DEPARTMENT OF MATHEMATICS

GUIDE TO MODULES

SECOND YEAR (BSc/MSci)
2017-2018

Notes and syllabus details on modules for students in their Second Year

For degree codings:

G100, G103  MATHEMATICS (BSc, MSci)
G104  MATHEMATICS WITH A YEAR IN EUROPE (MSci)
G102  MATHEMATICS WITH MATHEMATICAL COMPUTATION
G125  MATHEMATICS (PURE MATHEMATICS)
G1F3  MATHEMATICS WITH APPLIED MATHEMATICS/MATHEMATICAL PHYSICS
G1G3  MATHEMATICS WITH STATISTICS
G1GH  MATHEMATICS WITH STATISTICS FOR FINANCE
GG31  MATHEMATICS, OPTIMISATION AND STATISTICS
G1EB, G1EM  MATHEMATICS WITH EDUCATION (BSc, MSci)

NOTE that GG14, GG41, IG11 and GI43 MATHEMATICS AND COMPUTER SCIENCE are administered by the Department of Computing.

Prof David Evans
Director of Undergraduate Studies

May 2017

TO BE READ IN CONJUNCTION WITH THE UNDERGRADUATE HANDBOOK.
This information WILL be subject to alteration. Updated programmes can be viewed on the MathsCentral Blackboard site and online at:
https://www.imperial.ac.uk/natural-sciences/departments/mathematics/study/students/undergraduate/programme-information/
<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>SECOND YEAR OVERVIEW</td>
<td>2</td>
</tr>
<tr>
<td>ADVICE ON THE CHOICE OF OPTIONS</td>
<td>3</td>
</tr>
<tr>
<td>SECOND YEAR STRUCTURE</td>
<td>3</td>
</tr>
<tr>
<td>IMPERIAL HORIZONS</td>
<td>4</td>
</tr>
<tr>
<td>EXAMINATIONS</td>
<td>4</td>
</tr>
<tr>
<td>M2R GROUP PROJECT</td>
<td>4</td>
</tr>
<tr>
<td>PROGRESSION TO THE THIRD YEAR</td>
<td>4</td>
</tr>
<tr>
<td>HONOURS MARKS, YEAR TOTALS AND YEAR WEIGHTINGS</td>
<td>5</td>
</tr>
<tr>
<td>ECTS</td>
<td>5</td>
</tr>
<tr>
<td>SECOND YEAR MATHEMATICS SYLLABUSES:</td>
<td>6</td>
</tr>
<tr>
<td>COMPULSORY MODULES</td>
<td>6</td>
</tr>
<tr>
<td>OPTION MODULES</td>
<td>8</td>
</tr>
<tr>
<td>DEGREE COURSE CODING REQUIREMENTS</td>
<td>9</td>
</tr>
<tr>
<td>THIRD AND FOURTH YEAR</td>
<td>9</td>
</tr>
</tbody>
</table>
SECOND YEAR OVERVIEW

The programme of study in the Second Year extends and enhances major themes that feature in the First Year and takes place over three terms – Term 1 (also known as Autumn Term), Term 2 (also known as Spring Term) and Term 3 (also known as Summer Term).

The programme has ten examined mathematics modules that have up to 30 lectures or their equivalent. Seven of the modules are compulsory and students must then choose one module from the remaining three module options. (G1EB students do not have this choice and must instead take the module M2T Communicating Mathematics module which is not available to other students until the Third Year.) There is also the (compulsory) assessed M2R Group Project in Term 3, which involves written, oral and collaborative elements.

For the modules M2AA1, M2AA2, M2AA3, M2PM1, M2PM2, M2PM3, M2S1 the Department normally expects to run supporting problems classes. For other modules this provision is expected to be within the timetabled lectures – the normal expectation is that there should be a ‘lecture’/’class’ balance of about 5/1. The identification of particular class times within the timetabled periods is normally at the discretion of the lecturer, in consultation with the class and as appropriate for the module material.

