Notes and syllabus details for modules available to students in their Third Year

For degree codings:

- G100, G103  MATHEMATICS (BSc, MSci)
- G104  MATHEMATICS WITH A YEAR IN EUROPE (MSci)
- G102  MATHEMATICS WITH MATHEMATICAL COMPUTATION
- G125  MATHEMATICS (PURE MATHEMATICS)
- G1F3  MATHEMATICS WITH APPLIED MATHEMATICS/MATHEMATICAL PHYSICS
- G1G3  MATHEMATICS WITH STATISTICS
- G1GH  MATHEMATICS WITH STATISTICS FOR FINANCE
- GG31  MATHEMATICS, OPTIMISATION AND STATISTICS
- G1EB, G1EM  MATHEMATICS WITH EDUCATION (BSc, MSci)

NOTE that GG14, GG41, IG11 and GI43 MATHEMATICS AND COMPUTER SCIENCE are administered by the Department of Computing.

Professor David Evans
Director of Undergraduate Studies

May 2016.

TO BE READ IN CONJUNCTION WITH THE UNDERGRADUATE HANDBOOK.

This information WILL be subject to alteration. Updated programmes can be viewed online at: https://www.imperial.ac.uk/natural-sciences/departments/mathematics/study/students/undergraduate/programme-information/
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THIRD YEAR OVERVIEW

The Third Year programme takes place over three terms – Term 1 (also known as Autumn Term), Term 2 (also known as Spring Term) and Term 3 (also known as Summer Term).

After the first two years, which consist predominantly of compulsory ‘core’ mathematics, the Third Year has been designed to permit much student choice.

Students must take eight modules from a wide variety of selections from within the Department and from certain modules elsewhere. The modules specifically approved are listed below, but students may apply to the DUGS for permission to take any module offered in other departments, e.g. Physics or Computing. Each Mathematics module has up to 30 lectures or their equivalent. M2 option modules not taken in the Second Year are normally also available to Third Year students but only one of these may be taken in the Third Year and it counts for fewer ECTS (7 rather than 8).

Lecturing will take place during Term 1 and Term 2 with three hours per week, which usually includes some problems classes. The normal expectation is that there should be a 'lecture'class' balance of about 5/1. The identification of particular class times within the timetabled periods is at the discretion of the lecturer, in consultation with the class and as appropriate for the module material.

Some BSc students will prefer to remain broad in their interests in their Final Year of study, while others will specialise, either from personal preference, or in order to satisfy the requirements of their particular degree coding. Students registered for the MSci coding G103 Mathematics are advised not to specialise too narrowly at the Third Year stage and should retain some flexibility in their planning for the Fourth Year.

G103: The primary criterion for eligibility to remain on G103 is to achieve a year total of at least 600 in Second Year. Students who score 600+ in Second Year, 580+ in Third Year and pass all their Third Year modules, have the automatic right to continue on to the Fourth Year of the MSc degree. Anyone scoring less than 580 in their Third Year, or who fails a module, does not have this right and may be graduated with a BSc at the Department’s discretion (which is exercised only rarely).

Those who scored less than 600 in their Second Year may be allowed to remain on G103 at the Senior Tutor’s discretion but will have conditions set for their Third Year performance that take precedence over the rule above.

For further information on year totals, please see page 4.

G104: Students registered for G104 Mathematics with a Year in Europe spend their Third Year (of four) studying mathematics courses/project material at another European institution. On the rare occasion that a G104 student performs very poorly in their year away they may, at the discretion of the Senior Tutor, be transferred to the BSc G100 Mathematics degree or the BSc G101 Mathematics with a Year in Europe take M3 subjects in their Final Year. When this occurs, the weighting for each year is 1: 3 : 2 : 5.

G1EB: Students on the Mathematics with Education BSc must take Education options in the 1st term, and should select four 2nd term Mathematics options. Students on the MSci version G1EM must take the option M3T.

ADVICE ON THE CHOICE OF OPTIONS

Students are advised to read these notes carefully and to discuss their option selections with their Personal Tutor. An 'Option Fair’ will take place after exams in the Summer Term, where staff will answer questions on all available options.

It is anticipated that lecturers will give advice on suitable books at the start of each module. Students should contact the proposed lecturers if they desire any further details about module content in order to make their choice of course options. Students should also feel free to seek advice from Year Level Tutors and the Senior Tutor, the Heads of Section and the Director of Undergraduate Studies.
Course option choices should be registered on the designated website between the 8th June and the 1st of July, 2016. You will not be committed to taking those modules until the completion of your examination entry at the beginning of Term 2.

**NON-MATHEMATICS MODULES**

Third Year BSc students are permitted to take up to two Centre for Co-Curricular Studies/Business School options in their Third Year from the approved list below. MSci students may take one such option, at most, in each of their Third and Fourth Years.

