Imperial College London

Turbulent Entrainment of a Plume in a Stratified Environment

F9

Muhammad Danyal Khan and Dr Maarten van Reeuwijk, Civil and Environmental Engineering, Imperial College London

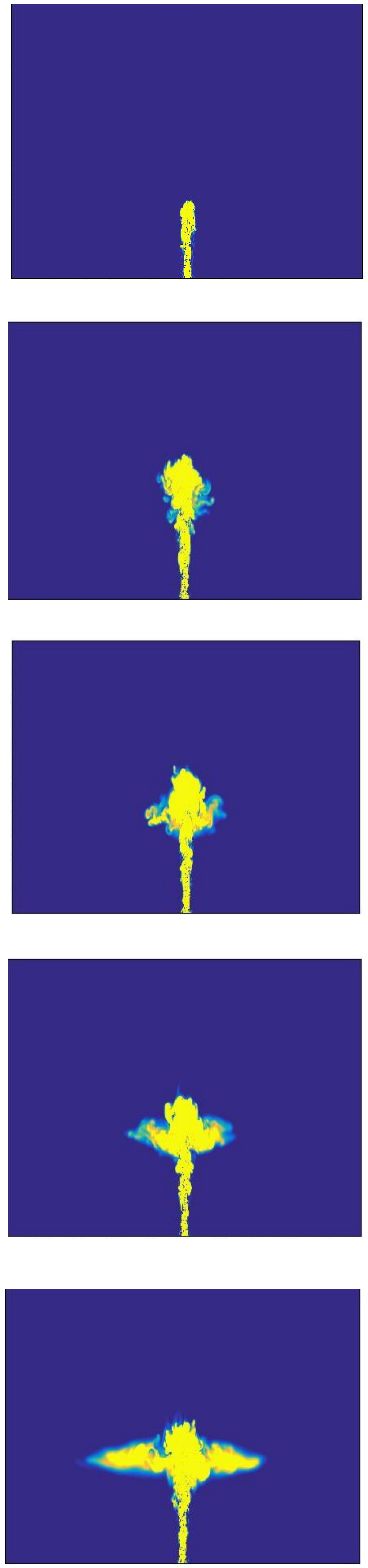

Figure 4: Simulation of a plume in a linearly stratified environment.

Introduction

A turbulent plume is a type of flow generated by the convection of less dense fluid released from a localised source in to a higher density environment. The difference in densities generates buoyancy which drives the flow. A key aspect of plume development is entrainment, entrainment describes the process of engulfment and mixing of fluid from the environment into the plume by large eddies, thus increasing its volume size with respect to height, as is shown in figure 1. Examples of plumes include thermal plumes from indoor heating systems, atmospheric pollution and volcanic eruptions.

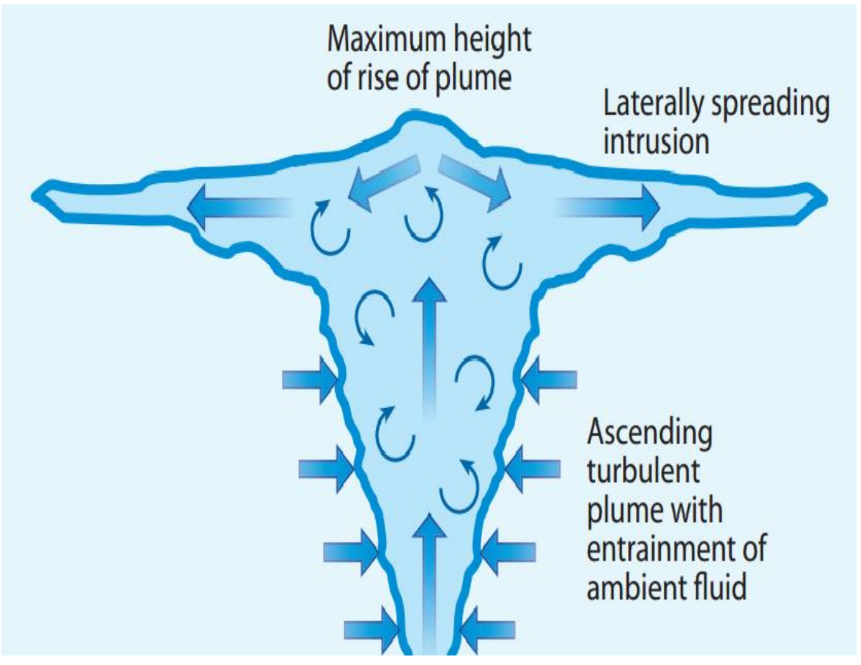

Figure 1: Schematic diagram of a plume (Woods, 2010)