The choice of 2nd Year option should not crucially restrict the availability of options in later years. It should be regarded as an opportunity to ‘taste’ special interests. This is especially true for the BSc degree codings G102, G125, G1F3, G1G3, G1GH, GG31, but apart from those on the G1EB coding, all students have a free choice. Note that one of the M2 optional modules can also feature as part of the Third Year choice if not taken in the Second Year.

Computation: There is no specifically computational module in the 2nd year.

It is normally required that students pass ALL programme elements (including the M2R Group Project) in order to proceed into the Third Year. Note also that it is normally required that students pass ALL programme elements in order to graduate. However, the College may condone a narrow fail in the Final Year of study in order to permit a student to graduate, but this cannot be guaranteed.

MSci Degrees: The first two years of study on the BSc and MSci degree codings are the same. Those students who were originally registered on the MSci coding and those who have transferred onto it are normally required to maintain a good level of performance in mathematics (at Upper Second Class level or better) in order to remain on the coding in the Third Year. Students on the MSci coding are also required to make satisfactory progress in the Third Year to remain on the Four Year programme.

G103: The primary criterion for eligibility to remain on G103 is to achieve a year total of at least 60 percent (College scale) in Second Year. Students who score 60 percent in Second Year, 58 percent in Third Year and pass all their Third Year modules, have the automatic right to continue on to the Fourth Year of the MSci degree. Anyone scoring less than 58 percent in their Third Year, or who fails a module, does not have this right and may be graduated with a BSc at the Department’s discretion (which is exercised only rarely).

Those who score less than 60 percent in their Second Year may be allowed to remain on G103 at the Senior Tutor's discretion but will have conditions set for their Third Year performance that take precedence over the rule above.

For further information on year totals, please see pages 4-5.

G104: Students registered for G104 Mathematics with a Year in Europe spend their Third Year (of four) studying mathematics modules/project material at another European institution. At the end of Second Year, students must be in a position to take advantage of the Third Year of the course, both mathematically and linguistically. Note: Whilst G104 students must pass the language examinations at the end of First and Second Year in order to stay on G104, language examination results do not directly contribute to mathematics degree Honours marks.
ADVICE ON THE CHOICE OF OPTIONS

Students are advised to read these notes carefully and to discuss their programme with their Personal Tutor.

It is anticipated that lecturers will give advice on suitable books at the start of each module. Students should contact the proposed lecturers if they desire any further details about module content in order to make their choice of course options. Students should also feel free to seek advice from Year Level Tutors and the Senior Tutor, the Undergraduate Liaison Officer and the Director of Undergraduate Studies. Information may also be available online and in the College Library.

Course option choices must be registered on the designated website between the 12th June and the 1st of July, 2017. You will not be committed to taking those modules until the completion of your examination entry at the beginning of Term 2.

SECOND YEAR COURSE STRUCTURE

Term 1: ALL 4 COMPULSORY

<table>
<thead>
<tr>
<th>Module</th>
<th>ECTS Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MULTIVARIABLE CALCULUS M2AA2</td>
<td>7</td>
</tr>
<tr>
<td>REAL ANALYSIS M2PM1</td>
<td>7</td>
</tr>
<tr>
<td>ALGEBRA 2 M2PM2</td>
<td>7</td>
</tr>
<tr>
<td>PROBABILITY AND STATISTICS 2 M2S1</td>
<td>7</td>
</tr>
</tbody>
</table>

Term 2: 3 COMPULSORY, 1 OPTIONAL

<table>
<thead>
<tr>
<th>Module</th>
<th>ECTS Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTRODUCTION TO NUMERICAL ANALYSIS M2AA3</td>
<td>7</td>
</tr>
<tr>
<td>COMPLEX ANALYSIS M2PM3</td>
<td>7</td>
</tr>
<tr>
<td>DIFFERENTIAL EQUATIONS M2AA1</td>
<td>7</td>
</tr>
<tr>
<td>COMMUNICATING MATHEMATICS M2T</td>
<td>7</td>
</tr>
<tr>
<td>OPTIONS (not G1EB)</td>
<td>7</td>
</tr>
<tr>
<td>ONE FROM NON-LINEAR WAVES M2AM</td>
<td>7</td>
</tr>
<tr>
<td>METRIC SPACES AND TOPOLOGY M2PM5</td>
<td>7</td>
</tr>
<tr>
<td>STATISTICAL MODELLING 1 M2S2</td>
<td>7</td>
</tr>
</tbody>
</table>