In addition, the department offers a few options, which are deemed to be ‘non-Mathematical’.

These may be taken as an alternative to a Centre for Co-Curricular Studies/Business School option. However, as they are Department of Mathematics modules, their ECTS value is 8 (rather than 6).

For 2016-17, these options are:

M3E Econometric Theory and Methods  
M3C High Performance Computing  
M3H History of Mathematics  
M3B Mathematics of Business  
M3T Communicating Mathematics

BSc students are permitted to take no more than two of these ‘non-Mathematical’ Modules options if one or both of the options is an ‘External’ module from the Centre for Co-Curricular Studies, from the Business School list of Options or (at the discretion of the Director of Undergraduate Studies) from another department. BSc students may take up to three ‘Less Mathematical’ Modules offered by the Department of Mathematics (M3E, M3C, M3H, M3B, M3T). Students on degree codes other than G100 should note the special requirements of these programmes and consult with the Director of Undergraduate Studies if they wish to take three of M3E, M3C, M3H, M3B, M3T.

Subject to the Department’s approval, students may take a mathematical module given outside the Department, e.g. in the Department of Physics. Students must obtain permission from the Director of Undergraduate Studies if they wish to consider such an option.

**MODULE ASSESSMENT AND EXAMINATIONS**

Each of the following modules is examined by one written examination (usually 2 hours):

M2AM, M2PM5, M2S2, M3A2, M3A4, M3A6, M3A7, M3A10, M3PA16, M3A21, M3F22, M3A25, M3A28, M3PA34, M3B, M3E, M3H, M3M3, M3M6, M3M7, M3M9, M3P5, M3P6, M3P7, M3P8, M3P10, M3P11, M3P12, M3P14, M3P15, M3P16, M3P17, M3P18, M3P19, M3P20, M3P21, M3P22, M3P23, M3P60, M3PA23, M3PA24, M3PA60, M3PA46, M3PA50, M3S1, M3S2, M3S4, M3S8, M3S9, M3S11, M3S14, M3S15, M3S16, M3S17.

Some of the modules may have an assessed coursework/progress test element, limited in most cases to 10% of overall module assessment. This will be made clear at the commencement of each module, particularly for any exceptions that have a more substantial assignment element (e.g. M3S2, M3S9, M3S16, M3S17 – each approximately 25%).

The modules M3A29, M3N7, M3N9, M3N10, M3S7, M3C and M3SC are examined solely by projects.

The module M3R is examined by a research project; an oral element forms part of the assessment.

The module M3T is examined by a journal of teaching activity, teacher’s assessment, oral presentation, and end of module report.

*Note: Students who take modules which are wholly assessed by project will be deemed to be officially registered on the module through the submission of a specified number of pieces of assessed work for that module. Thus, once a certain point is reached in these modules, a student will be committed to*
Completing it. In contrast, students only become committed to modules with summer examinations when they enter for the examinations in February.

Students who do not obtain Passes in examinations at the first attempt will be expected to attend resit examinations where appropriate. Third Year students have resit opportunities the following May/June (NOT normally in September). Two resit attempts are normally available to students; however, MSci students who fail a module in their Third Year only have one resit opportunity to be able to progress to the Fourth Year.

Note: Resits may not be offered for modules assessed solely by project.

Resit examinations are for Pass credit only – a maximum mark of 30 will be credited. Once a Pass is achieved, no further attempts are permitted.

Students who have not achieved the required Passes by the beginning of the new academic year are required by College to spend a year out of attendance. During this time they are not considered College students. This may create a number of issues and hold visa implications.

PROGRESSION TO THE FOURTH YEAR AND GRADUATION

It is normally required that MSci students pass all course components in order to proceed into the Fourth Year.

It is normally required that BSc students pass all course components in order to graduate. However, the College may condone a narrowly failed module in the Final Year of study. The Examination Board may also graduate students under exceptional circumstances who have one or more badly failed module, provided the overall average mark is high enough.

The total of Honours marks for examinations, assessed coursework, progress tests, assignments and projects, with the appropriate year weightings, is calculated and recommendations are made to the Examiners’ Meeting (normally held at the end of June) for consideration by the Academic Staff and External Examiners. Degrees are formally decided at this meeting.

Students at graduation may be awarded Honours degrees classified as follows: First, Second (upper and lower divisions) and Third, with a good Final Year being viewed favourably by the External Examiners for borderline cases.

Rarely, circumstances may require the Department to graduate an MSci student with a BSc.

Further information on degree classes can be found online in the Scheme for the Award of Honours at: https://www.imperial.ac.uk/natural-sciences/departments/mathematics/study/students/undergraduate/programme-information/

In general, applications for postponement of consideration for Honours will NOT be granted by the Department except in special cases, such as absence through illness.

