

# Flexible Magneto-Inductive Resonators and Waveguides

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## Abstract

A flexible metamaterial RF detector for magnetic resonance imaging is described. The circuit consists of a polygonal arrangement of magnetically coupled L-C resonators with rectangular inductors, which supports magneto-inductive waves. The elements are mechanically linked to allow adjacent elements to rotate as the ring is flexed. The pivot is optimised to hold the nearest neighbour coupling coefficient  $\kappa_1$  invariant to small changes in angle so resonances are unaffected. Theory is developed to find the optimum pivot and verified using PCB elements. The method is also applicable to flexible waveguides.

## 1. Introduction

The birdcage coil is a workhorse of magnetic resonance imaging, providing an extended volume with uniform performance in transmission and reception [1]. Its behaviour has been explained using ladder networks [2], and the performance of non-circular coils, which can have improved filling factor in head imaging, has been optimised. In metamaterials, waves propagating in arrays of low frequency resonators by purely magnetic coupling are termed “magneto-inductive” or MI waves [3, 4]. MI waveguides consist of a regular arrangement of L-C resonators, coupled to its nearest neighbour by mutual inductance  $M_1$ . The properties of the wave are determined by the coupling coefficient  $\kappa_1 = 2M_1/L$  (and by smaller non-nearest neighbour terms). When the loops form a polygonal ring, as shown in Fig. 1a, the MI wave will satisfy standard resonance conditions, namely that the round trip phase shift must be an integer multiple of  $2\pi$ . The primary mode can be coupled to the field of a rotating dipole, if the resonant frequency is correct, so a MI ring is analogous to a birdcage [5]. However, the lack of rigid connections in the MI approach should allow construction of a flexible coil that can be adjusted to the subject as shown in Fig. 1b. Here we present a mechanical linkage that can hold  $\kappa_1$  constant, so that the resonant frequencies are maintained as the coil is flexed.

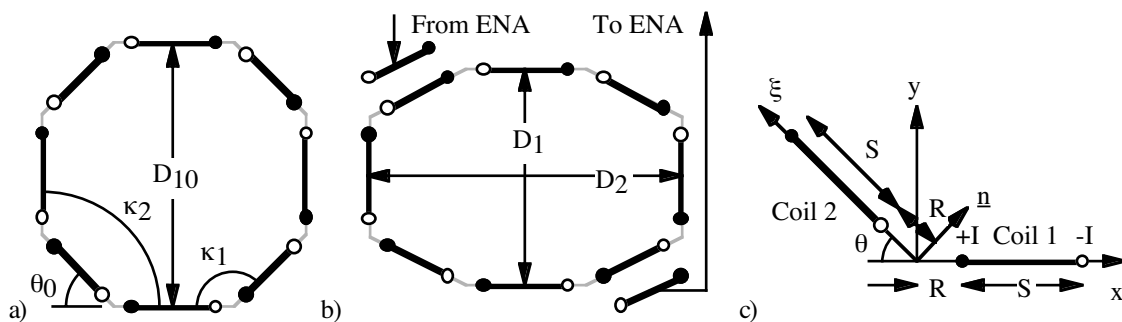


Fig.1 a) Octagonal MI ring with rectangular elements; b) symmetrically distorted ring; c) single vertex.

## 2. Flexible link design

Fig. 1a really requires 3D analysis to account for end effects. Here we use the approximate vertex geometry in Fig. 1c, in which each inductor is modelled by two infinite parallel wires of radius  $A$  and separation  $S$  and adjacent coils subtend an angle  $\theta$  at a centre  $R$  away. By standard methods, we can obtain the per unit length mutual inductance  $M_1$  as:

$$M_1 = -(\mu_0/4\pi)_{R+A} [\log_e \{ [\xi + R \cos(\theta)]^2 + R^2 \sin^2(\theta) \}]^{R+S-A} + (\mu_0/4\pi)_{R+A} [\log_e \{ [\xi + (R+S) \cos(\theta)]^2 + (R+S)^2 \sin^2(\theta) \}]^{R+S-A} \quad (1)$$

The per unit length self-inductance  $L$  may be found by similar methods, as:

$$L = (\mu_0/\pi) \log_e\{(S - A)/A\} \quad (2)$$

The nearest neighbour coupling coefficient  $\kappa_1$  may be estimated using Equations 1 and 2, and the mutual inductance  $M_2$  between second neighbours may also be found in a similar way.