Term 3: REVISION CLASSES/EXAMINATIONS

<table>
<thead>
<tr>
<th>Module</th>
<th>ECTS Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>GROUP PROJECT M2R</td>
<td>5</td>
</tr>
</tbody>
</table>
**IMPERIAL HORIZONS**

The College has created the ‘Imperial Horizons’ programme to broaden students’ education and enhance their career prospects. This programme is open to all undergraduate students.

The Department of Mathematics always endeavours to avoid timetabling Mathematics modules during the times allocated for Horizons modules.

*Note that modules on this programme do not contribute to degree Honours marks but they do have an ECTS value of 6.*

Further information about the ‘Horizon’ programme can be viewed online at: [http://www3.imperial.ac.uk/horizons](http://www3.imperial.ac.uk/horizons)

**EXAMINATIONS**

In May/June there is one written examination in each of M2AM, M2AA1, M2AA2, M2AA3, M2PM1, M2PM2, M2PM3, M2PM5, M2S1, M2S2. The modules generally have an assessed coursework/progress test element which is limited to ~10% of the overall assessment in each case.

Students who do not obtain Passes in examinations at the first attempt will be expected to attend resit examinations where appropriate. September resits are available to Second Year students followed by resit opportunities the following May/June. Two resit attempts are normally available to students.

*Note: Resits may not be offered for modules assessed solely by project.*

Resit examinations are for Pass credit only – a maximum mark of 40 percent (College scale) will be credited. Once a Pass is achieved, no further attempts are permitted.

Students who have not achieved the required Passes by the beginning of the new academic year are required by College to spend a year out of attendance. During this time they are not considered College students. This may create a number of issues and hold visa implications.

**M2R GROUP PROJECT AND MINI-CONFERENCE**

After the examinations in Term 3 there is the M2R Group Project, which counts in the assessment of Honours and must be passed for progression into the Third Year. The M2R Group Project has oral and collaborative elements and it counts for marks up to the equivalent of half of one of the lectured modules.

Students are asked to select a broad subject area in which they wish to do a project and are allocated an appropriate group and a staff supervisor. A written project must be produced, for which all group members usually get the same mark. All groups then present their work orally at a mini-conference towards the end of term. An individual oral mark is then added to the written mark.

There is normally no resit opportunity within the session for the M2R Group Project

**PROGRESSION TO THE THIRD YEAR**

It is normally required that students pass ALL programme elements (including the M2R Group Project) in order to proceed into the Third Year.
MARKS, YEAR TOTALS AND YEAR WEIGHTINGS

What follows is a brief summary – more details of these topics can be found in the Scheme for the Award of Honours online at: https://www.imperial.ac.uk/natural-sciences/departments/mathematics/study/students/undergraduate/programme-information/

(information for 2017-2018 will be updated over the summer of 2017).
Within the Department each total module assessment is rescaled so that overall performances in different modules may be compared. From 2017-18, all marks will be computed on the College scale, rather than the Mathematics scale used in previous years. The rescaling onto the scale 0 – 100 marks is such that 40 then corresponds to the lowest Pass mark and 70 corresponds to the lowest First Class performance.

Marks from the eight lectured modules and the M2R project taken in the second year are combined (with M2R weighted half that of a lectured module) into a year total expressed as a percentage.

For three year BSc codings, the 1st : 2nd : 3rd year weightings are 1 : 3 : 5.

For the four year MSci codings G103, G104, G1EM, the year weightings are 1 : 3 : 4 : 5.

(For G104 students who first enrol in the Department from 2017-18 onwards, the year weightings will be 1 : 3 : 3 : 5.)