Information about Commemoration (Graduation) ceremonies can be found online at: http://www3.imperial.ac.uk/graduation

HONOURS MARKS, YEAR TOTALS AND YEAR WEIGHTINGS

What follows is a brief summary – more details of these topics can be found online at: https://www.imperial.ac.uk/natural-sciences/departments/mathematics/study/students/undergraduate/programme-information/
Within the Department each total module assessment is rescaled so that overall performances in different modules may be compared. The rescaling onto the scale 0 – 100 Honours marks is such that 30 then corresponds to the lowest Pass Honours mark and 75 corresponds to a clear First Class performance.

Note that Registry report individual module performances on transcripts using the College Scale. On this scale a Pass mark is reported as 40 and the lowest First Class mark is 70.

Information on the Mathematics and College scales can be found in the Scheme for the Award of Honours.

The total Third year Honours mark available is 800, made up as \(8 \times 100\) lecture modules.

**Note:** For uniformity, the total Honours marks for each year are scaled out of 1000 and are known as a year total.

For three year BSc codings, the 1\(^{st}\) : 2\(^{nd}\) : 3\(^{rd}\) year weightings are 1 : 3 : 5.

For the four year MSci codings G103, G104, G1EM, the year weightings are 1 : 3 : 4 : 5.

The differences in year weighting reflect the increasing level of mathematical complexity.

**ECTS**

To comply with the European ‘Bologna Process’, degree programmes are required to be rated via the ECTS (European Credit Transfer System) – which is based notionally on hour counts for elements within the degree.

Each Third Year mathematics module has an ECTS value of 8. Centre for Co-Curricular Studies/Business School modules have an ECTS value of 6. Each Second Year mathematics module has an ECTS value of 7 with M2R having an ECTS value of 5. First Year mathematics modules have an ECTS value of 6.5 except for M1R which has an ECTS value of 4.5 and M1C which has an ECTS value of 4.

<table>
<thead>
<tr>
<th>MSci students who wish to increase their ECTS counts from roughly 240 to 270 must undertake additional study over the summer vacations of their Second and Third Years. Contact the Director of Undergraduate Studies for further information.</th>
</tr>
</thead>
</table>

Details can be viewed online at: [https://www.imperial.ac.uk/natural-sciences/departments/mathematics/study/students/undergraduate/programme-information/](https://www.imperial.ac.uk/natural-sciences/departments/mathematics/study/students/undergraduate/programme-information/)

**THIRD YEAR MODULE LIST**

Note that not all of the individual modules listed below are offered every session and the Department reserves the right to cancel a particular module if, for example, the number of students wishing to attend that module does not make it viable. Similarly, some modules are occasionally run as 'Reading Modules'.

M2 optional modules not taken in the Second Year are normally also available to Third Year students but only one of these may be taken in the Third Year and it counts for fewer ECTS (7 rather than 8).

Modules marked * are also available in M4 form for Fourth Year MSci students (which typically involves taking a longer Examination). When a module is offered it is usually, but not always, available in both forms. **No student may take both the M3 and M4 forms of a module.**

M3E, M3B, M3H and M3C are also available to Fourth Year students but function like a Centre for Co-Curricular Studies/Business School option, except that their ECTS value is 8. M3T may only be taken in year 4 by returning G104 students.