If  $S \gg R$  and  $R > A$  (as is generally the case) the mutual inductance may be approximated as:

$$M_1 \approx = -(\mu_0/2\pi) \log_e\{S/[4R \cos^2(\phi)]\} \quad (3)$$

Here,  $\phi = \theta/2$ . The coupling coefficient is a function of  $R$  and  $\phi$ . However, if  $R$  is altered to compensate for changes in  $\phi$ ,  $\kappa_1$  may be held constant. The required variation is  $R(\phi) = R_0/\cos^2(\phi)$ , where  $R_0 = R(0)$ , and must be obtained automatically using a linking mechanism.

The mechanism must be compact to avoid interference with the subject, and low-cost since an identical unit is needed at each vertex. We have therefore developed a simple pivot that holds  $\kappa_1$  constant over a limited range. Fig. 2a shows the design. The optimised pivot  $P$  is arranged so that its radius arm (of length  $r$ ) makes an angle  $\phi_0$  to the inner conductor of each coil, when the design half-angle at the apex  $Q$  is also  $\phi_0$ . Using small angle approximations, it is simple to show that the desired variation in  $R$  is obtained. The length of radius arm is  $r = R_0/\cos(2\phi_0) = R_0/\cos(\theta_0)$ , and the joint may be formed simply by placing a hinge at a height  $H = R_0 \tan(2\phi_0) = R_0 \tan(\theta_0)$  above the plane of the coils. Fig. 2b shows predictions for the angular variation in coupling coefficient for two cases: i) a sub-optimal pivot at the apex point  $Q$ , and ii) an optimised pivot at  $P$ , assuming  $\theta_0 = 45^\circ$  for an 8-element ring, and the parameters  $A = 0.5$  mm,  $S = 45$  mm,  $R_0 = 2$  mm. When the pivot is at  $Q$ ,  $\kappa_1$  varies continually. However, for a pivot at  $P$ ,  $\kappa_1$  is stationary near  $\theta = 45^\circ$ , albeit at a lower value. The working range is at least  $\pm 10^\circ$ , which should allow considerable distortion of a ring. For these parameters, the nearest neighbour coefficient  $\kappa_2$  is smaller, by a factor of  $\approx 10$ .

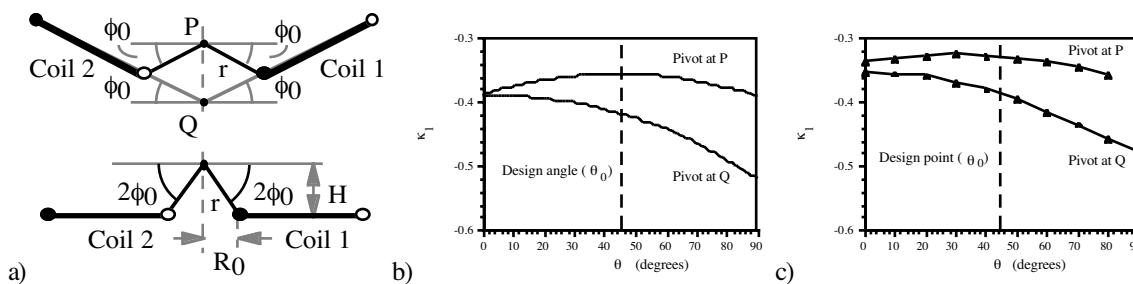


Fig. 2 a) Optimised pivot design; b) example variation of  $\kappa_1$  with  $\theta$  with hinge at i)  $Q$  and ii)  $P$ , for an octagonal ring; c) corresponding experimental result.

### 3. Experimental verification

Experiments were conducted with resonators formed from non-magnetic FR-4 printed circuit board inductors, using a network analyser for characterisation. Single turn rectangular coils with a span  $S = 45$  mm, an aspect ratio of 4 : 1, and a track width and thickness of 1 mm and 85  $\mu\text{m}$ , respectively, gave an inductance of 0.45  $\mu\text{H}$ . With the addition of surface mount capacitors a resonant frequency of  $f_0 \approx 49$  MHz and a Q-factor of 95 were obtained. Mutual inductances and coupling coefficients were determined by the split resonance method. Hinges were constructed as flexure elements, by machining a V-shaped groove into a short section of Delrin. Hinges were attached to the PCBs by plastic bolts, using thin spacers to set the value of  $H$ . Fig. 2c shows the angular variation of  $\kappa_1$  with the spacer thickness optimised for a design angle of  $\theta_0 = 45^\circ$ . The corresponding result obtained with a pivot located at the apex point  $Q$  is also shown. The qualitative agreement with Fig. 2b is excellent, but there are some differences that arise from the shape of the conducting tracks on the PCB, which distort the local magnetic field. The coupling coefficients are generally smaller, and thinner spacers were required than would be expected from the analytic prediction for the height  $H$ .

