathbf{R} \quad \text{subject to} \quad \mathbf{R} \in \mathcal{R}^{DPC}(P) \quad (11)$$

where $\boldsymbol{\alpha} = [\alpha_1, \dots, \alpha_K]$ is the vector of QoS weights. In [10] an algorithm is proposed to optimally solve (11) by exploiting the duality with the uplink channel [8].

C Simulation methodology

In this subsection, we describe the simulation methodology for determining the throughput for both ZF and DPC, where the throughput is defined as the average sum rate of the K users.

¹Note that the achievable rate region of the downlink transmission defined by the channel matrix \mathbf{H} and the noise variances $[\sigma_1^2, \dots, \sigma_K^2]$ is the same of the one of the downlink transmission defined by the channel $\mathbf{\Sigma}$ and the noise variances $[1, \dots, 1]$.

We describe a methodology that assumes M -antenna coordination is applied throughout the entire network. However, instead of having to run a multicell simulation where coordination occurs in each hexagonal coordination region, we propose a much simpler procedure that requires coordination to be simulated in only the center coordination region. This procedure occurs in multiple rounds. To simplify the discussion, we use the term "cell" to refer to the hexagonal coordination region.

For the first round, consider a single isolated cell. In other words, we assume there is no intercell interference, and $\sigma_1 = \dots = \sigma_K = 1$. We randomly drop K users with a spatially uniform distribution in this region. For each user, we also generate a shadow fading realization. We fix the user location and shadow fading realizations for F frames. Over these F frames, we generate i.i.d Rayleigh fading realizations for each frame and determine the maximum weighted sum rate according to either (8) or (11). The generation of QoS weights is described in Section V, and the mean rate of each user is calculated after a steady state has been reached after $F/2$ frames. At the end of F frames, we generate a new set of user positions and shadow fadings, and the procedure is repeated for a total of D sets of F frames. The sum average rate metric is simply the sum of the K users' mean rates. For each frame after steady state, we record the transmit covariance; therefore we collect a total of $DF/2$ transmit covariances corresponding to the steady state performance.

On successive rounds, we account for the effects of intercell interference. For a given frame, we account for this interference by assigning to each cell a random member from the set of $DF/2$ transmit covariances from the previous round. In doing so, we treat each interfering cell as if it was an isolated cell serving K users as in the first round. Then for the k th user in the center cell, we can compute the total thermal noise plus interference variance σ_k based on the transmit covariances from the interfering cells and channel realizations for this user. The sum average rate and transmit covariances for the center cell can be computed as before.

This procedure should be repeated for successive rounds until the statistics of the interference are stabilized. However, in our simulations, two rounds are sufficient for stabilization.

IV SECTORIZATION

In this section we consider a sectorized system, where there is single antenna in each sector, and there are either 3 or 12 sectors per cell.

A System architecture

For the case of 3 sectors, the response of each element is given by (2) with $\theta_{3dB} = \frac{70}{180}\pi$ and $A_m = 20$ dB. For the case of 12 sectors, the response is narrower, and the sidelobe power is 6dB lower: $\theta_{3dB} = \frac{17.5}{180}\pi$ and $A_m = 26$ dB.

B Transmission scheme

Transmission for the sectorized systems is assumed to be coordinated in the sense that the scheduled transmission among sectors with co-located antennas is performed jointly. If the num-

ber of users per sector is large, then coordinated transmission would not have much advantage over the conventional transmission technique where each sector operates autonomously. However, if the number of users per sector is smaller, then there are some gains over autonomous transmission.

Each sector is assumed to transmit at full power, and each user measures its received signal power from all sectors. Each user feeds back its vector of received signal powers, so each base can compute the SINRs of all users associated with its sectors. The problem is to determine the optimum allocation of users to sectors in order to maximize the weighted sum rate among users in a cell. We let \mathbf{s} be the $N_s \times 1$ sector allocation vector, where $s(i), i = 1, \dots, N_s$ indicates the user allocated to the i th sector and N_s is the number of sectors, and let $\gamma_{\mathbf{s}}(k)$ the SINR of the k th user under the sector allocation \mathbf{s} . The optimal sector allocation is given by the solution of the following problem

$$\mathbf{s}^* = \arg \max_{\mathbf{s}} \sum_{i=1}^{N_s} \alpha(s(i)) \log_2(1 + \gamma_{\mathbf{s}}(s(i))) \quad (12)$$

In general, solving (12) requires a brute force search over all possible sets of N_s users. The number of set to consider is $\frac{K!}{(K-N_s)!}$, so the complexity of this search becomes unacceptably high for large K . We therefore use a suboptimal greedy algorithm which is analogous to the one presented in Section III.

C Simulation methodology

The central cell throughput is evaluated using a similar methodology as for the ZF and DPC case. However, we need only a single round to evaluate the throughput, because the effect of the multicell interference is accounted for in the SINR expression (12).

V SCHEDULING METHODOLOGY

In this section, we describe how to generate the quality of service (QoS) weights $\alpha_1, \dots, \alpha_K$ used in calculating the weighted sum rate metric used by all transmission techniques. For a given set of QoS weights, recall that DPC numerically determines the optimal subset of users to be served. On the other hand, both ZF and sectorization rely on a greedy algorithm for choosing the subset of users.