The groupings of modules below have been organised to indicate some natural affinities and connections.
<table>
<thead>
<tr>
<th>Module Codes</th>
<th>Module Titles</th>
<th>Terms</th>
<th>Honours Marks</th>
<th>ECTS Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>M2AM</td>
<td>Non-linear Waves</td>
<td>2</td>
<td>100</td>
<td>7</td>
</tr>
<tr>
<td>M3A2*</td>
<td>Fluid Dynamics 1</td>
<td>1</td>
<td>100</td>
<td>8</td>
</tr>
<tr>
<td>M3A10*</td>
<td>Fluid Dynamics 2</td>
<td>2</td>
<td>100</td>
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</tr>
<tr>
<td>M3A28*</td>
<td>Introduction to Geophysical Fluid Dynamics</td>
<td>2</td>
<td>100</td>
<td>8</td>
</tr>
<tr>
<td>M3M7*</td>
<td>Asymptotic Analysis</td>
<td>1</td>
<td>100</td>
<td>8</td>
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<tr>
<td>M3A10*</td>
<td>Fluid Dynamics 2</td>
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<tr>
<td>M3A28*</td>
<td>Introduction to Geophysical Fluid Dynamics</td>
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<tr>
<td>M3M7*</td>
<td>Asymptotic Analysis</td>
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<td>M3PA48*</td>
<td>Dynamics of Games</td>
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<tr>
<td>M3PA23*</td>
<td>Dynamical Systems</td>
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<tr>
<td>M3PA24*</td>
<td>Bifurcation Theory</td>
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<td>M3PA16*</td>
<td>Geometric Mechanics</td>
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<tr>
<td>M3PA34*</td>
<td>Dynamics, Symmetry and Integrability</td>
<td>2</td>
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<tr>
<td>M3PA50*</td>
<td>Introduction to Riemann Surfaces and Conformal Dynamics</td>
<td>2</td>
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<tr>
<td>M3F22*</td>
<td>Mathematical Finance: An Introduction to Option Pricing</td>
<td>1</td>
<td>100</td>
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<td>M3A49*</td>
<td>Mathematical Biology</td>
<td>1</td>
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<tr>
<td>M3A4*</td>
<td>Mathematical Physics 1: Quantum Mechanics</td>
<td>2</td>
<td>100</td>
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<tr>
<td>M3A6*</td>
<td>Special Relativity and Electromagnetism</td>
<td>1</td>
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<tr>
<td>M3A7*</td>
<td>Tensor Calculus and General Relativity</td>
<td>2</td>
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<tr>
<td>M3A29*</td>
<td>Theory of Complex Systems</td>
<td>2</td>
<td>100</td>
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<tr>
<td>M3M3*</td>
<td>Introduction to Partial Differential Equations</td>
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<tr>
<td>M3M6*</td>
<td>Methods of Mathematical Physics</td>
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<tr>
<td>M3M9*</td>
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<td>1</td>
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<tr>
<td>M3N7*</td>
<td>Numerical Solution of Ordinary Differential Equations</td>
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<td>100</td>
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<tr>
<td>M3N9*</td>
<td>Computational Linear Algebra</td>
<td>1</td>
<td>100</td>
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<tr>
<td>M3N10*</td>
<td>Computational Partial Differential Equations 1</td>
<td>2</td>
<td>100</td>
<td>8</td>
</tr>
<tr>
<td>M3SC*</td>
<td>Scientific Computation</td>
<td>2</td>
<td>100</td>
<td>8</td>
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<tr>
<td>M2PM5</td>
<td>Metric Spaces and Topology</td>
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<td>7</td>
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<tr>
<td>M3P6*</td>
<td>Probability</td>
<td>2</td>
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<td>M3P7*</td>
<td>Functional Analysis</td>
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<td>Fourier Analysis and Theory of Distributions</td>
<td>2</td>
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<td>M3P19*</td>
<td>Measure and Integration</td>
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<tr>
<td>M3P60*</td>
<td>Geometric Complex Analysis</td>
<td>1</td>
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<tr>
<td>M3P5*</td>
<td>Geometry of Curves and Surfaces</td>
<td>1</td>
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<tr>
<td>M3P20*</td>
<td>Geometry 1: Algebraic Curves</td>
<td>1</td>
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</table>

**FLUIDS**

**DYNAMICS**

**FINANCE**

**BIOLOGY**

**MATHEMATICAL PHYSICS**

**NUMERICAL ANALYSIS/COMPUTATION**

**PURE MATHEMATICS**

**ANALYSIS**

**GEOMETRY**
**ALGEBRA AND DISCRETE MATHEMATICS**

<table>
<thead>
<tr>
<th>Module Codes</th>
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<th>Terms</th>
<th>ECTS Values</th>
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<td>M3P21*</td>
<td>Geometry 2: Algebraic Topology</td>
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<td>M3P10*</td>
<td>Group Theory</td>
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<td>M3P12*</td>
<td>Galois Theory</td>
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<td>M3P13*</td>
<td>Group Representation Theory</td>
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<td>M3P14*</td>
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**NUMBER THEORY**

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<td>M3P14*</td>
<td>Number Theory</td>
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<td>M3P15*</td>
<td>Algebraic Number Theory</td>
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**STATISTICS**

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<tr>
<td>M2S2</td>
<td>Statistical Modelling 1</td>
<td>2</td>
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<td>M3S1*</td>
<td>Statistical Theory</td>
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<td>M3S2*</td>
<td>Statistical Modelling 2</td>
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<td>M3S4*</td>
<td>Applied Probability</td>
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<td>M3S8*</td>
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<td>M3S11*</td>
<td>Games, Risks and Decisions</td>
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<td>M3S14*</td>
<td>Survival Models and Actuarial Applications</td>
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<td>M3S16*</td>
<td>Credit Scoring</td>
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<tr>
<td>M3S17*</td>
<td>Quantitative Methods in Retail Finance</td>
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**PROJECT (Available Only to Final Year BSc Students)**

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<tr>
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<th>Terms</th>
<th>Honours Marks</th>
<th>ECTS Values</th>
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<tr>
<td>M3R</td>
<td>Research Project in Mathematics</td>
<td>2 + 3</td>
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**OTHER MATHEMATICAL OPTIONS**