Figure 2: Chile's Calbuco volcanic eruption (Guardian, 2015)

Understanding entrainment is important because it can provide powerful predictive properties of a plume, such as its volume flux. This is useful particularly in light of the plumes produced as a result of the volcanic eruptions in Chile in 2015 and Iceland in 2010, which significantly disrupted the surrounding as well as airborne infrastructure. The physical processes involved in the mechanism of entrainment are looked into by analysing the behaviour of plumes through direction numerical simulations (DNS).

Plume Basics

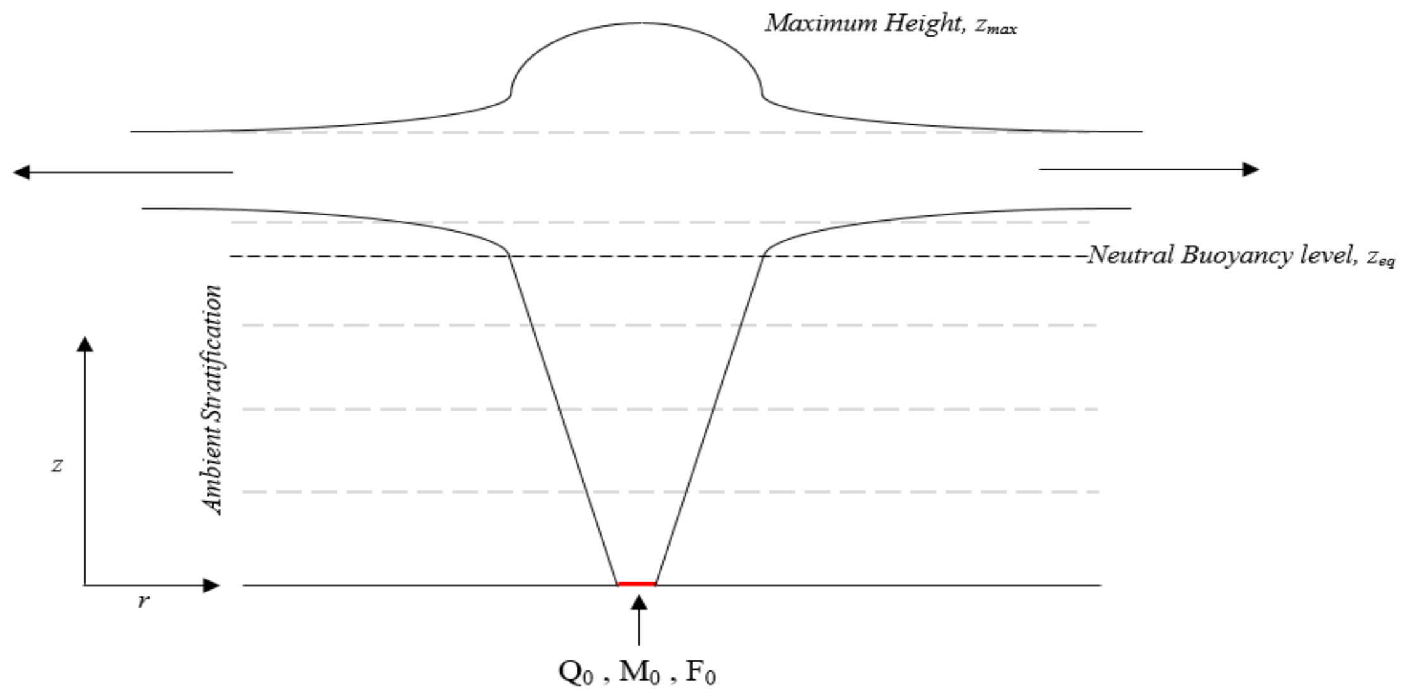

Figure 3

Plume behaviour depends on the type of environment it is in. If the environment is uniform in temperature, the plume rises indefinitely. If the plume has an increasing temperature field with respect to height (i.e. linearly stratified) , the plume rises to a finite height, where its buoyancy is equal to that of the environment, this level is called the neutral buoyancy level. The plume overshoots beyond this point to a maximum height due to its momentum however its behaviour is significantly different. Thus the entrainment analysis has been split into two segments: below and above the neutral buoyancy level.