For the weight calculation we use the proportional fair scheduling (PFS) algorithm [11]: the users are scheduled by taking into account the ratio between the instantaneous achievable rate and the average rate. Indeed, the QoS weight α_k for the k th user is the reciprocal of the user's average windowed rate (note that the instantaneous achievable rate term in (7), (10) and (12) is given by $\log(\cdot)$). In other words, if we let $\bar{R}_k(f)$ be the average rate of the k th user on frame f , then the average rate on next frame is updated using a sliding window average based on a forgetting factor γ : $\bar{R}_k(f+1) = (1-\gamma)\bar{R}_k(f) + \gamma R_k(f)$, where $R_k(f)$ is the rate received by user k during frame f . This value is zero if the user is not served. The QoS weight is simply $\alpha_k(f) = 1/\bar{R}_k(f)$.

Note that in general, we do not include the frame index f for the QoS weight. The QoS weight for the k th user is initialized using an estimate of its average achievable rate $\bar{R}_k(1) = \left(\log(1 + [d_{\min}/d_0]^{-n} \Gamma)\right)^{-1}$, where d_{\min} is the distance from the user to the closest base antenna.

The PFS algorithm is applicable for scheduling the single best user out of K users during a frame. It ensures fairness in the sense that over the long term, each user gets served a fraction $1/K$ of the frames. It is also optimal in the sense that it maximizes the sum log of user rates. Our scheduler is more general in that multiple users could be served during a frame. It is reasonable to use the average rate reciprocal as the QoS weight since the same fairness mechanisms at work in PFS apply here. Namely, users that have been starved will have higher QoS weights, and users that have been served recently will have lower QoS weights. In Section VI, we show empirically that these mechanisms provide fairness; however, analytic optimality of this scheduler for multiuser service has not been shown.

VI SIMULATION RESULTS

We simulate a cellular system consisting a central cell and two rings of surrounding cells, each with $K = 20$ users. For the DPC and ZF the transmitter perfectly knows the channel state information; for the sectorized system, the transmitter only knows the SINR of the users. We assume a pathloss coefficient $n = 4$, shadowing with standard deviation 8dB, $F = 100$ frames per user drop, $D = 10$ drops, forgetting factor $\gamma = 0.1$.

In Figure 1 we compare DPC, ZF and sectorized system (3 and 12 sectors) in terms of sum-rate vs reference SNR, where the reference SNR is defined as the average SNR in a vertex of the cell. We note that the centralized system architecture per-

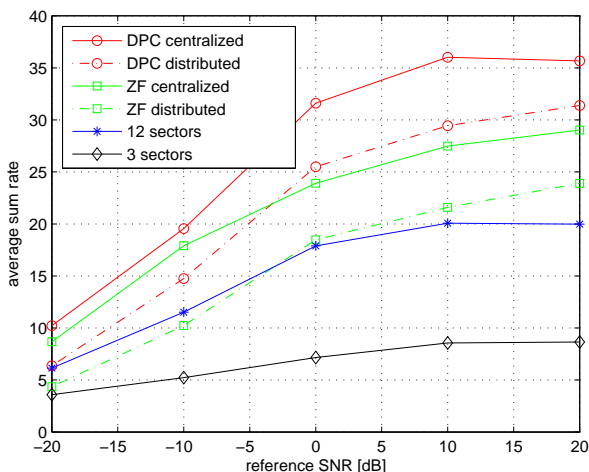


Figure 2: Comparison between DPC, ZF and sectorized system (3 and 12 sectors) in terms of sum-rate vs reference SNR.

forms better than the distributed system architecture for both DPC and ZF. This is due to the fact that the interference between sectors of the co-located linear arrays for the distributed architecture is more detrimental than the intercell interference

for the centralized architecture. When the centralized architecture is used, compared to a baseline sectorized system with 12 (3) sectors, the interference-limited throughput of the ZF and DPC are respectively a factor 1.5 (3.5) and 1.75 (4) higher. Note that as K increases, the performance gain of the 12-sector system versus the 3-sector system approaches 4, as one might expect due to the linear throughput gain of sectorization.

VII CONCLUSIONS

We have studied the potential improvement in downlink throughput of cellular systems using limited network coordination to mitigate intercell interference. We studied ZF and DPC precoding techniques under distributed and centralized architectures. For either precoding technique, the centralized architecture performs uniformly better over the range of powers considered. DPC and ZF provide respective gains of up to 1.75 and 1.5 over the 12-sector baseline and gains of up to 4.5 and 3.5 over the 3-sector baseline. Sectorization using 12 sectors is a cost-efficient technique for improving the performance of a conventional 3-sector system. Additional throughput gains can be achieved under ideal conditions using limited network coordination under the centralized architecture. However, the impact of practical considerations such as imperfect channel knowledge at the transmitter should be studied.

ACKNOWLEDGEMENT

This work has been partly supported by the IST project I027310 MEMBRANE.

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