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<th>Module Titles</th>
<th>Terms</th>
<th>Honours Marks</th>
<th>ECTS Values</th>
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<td>M3E</td>
<td>Econometric Theory and Methods</td>
<td>1</td>
<td>100</td>
<td>8</td>
</tr>
<tr>
<td>M3T</td>
<td>Communicating Mathematics</td>
<td>2 + 3</td>
<td>100</td>
<td>8</td>
</tr>
<tr>
<td>M3H</td>
<td>History of Mathematics</td>
<td>2</td>
<td>100</td>
<td>8</td>
</tr>
<tr>
<td>M3B</td>
<td>Mathematics of Business &amp; Economics</td>
<td>2</td>
<td>100</td>
<td>8</td>
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<tr>
<td>M3C</td>
<td>High Performance Computing</td>
<td>1</td>
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**THIRD YEAR MATHEMATICS SYLLABUSES**

Most modules running in 2016-2017 will also be available in 2017-2018, although there can be no absolute guarantees.

**APPLIED MATHEMATICS/MATHEMATICAL PHYSICS/NUMERICAL ANALYSIS**

**M2AM    NON-LINEAR WAVES**

Professor D. Papageorgiou

Term 2
This module considers the dynamics of a continuous medium or fluid. One dimensional flows and waves are considered in detail to model gas dynamics and water waves as well as models of traffic flow. Shock formation and propagation in single or two by two systems of conservation laws are developed and solutions constructed for different problems. The course concludes with the theory of water waves including progressing and standing waves.

The continuum hypothesis and fluid particles.
Eulerian and Lagrangian descriptions of 1D fluid motion.
Fluid acceleration, material derivative.
Simple problems in finding position, given velocity and vice versa.
Conservation of mass, equation of continuity.
Pressure and gravitational forces.
Conservation of momentum – Euler’s equation.
Non-linear solutions leading to kinematic wave equation and general implicit solution.
Wave steepening. The Burgers equation.
Method of characteristics.
Shocks and weak solutions: Rankine-Hugoniot conditions, shock evolution equation.
Ideal gas dynamics: linear and non-linear problems leading to same equations as above.
Application to traffic flow (or something similar).
Application to physiological flows, river flows and hydraulic jumps – all leading to similar equations to those already studied.
Shallow water waves, systems of hyperbolic PDEs, Riemann invariants.
Dam break problems. Advancing and receding piston problems.
The equations of water waves.
Gravity-capillary water waves.
Dispersion relations, wave-packets, group velocity.
Standing waves, travelling waves and particle paths.

**FLUIDS**

**M3A2** **FLUID DYNAMICS 1**

Professor A. Ruban

**Term 1**

This module is an introduction to the Fluid Dynamics. It will be followed by Fluid Dynamics 2 in Term 2.

Fluid Dynamics deals with the motion of liquids and gases. Being a subdivision of Continuum Mechanics the fluid dynamics does not deal with individual molecules. Instead an ‘averaged’ motion of the medium is of interest. Fluid dynamics is aimed at predicting the velocity, pressure and temperature fields in flows past rigid bodies. A theoretician achieves this goal by solving the governing Navier-Stokes equations. In this module a derivation of the Navier-Stokes equations will be presented, followed by description of various techniques to simplify and solve the equation with the purpose of describing the motion of fluids at different conditions.

**Aims of this module:**
To introduce students to fundamental concepts and notions used in fluid dynamics. To demonstrate how the governing equations of fluid motion are deduced, paying attention to the restriction on their applicability to real flows. Then a class of exact solutions to the Navier-Stokes equations will be presented. This will follow by a discussion of possible simplifications of the Navier-Stokes equations. The main attention will be a wide class of flows that may be treated as Inviscid. To this category belong, for example, aerodynamic flows. Students will be introduced to theoretical methods to calculate inviscid flows past aerofoils and other aerodynamic bodies. They will be shown how the lift force produced by an aircraft wing may be calculated.

**Content:**
M3A10*  FLUID DYNAMICS 2

Professor P. Hall
Term 2

Prerequisites: Fluid Dynamics 2 is a continuation of the module Fluid Dynamics 1 given in Term 1.

In Fluid Dynamics 1 the main attention was with exact solutions of the Navier-Stokes equations governing viscous fluid motion. The exact solutions are only possible in a limited number of situations when the shape of the body is rather simple. A traditional way of dealing with more realistic shapes (like aircraft wings) is to seek possible simplifications in the Navier-Stokes formulation. We shall start with the case when the internal viscosity of the fluid is very large, and the Navier-Stokes equations may be substituted by the Stokes equations. The latter are linear and allow for simple solutions in various situations. Then we shall consider the opposite limit of very small viscosity, which is characteristic, for example, of aerodynamic flows. In this case the analysis of the flow past a rigid body (say, an aircraft wing) requires Prandtl’s boundary-layer equations to be solved. These equations are parabolic, and in many situations may be reduced to ordinary differential equations. Solving the Prandtl equations allows us to calculate the viscous drag experienced by the bodies. The final part of the module will be devoted to the theory of separation of the boundary layer, known as Triple-Deck theory.