A plume's volume (Q), momentum (M) and buoyancy fluxes (F) and kinetic energy (M^2/Q)are dictated by the following classical plume theory equations:

$$\frac{dQ}{dz} = 2\alpha M^{1/2}, \qquad \frac{dM}{dz} = \frac{FQ}{M}, \qquad \frac{dF}{dz} = -N^2 Q, \qquad \frac{d}{dz}\left(\frac{4}{3}\frac{M^2}{Q}\right) = \delta_{PB}\frac{M^{5/2}}{Q^2} + 2F$$

Here, N is the buoyancy frequency, which represents the state of the stratification of the environment. The MTT model (Morton et al, 1956) utilises the first three equations, with the assumption that α, the entrainment coefficient, is constant. The PB model (Priestley & Ball, 1955) utilises the latter three sets of equations, assuming that δ_{PB} is constant, this leads to α, under the confines of this particular model, as not being constant. The validity of both models are tested on plumes in a linearly stratified environment using a newly developed relation called the entrainment relation (van Reeuwijk & Craske, 2015). The entrainment relation is an energy consistent relationship between the entrainment coefficient and three physical factors of a plume, namely, the turbulent production, α_{prod}, the net buoyancy effect, α_{buoy} and the vertical velocity radial profile shape, α_{shape}.

Direct Numerical Simulation

The three plumes that were analysed were developed through direct numerical simulations, this involves the numerical integration of the full Navier-Stokes equations up to the smallest scale. Thus no turbulence modelling is required. An example of one of the plume simulations is shown on the left.

Entrainment below the Neutral Buoyancy Level

The volume flux of one of the DNS plumes is displayed in figure 5 along with the predicted fluxes of various other entrainment models. It is clear that the volume flux of the models agree very well below the neutral buoyancy level, however above it there is a greater degree of discrepancy.

The predicted values of the entrainment coefficient, α, using the MTT model (dashed **yellow**), the PB model (dashed **red**) and the entrainment relation (solid **blue**) is also displayed in figure 6 for three simulations. All of which indicate that the PB model is a more suitable fit for the plume.