These differences in year weighting reflect the increasing level of mathematical complexity.

ECTS

To comply with the European 'Bologna Process', degree programmes are required to be rated via the European Credit Transfer System (ECTS) – which is based notionally on hour counts for elements in the degree.

Each Second Year mathematics module has an ECTS value of 7 with M2R having an ECTS value of 5. First Year mathematics modules have an ECTS value of 6.5 except for M1R which has an ECTS value of 4.5 and M1C which has an ECTS value of 4. Language modules, taken by G104 Mathematics with a Year in Europe students, have an ECTS value of 6.

ECTS ratings of Department of Mathematics courses and degrees can be viewed online at: http://www.imperial.ac.uk/natural-sciences/departments/mathematics/study/students/undergraduate/programme-information/

| MSci students who wish to increase their ECTS counts from roughly 240 to 270 must undertake additional study over the summer vacations of their Second and Third Years. Contact the Director of Undergraduate Studies for further information. |

SECOND YEAR MATHEMATICS SYLLABUSES

COMPULSORY MODULES

M2AA1  DIFFERENTIAL EQUATIONS
Prof S. van Strien
Term 2

Differential equations play a key role in both pure and applied mathematics and also in many applications in physics, engineering, biology, economics, etc. Usually it is impossible to solve differential equations explicitly. In this course it is shown when solutions exist, and information is obtained about these solutions without necessarily having an explicit expression for these solutions.

Topics:
* Contraction theorem: a general method in analysis for proving that solutions exist
* Existence and uniqueness of solutions of ODEs
* Linear systems
* Power series solutions
* Boundary value problems and Sturm-Liouville problems
* Nonlinear theory
* Dynamical Systems

M2AA2 MULTIVARIABLE CALCULUS

Dr A. Walton
Term 1

The calculus of functions of several independent variables is developed in this module, together with an introduction to partial differential equations. The importance of these ideas is emphasized by the inclusion of a number of applications in Science and Engineering.

• Fourier series: orthonormal systems, periodic functions, even and odd functions, fullrange series, the Gibbs phenomenon, Parseval's theorem, half-range series, integration and differentiation of Fourier series, exponential form.
• Fourier transforms: exponential, cosine and sine transforms, elementary properties, convolution theorem, energy theorem, Dirac delta function.
• Partial differential equations: the wave equation: method of separation of variables, use of Fourier transforms, D’Alembert’s solution; the heat equation: solution using same methods as above; Laplace and Poisson equation: types of boundary conditions, uniqueness of solution, point sources, Green’s functions, method of images.

M2AA3 INTRODUCTION TO NUMERICAL ANALYSIS

Dr R. Nurnberg
Term 2

An important applied analysis module leading also to an introduction to some of the standard algorithms in numerical analysis.

Orthogonality: Inner/outer products on \( \mathbb{R}^n \), review linear dependence/independence, orthogonal/orthonormal vectors, classical Gram-Schmidt, orthogonal matrices, Givens rotations, QR factorization, Cauchy-Schwartz inequality.

Gradients/Hessians: Taylor series for \( f: \mathbb{R}^n \rightarrow \mathbb{R} \), classification of stationary points, positive definite matrices, generalised inner products on \( \mathbb{R}^n \), Cholesky factorization of symmetric positive definite matrices.

Least Squares Problems: Normal equations and QR approach for overdetermined systems \( Ax=b \), inner products on a general vector space, least squares problems, Gram matrices, Hilbert matrix.
Orthogonal Polynomials: Monic orthogonal polynomials, three term recurrence relationship, best approximation by polynomials in inner product induced norms, Chebyshev and Legendre polynomials. Fourier series.

Polynomial Interpolation: Lagrange, Newton and canonical forms of interpolating polynomial, existence and uniqueness, divided differences, error analysis, Runge phenomenon.

Mini-Max Approximation: Best approximation by polynomials in the uniform sense, Chebyshev equioscillation theorem, optimal interpolation points – zeros of Chebyshev polynomial.