Aims of the module:
To introduce the students to various aspects of Viscous Fluid Dynamics, and to demonstrate the power (and beauty) of modern mathematical methods employed when analysing fluid flows. This includes the Method of Matched Asymptotic Expansions, which was put forward by Prandtl for the purpose of mathematical description of flows with small viscosity. Now this method is used in all branches of applied mathematics.

Content:
Dynamic and Geometric Similarity of fluid flows. Reynolds number and Strouhal number.
Large Reynolds Number Flows: the notion of singular perturbations. Method of matched asymptotic expansions. Prandtl’s boundary-layer equations. Prandtl’s hierarchical concept. Displacement thickness of the boundary layer and its influence on the flow outside the boundary layer.
Triple-Deck Theory: The notion of boundary-layer separation. Formulation of the triple-deck equations for a flow past a corner. Solution of the linearised problem (small corner angle case).

M3A28*  INTRODUCTION TO GEOPHYSICAL FLUID DYNAMICS

Dr P. Berloff
Term 2

Prerequisites: Students will be expected to have had grounding in classical and elementary fluid mechanics (inviscid and viscous), waves, vortices, vector calculus and partial differential equations. Studies in waves, vortices and hydrodynamic instabilities will be of use.
• Overview of physical phenomena
• Governing equations of motion
  — Rotation and sphericity
  — Geostrophic and hydrostatic balances
  — Boussinesq approximation
• Rotating shallow-water equations
  — Linear geostrophic and ageostrophic waves (including equatorial and Kelvin)
  — Geostrophic adjustment
  — Stokes drift
• Potential vorticity
• Quasigeostrophy
• Rossby waves
• Barotropic and baroclinic instabilities
  — Kelvin-Helmholtz instability
  — Eady and Phillips problems
• Turbulent baroclinic zonal jet
  — Eddy fluxes
  — Non-acceleration theorem
• Incompressible turbulence
  — The closure problem
  — Kolmogorov theory
  — Two-dimensional turbulence
  — Coherent vortices
• Ekman boundary layers
• Western boundary layers

M3M7* ASYMPTOTIC ANALYSIS

Professor X. Wu
Term 1


DYNAMICS

M3PA48* DYNAMICS OF GAMES

Prof D. Turaev
Term 1

Contents of the module.
Recently there has been quite a lot of interest in modelling learning. The settings to which these models are applied is wide-ranging. Examples are (i) how populations in biology optimise their strategies and (ii) how people pick what actions to take in a competitive environment.

This module is aimed at discussing a number of such models in which learning evolves over time, and which have a game theoretic background. The module will use tools from the theory of dynamical systems, and will aim to be rigorous. Topics will include replicator dynamics and best response dynamics.

Prerequisites. M2AA1 (Differential Equations).

A significant amount of the material will use dynamical systems tools. Related modules are M3PA46 (Chaos and
Fractals) or M3PA23 (Dynamical Systems). However, it is not necessary to have taken these modules, nor is it necessary to have any background in game theory. In spite of its name, the module style will be rather Pure.

M3PA23* DYNAMICAL SYSTEMS

Dr M. Rasmussen
Term 1

The theory of Dynamical Systems is an important area of mathematics which aims at describing objects whose state changes over time. For instance, the solar system comprising the sun and all planets is a dynamical system, and dynamical systems can be found in many other areas such as finance, physics, biology and social sciences. This course provides a rigorous treatment of the foundations of discrete-time dynamical systems, which includes the following subjects:

- Periodic orbits
- Topological and symbolic dynamics
- Chaos theory
- Invariant manifolds
- Statistical properties of dynamical systems

M3PA24* BIFURCATION THEORY

Dr D. Turaev
Term 2

This module serves as an introduction to bifurcation theory, concerning the study of how the behaviour of dynamical systems (ODEs, maps) changes when parameters are varied.

The following topics will be covered:

1) Bifurcations on a line and on a plane.
2) Centre manifold theorem; local bifurcations of equilibrium states.
3) Local bifurcations of periodic orbits – folds and cusps.
4) Homoclinic loops: cases with simple dynamics, Shilnikov chaos, Lorenz attractor.
5) Saddle-node bifurcations: destruction of a torus, intermittency, blue-sky catastrophe.
6) Routes to chaos and homoclinic tangency.

M3PA16* GEOMETRIC MECHANICS

Professor D. Holm
Term 1

This module on geometric mechanics starts with Fermat's principle, that light rays follow geodesics determined from a least action variational principle. It then treats subsequent developments in mechanics by Newton, Euler, Lagrange, Hamilton, Lie, Poincaré, Noether, and Cartan, who all dealt with geometric optics.