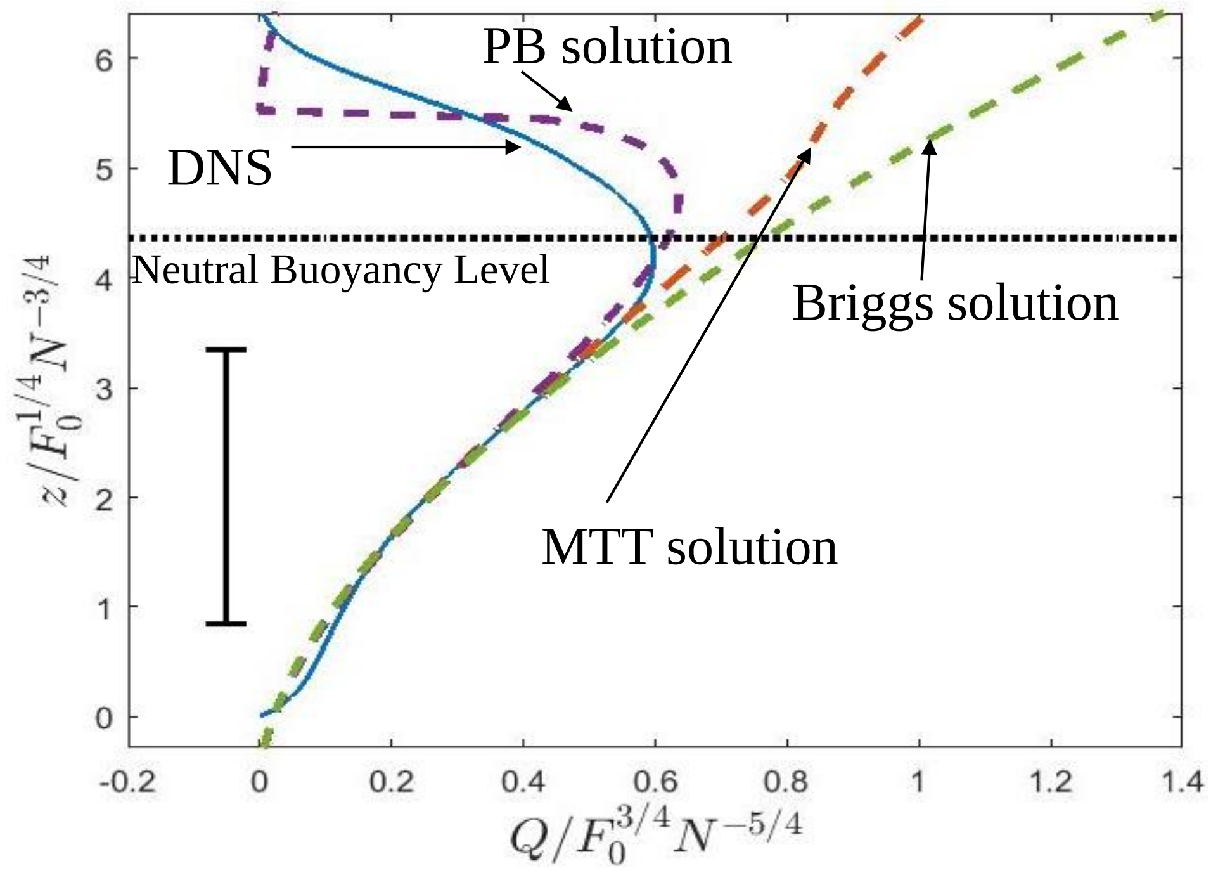

Figure 5: The volume fluxes

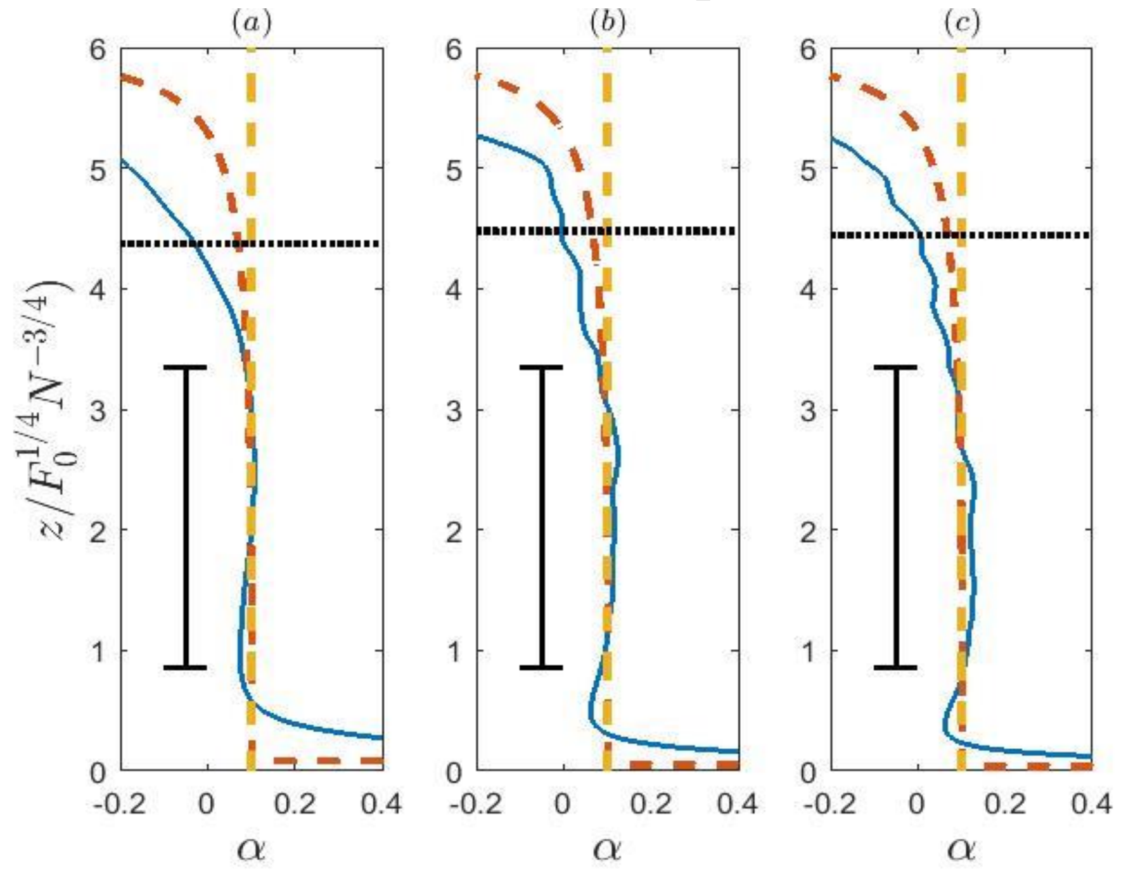

Figure 6: The entrainment coefficients

Entrainment above the Neutral Buoyancy Level

With regards to entrainment above the neutral buoyancy level, it was discovered, through the examination of figure 7, that the entrainment coefficient is proportional to its net buoyancy effect component, $\alpha \propto \alpha_{buoy}$. Figure 7 displays α using the first classical plume equation (solid **blue**), the entrainment relation (solid **red**) and its components α_{prod} (dashed **yellow**), α_{buoy} (dashed **purple**) and α_{shape} (dashed **green**). In conjunction with the PB model, the aforementioned relationship was utilised to predict the fluxes of the plume. The results are in good agreement to the DNS, an example of the volume flux with the DNS is provided in figure 8.