Octagonal ring resonators with an undistorted diameter  $D_{10} = 115$  mm were constructed from eight identical PCB elements linked by hinges (Figs. 3a and 3b), and their resonances were located using

weak inductive probes using the arrangement shown in Fig. 1b. Five resonances were observed, as shown in the transfer characteristics of Fig. 3c and 3d. As the ring is flexed, the majority of the resonances are entirely unaffected. As the distortion increases from state 1 ( $D_1/D_{10} = 1$ ; undistorted as in Fig 3a) to 4 ( $D_1/D_{10} = 0.74$ ; heavily distorted as in Fig 3b), the important primary resonance gradually separates into two orthogonal modes. However, the splitting is small compared with the corresponding result obtained without the compensating hinges, thus validating the general approach. Further performance improvements would be obtained by increasing the magnitude of the primary coupling coefficient compared with the higher-order coupling coefficients, whose variations are not compensated by the hinge mechanism. For example, the use of PCBs whose conductor separation  $S$  is even greater compared with the apex separation  $R_0$  will increase  $\kappa_1$  without changing  $\kappa_2$ . Similarly, the use of higher-order polygons will reduce  $\kappa_2$  without significantly altering  $\kappa_1$ .

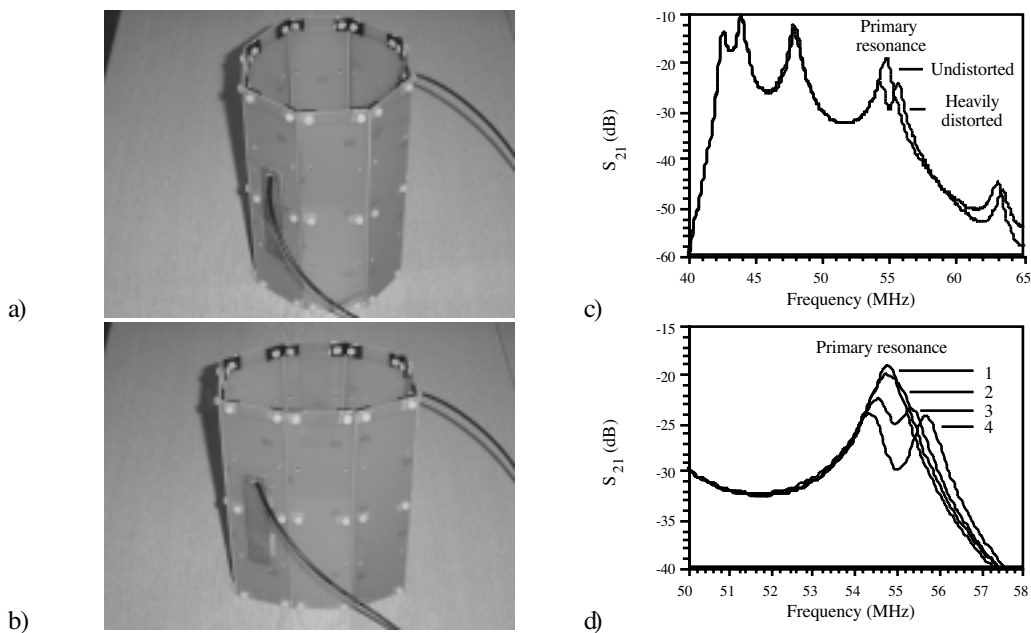


Fig. 3 a) Undistorted and b) heavily distorted octagonal MI ring resonator; c) frequency variation of  $S_{21}$  for different amounts of distortion; d) detail in vicinity of primary resonance.

#### 4. Conclusions

A design has been presented for a mechanical linkage for magneto-inductive waveguides that allows the coupling coefficient to be preserved as the guide is flexed. Preliminary results show performance that may be useful in birdcage type ring resonators for magnetic resonance imaging. However, similar links could be used to prevent reflections in other flexible MI waveguides, for example that route signals round a bend.

#### References

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