The module will explicitly illustrate the following concepts of geometric mechanics:

* Configuration space, variational principles, Euler-Lagrange equations, geodesic curves,
* Legendre transformation, phase space, Hamilton’s canonical equations,
* Poisson brackets, Hamiltonian vector fields, symplectic transformations,
* Lie group symmetries, conservation laws, Lie algebras and their dual spaces,
* Divergence free vector fields, momentum maps and coadjoint motion.
All of these concepts from geometric mechanics will be illustrated with examples, first for Fermat’s principle and then again for three primary examples in classical mechanics: (1) motion on the sphere, (2) the rigid body and (3) pairs of \( n:m \) resonant oscillators.


**M3PA34* DYNAMICS, SYMMETRY AND INTEGRABILITY**

**Professor D. Holm**  
**Term 2**

The following topics will be covered:

* Introduction to smooth manifolds as configuration spaces for dynamics.  
* Transformations of smooth manifolds as flows of smooth vector fields.  
* Introduction to differential forms, wedge products and Lie derivatives.  
* Adjoint and coadjoint actions of matrix Lie groups and matrix Lie algebras  
* Action principles on matrix Lie algebras, their corresponding Euler-Poincaré ordinary differential equations and the Lie-Poisson Hamiltonian formulations of these equations.  
* EPDiff: the Euler-Poincaré partial differential equation for smooth vector fields acting on smooth manifolds  
* The Hamiltonian formulation of EPDiff: Its momentum maps and soliton solutions  
* Integrability of EPDiff: Its bi-Hamiltonian structure, Lax pair and isospectral problem, as well as the relationships of these features to the corresponding properties of KdV.

**M3PA50* INTRODUCTION TO RIEMANN SURFACES AND CONFORMAL DYNAMICS**

**Dr F. Bianchi**  
**Term 2**

This elementary course starts with introducing surfaces that come from special group actions (Fuchsian / Kleinian groups). It turns out that on such surfaces one can develop a beautiful and powerful theory of iterations of conformal maps, related to the famous Julia and Mandelbrot sets. In this theory many parts of modern mathematics come together: geometry, analysis and combinatorics.


Syllabus:

Part 1: Discrete groups, complex Mobius transformations, Riemann surfaces, hyperbolic metrics, fundamental domains.  

Recommended texts:

1) *Kleinian Groups* by Berbard Maskit,  
2) *The Geometry of Discrete Groups* by Alan F. Beardon,  
3) *Dynamics in one complex variable* by John Milnor,  
4) Riemann surfaces, dynamics and geometry, lecture notes by Curtis McMullen.
FINANCE

M3F22* MATHEMATICAL FINANCE: AN INTRODUCTION TO OPTION PRICING

Professor N.H. Bingham
Term 1

Prerequisites: Differential Equations (M2AA1), Multivariable Calculus (M2AA2), Real Analysis (M2PM1) and Probability and Statistics 2 (M2S1).

The mathematical modeling of derivatives securities, initiated by the Louis Bachelier in 1900 and developed by Black, Scholes and Merton in the 1970s, focuses on the pricing and hedging of options, futures and other derivatives, using a probabilistic representation of market uncertainty. This module is a mathematical introduction to this theory, which uses a wide array of tools from stochastic analysis, which are covered in the module in a self-contained manner: Brownian motion, stochastic integration, Ito calculus and parabolic partial differential equations.

Outline:
Filtrations and information. Conditional expectation.
Brownian motion. Simulation of Brownian motion.
Gaussian properties Markov property, martingale property.
Relation with the heat equation. Feynman-Kac formula. Quadratic variation.
Bachelier's model. The Black-Scholes model.
The Ito stochastic integral: definition, properties. Ito processes.
Arbitrage strategies. Arbitrage-free markets.
The Ito formula. Applications of the Ito formula.
Stochastic exponentials. Lévy's theorem.
Options and derivative securities. Call and put options.
Sensitivity analysis of an option: Delta, Gamma and Theta.
Arbitrage pricing of derivative securities: a one period example.
Dynamic hedging of options.
The Black-Scholes partial differential equation. Relation with heat equation.
The Black-Scholes formula.
Dynamic hedging in presence of uncertain volatility. Gamma exposure.

BIOLOGY

M3A49* MATHEMATICAL BIOLOGY

Dr F. Tettamanti-Boshier
Term 1

The aim of the module is to describe the application of mathematical models to biological phenomena. A variety of contexts in human biology and diseases are considered, as well as problems typical of particular organisms and environments.

The syllabus includes topics from:

*Epidemiology - the spread of plagues.
*Reaction-Diffusion models: Turing mechanism for pattern formation.
How the leopard got his spots (and sometimes stripes).
*Enzyme Kinetics and chemical reactions: Michaelis-Menten theory.
Hormone cycles, neuron-firing.