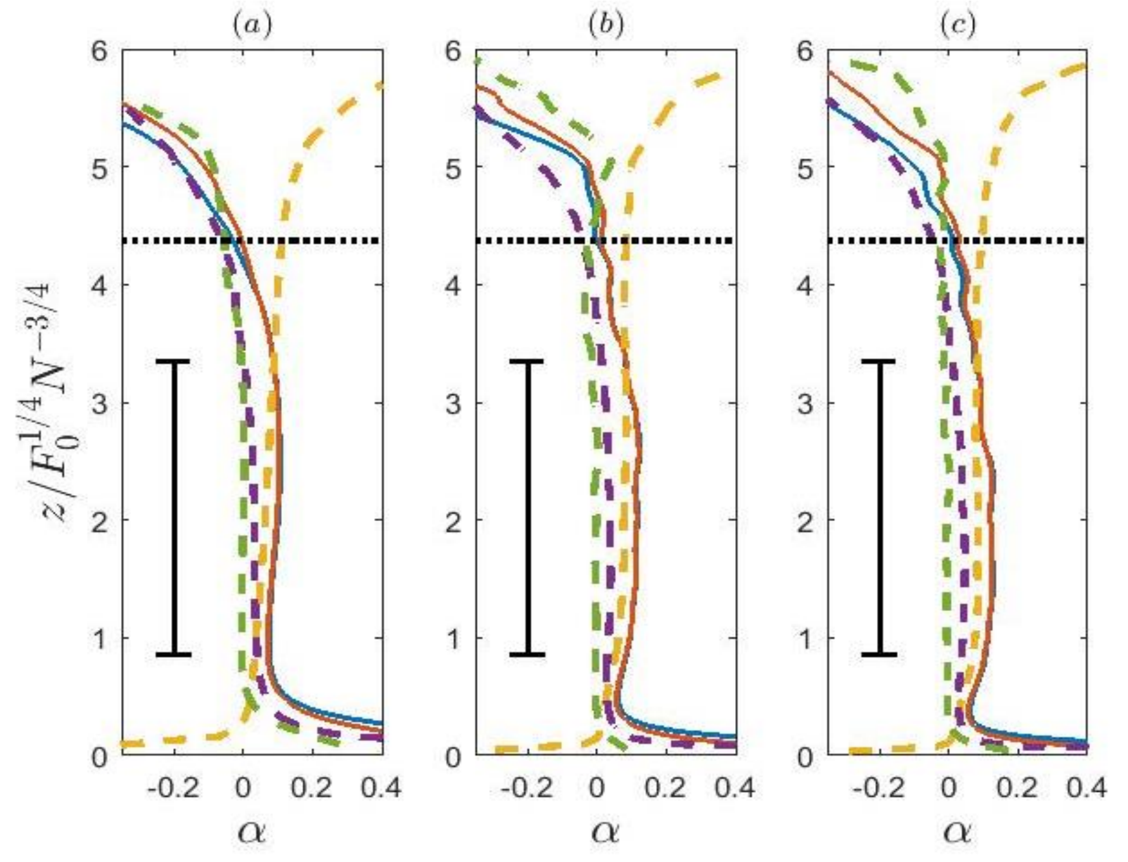

Figure 7: The entrainment coefficients and its components

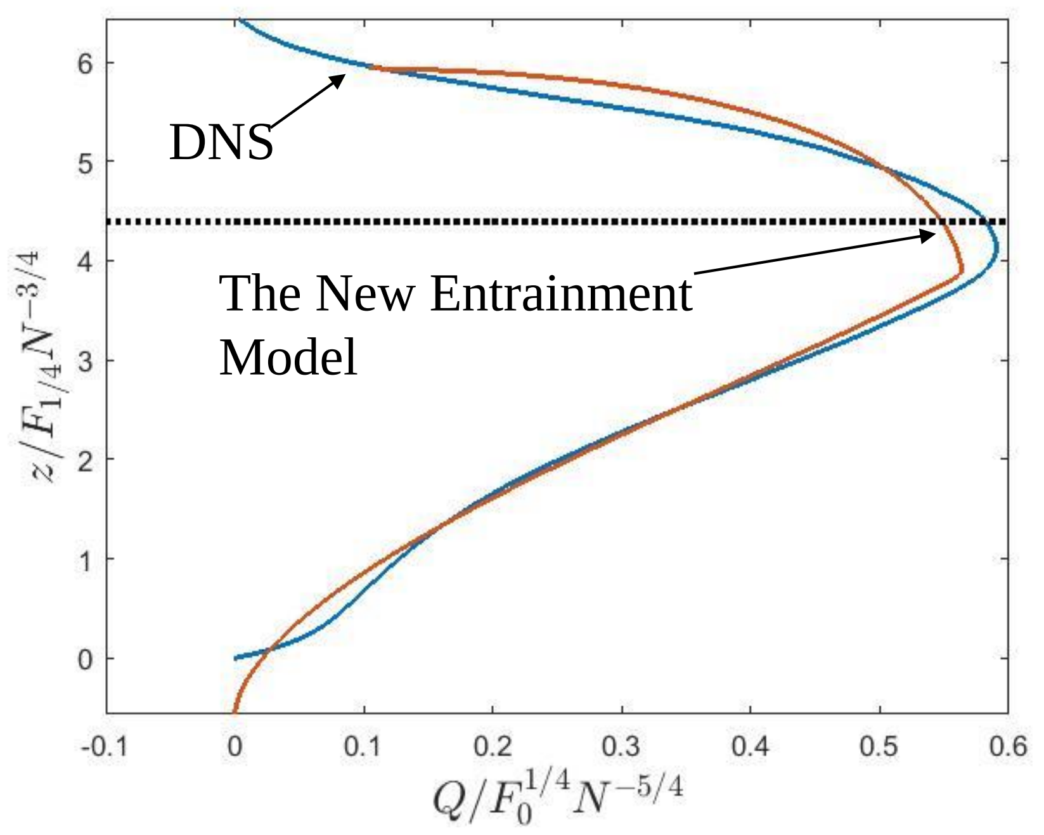

Figure 8: The volume flux based on the new entrainment model

Conclusion

Through the entrainment relation it was found that the PB model is the most suitable in modelling the entrainment of a plume below the neutral buoyancy level. Above the neutral buoyancy level, it was found that $\alpha \propto \alpha_{buoy}$. Thus, along with the PB model, an updated model was produced to generate more accurate predictions of the flux behaviour of a plume.

Acknowledgments

I would firstly like to thank Dr Maarten van Reeuwijk who under his exceptional guidance, I was able to enjoy and complete this project. I would also like to thank John Craske, who helped me understand many of the concepts that have been discussed in this paper.

References

Guardian, The. 2015 (April). *Chile's calbuco volcano erupts-in pictures.* Online

Morton, B. R., Taylor, G., & Turner, J. S. 1956. Turbulent gravitational convection from maintained and instantaneous sources. *Math. and phys. sciences proceedings of the royal society of London, series a, mathematical and physical sciences (1934-1990), Vol. 234(1196) 1-23.*

Priestley, C. G. B. & Ball, F. K. 1955. Continuous convection from an isolated source of heat. *Quart. journal of the royal meteorological society* 81(348).

Van Reeuwijk, M. & Craske, J. 2015. Energy-consistent entrainment relations for jets and plumes. *Under consideration for publication in journal of fluid mechanics*.

Woods, Andrew W. 2010. Turbulent plumes in nature. Annual review of fluid mechanics, 42(1), 391-412.