Gaussian Quadrature: Optimal sampling points – zeros of corresponding polynomial. Visualisation through Matlab commands and graphics.

**M2PM1  REAL ANALYSIS**

TBA

Term 1

*The aim of this module is to provide a rigorous foundation for the differential and integral calculus.*

Review of differentiation of functions in one variable: Rolle’s theorem, mean value theorem, Taylor’s theorem.

Uniform continuity and convergence: definitions and applications.

Integration in one dimension: the Darboux integral; the fundamental theorem of calculus; interchanging limits; applications.

Differentiation of functions from R^n to R^m: definition, directional derivatives, Schwartz theorem; inverse function theorem.

Multi-variable integration: the Darboux integral on R^n; Green’s theorem in the plane.

**M2PM2  ALGEBRA 2**

Prof M. Liebeck

Term 1

*A continuation from Algebra I in Groups and Linear Algebra.*

Groups: Homomorphisms, isomorphisms, normal subgroups, first isomorphism theorem. Fundamental theorem of abelian groups, groups of small order. Group actions, Burnside’s lemma, applications.

Linear algebra: Direct sums, quotient spaces. Determinants, minimal polynomials, Cayley-Hamilton theorem.

Canonical forms of matrices, Jordan normal form, rational canonical form. Further topics in ring theory.

**M2PM3  COMPLEX ANALYSIS**

Prof A. Laptev

Term 2

*This module concerns continuity, differentiation and integration of complex valued functions, with applications to real integration.*

The complex numbers; algebraic and metric structure. Sequences and series of complex numbers. Power series. Exponential, Trigonometric and Logarithmic functions. Complex functions; continuity, differentiability, behaviour on compact sets.

Functions defined by power series. Line integrals, theorem of the primitive, the ML inequality.

Cauchy’s Theorem for a triangle, a star domain; computation of real integrals. Local theory for analytic functions; uniform convergence of Power Series, Cauchy’s Integral Formula, Taylor’s Theorem, Cauchy Estimates. Liouville’s Theorem.


Classification of Singularities. The Residue Theorem; more real integrals!
M2S1  PROBABILITY AND STATISTICS 2

TBA
Term 1

This module extends the ideas met in M1S and introduces the distributional results needed for the study of statistical inference and applied probability.


OPTIONAL MODULES

M2AM  NON-LINEAR WAVES

Prof D. Papageorgiou
Term 2

This module considers the dynamics of a continuous medium or fluid. One dimensional flows and waves are considered in detail to model gas dynamics and water waves as well as models of traffic flow. Shock formation and propagation in single or two by two systems of conservation laws are developed and solutions constructed for different problems. The course concludes with the theory of water waves including progressing and standing waves.

The continuum hypothesis and fluid particles.
Eulerian and Lagrangian descriptions of 1D fluid motion.
Fluid acceleration, material derivative.
Simple problems in finding position, given velocity and vice versa.
Conservation of mass, equation of continuity.
Pressure and gravitational forces.
Conservation of momentum – Euler’s equation.
Non-linear solutions leading to kinematic wave equation and general implicit solution.
Wave steepening. The Burgers equation.
Method of characteristics.
Shocks and weak solutions: Rankine-Hugoniot conditions, shock evolution equation.
Ideal gas dynamics: linear and non-linear problems leading to same equations as above.
Application to traffic flow (or something similar).
Application to physiological flows, river flows and hydraulic jumps – all leading to similar equations to those already studied.
Shallow water waves, systems of hyperbolic PDEs, Riemann invariants.
Dam break problems. Advancing and receding piston problems.
The equations of water waves.
Gravity-capillary water waves.
Dispersion relations, wave-packets, group velocity.
Standing waves, travelling waves and particle paths.

M2PM5  METRIC SPACES AND TOPOLOGY

Prof T. Coates
Term 2
This module extends various concepts from analysis to more general spaces.

Metric spaces. Convergence and continuity. Examples (Euclidean spaces, function spaces; uniform convergence). The open sets in a metric space; equivalent metrics. Convergence and continuity in terms of open sets: topological spaces. Subspaces. Hausdorff spaces. Sequential compactness; compactness via open covers; compact spaces; determination of compact subspaces of \( \mathbb{R}^n \). Completeness in metric spaces. Relationship between compactness and completeness. Connected and path connected spaces; equivalence of these notions for open sets in \( \mathbb{R}^n \). Winding numbers, definition of fundamental group, its computation for the circle. Example: proof of fundamental theorem of algebra.

M2S2  STATISTICAL MODELLING 1

Dr B. Calderhead
Term 2

Traditional concepts of statistical inference, including maximum likelihood, hypothesis testing and interval estimation are developed and then applied to the linear model that arises in many practical situations.

Maximum likelihood estimation, likelihood ratio tests and their properties, confidence intervals.
Linear models - including non-full rank models: estimation, confidence intervals and hypothesis testing. The analysis of variance.

DEGREE COURSE CODING REQUIREMENTS

All modules within the Department are registered for G100, G103 Mathematics. To qualify for any degree a student must satisfy the overall College requirements.

As well as the regular G100 degree, the department offers several specialist degree codings. To qualify for the BSc codings G102, G125, G1F3, G1G3, G1GH, GG31, a suitable number of modules must eventually be passed from subsets of the general list as follows:

- **G102** 4 from Mathematics with Mathematical Computation
  M3N3, M3N4, M3N7, M3N9, M3N10, M3SC, M3R, M3C, M3A50.

- **G125** 6 from Mathematics (Pure Mathematics)
  M2PM5, M3P5, M3P6, M3P7, M3P8, M3P10, M3P11, M3P12, M3P14,
  M3P15, M3P17, M3P18, M3P19, M3P20, M3P21, M3P22, M3P23,
  M3P60, M3P65, M3PA50, M3R.

- **G1F3** 6 from Mathematics with Applied Mathematics/Mathematical Physics
  M2AM, M3A2, M3A4, M3A6, M3A10, M3PA16, M3F22, M3A28, M3A29,
  M3A49, M3A50, M3A52, M3PA34, M3M6, M3M7, M3M11, M3PA23,
  M3PA24, M3PA48, M3SC, M3R.

- **G1G3** 6 from Mathematics with Statistics
  M2S2, M3S1, M3S2, M3S4, M3S8, M3S9, M3S10, M3S11, M3S12,
  M3S14, M3S16, M3S17, M3R.

- **G1GH** 6 from Mathematics with Statistics for Finance
  M2S2, M3F22, M3S1, M3S2 M3S4, M3S7, M3S8, M3S9, M3S11,
It is generally possible to swap between the above BSc codings, subject to the stated requirements, at a fairly late stage.

As part of the continuing review of the undergraduate programme of study, amendments to this list can be expected, including changes in module numbering. **Not all of the individual modules listed are offered every session. The above are the normal requirements – the Department has the discretion to modify them.**

**THIRD AND FOURTH YEAR**

In the 3\textsuperscript{rd} and 4\textsuperscript{th} years there is a vast selection of optional modules, though the Third and Fourth Year syllabuses substantially overlap. Third and fourth year examinations are normally 2 and 2.5 hours respectively.

For details of the current Third and Fourth Year programmes, see the relevant online documentation on the MathsCentral Blackboard site and online at:
http://www.imperial.ac.uk/natural-sciences/departments/mathematics/study/students/undergraduate/programme-information/

**M4R Project:** In the 4\textsuperscript{th} year of an MSci, a substantial M4R project is required, equivalent to 2 lectured modules.

**G104:** For the Mathematics with a Year in Europe course G104 the Third Year is spent studying at a host institution elsewhere in Europe. On the rare occasion that a G104 student performs very poorly in their year away, they may, at the discretion of the Senior Tutor, be transferred onto a BSc on their return.

**G1EM:** For the Maths with Education MSci, the project is only half the M4R length, and takes place only in the second term. During the first term students take Education modules rather than Mathematics ones.