*Mass transport; Taylor dispersion.

*Biomechanics: Blood circulation, animal locomotion: swimming, flight.
Effects of scale and size.

*Other particular problems from biology.

**MATHEMATICAL PHYSICS**

**M3A4**  **MATHEMATICAL PHYSICS 1: QUANTUM MECHANICS**

Dr S. Jevtic
Term 2

Quantum mechanics is one of the most successful theories in modern physics and has an exceptionally beautiful underlying mathematical structure. It provides the basis for many areas of contemporary physics, including atomic and molecular, condensed matter, high-energy particle physics, quantum information theory, and quantum cosmology, and has led to countless technological applications.

This module aims to provide an introduction to quantum phenomena and their mathematical description. Quantum theory combines tools and concepts from various areas of mathematics and physics, such as classical mechanics, linear algebra, probability theory, numerical methods, analysis and even geometry. However, most of the concepts are basic, and little background knowledge is required before we can put them to practical use.

Core topics: the mathematics and foundations of quantum mechanics; Schrodinger equation and wave functions; quantum dynamics; one-dimensional systems; harmonic oscillator; angular momentum; spin-1/2 systems; multiparticle systems; entanglement. Additional topics may include the hydrogen atom and approximation methods.

**M3A6**  **SPECIAL RELATIVITY AND ELECTROMAGNETISM**

Dr G. Pruessner
Term 1

This module presents a beautiful mathematical description of a physical theory of great theoretical and technological importance. At every stage reference is made to experimental results and applications.

Special relativity: Einstein’s postulates, Lorentz transformation and its consequences, four vectors, dynamics of a particle, mass-energy equivalence, collisions.
Electromagnetism: Coulomb’s law and its consequences, the magnetic field, Biot-Savart law, field tensors, Lorentz force law, Faraday’s law, Maxwell’s equations, Poynting vector and energy-momentum conservation, radiation.

**M3A7**  **TENSOR CALCULUS AND GENERAL RELATIVITY**

Dr R. Barnett
Term 2

The mathematical description of a theory, which is fundamental to gravitation and to behaviour of systems at large scales.

Tensor calculus including Riemannian geometry; principle of equivalence for gravitational fields; Einstein’s field equations and the Newtonian approximation; Schwarzschild’s solution for static spherically symmetric systems;
the observational tests; significance of the Schwarzschild radius; black holes; cosmological models and ‘big bang’ origin of the universe; the early universe.

M3A29* THEOREY OF COMPLEX SYSTEMS

Professor H. Jensen
Term 2

Objective: To become familiar with the subject matter of Complexity Sciences, its methodology and mathematical tools.

Prerequisites: Curiosity and an interest in being able to understand the complex world surrounding us. Standard undergraduate mathematics (such as calculus, linear algebra). Some familiarity with computing (e.g. matlab or other programming language). A little familiarity with statistical mechanics may be helpful.

This module will provide the basic foundation in terms of concepts and mathematical methodology needed to analyse and model complex systems.

1) Simple functional integration: to discuss the emergent vortex solutions in terms extremal configurations for the partition integral of the 2D XY model.
2) Record statistics and record dynamics: to discuss the statistics of intermittent slowly decelerating dynamics as observed in models of evolution and many other complex systems. Relations to extreme value statistics.
3) Branching processes: to present a mean field discussion of avalanche dynamics in models of complex systems such as the sand pile, forest fires and more recent models of fusions of banks.
4) The Kuramoto transition to synchronisation as an example of collective cooperative dynamical behaviour of potential relevance to brain dynamics.
5) Intermittency in low (non-linear maps) and high dimensional systems (e.g. Tangled Nature model) and relation to renormalisation theory (low dim.) and mean field stability analysis (high dim).

Assessment: Two mini projects.

M3M3* INTRODUCTION TO PARTIAL DIFFERENTIAL EQUATIONS

Dr E. Zatorska
Term 1

1. Basic concepts: PDEs, linearity, superposition principle. Boundary and Initial value problems.
3. Linear and Quasilinear first order PDEs in two independent variables. Well-posedness for the Cauchy problem. The linear transport equation. Upwinding scheme for the discretization of the advection equation.

M3M6* METHODS OF MATHEMATICAL PHYSICS

Dr J. Marshall
Term 1

Complex integration [revision]

Wiener-Hopf technique continued [principal part integral – definition and examples, analytic properties of fns defined via a Cauchy-type integral on finite smooth contours, investigation of limits – Plemelj formulae, Hilbert problem (possibly limited to inversion formula possibly extended to the general case), log kernels].

Orthogonal polynomials [polynomials solutions to 2nd order differential equations, orthogonality, Gramm-Schmidt process, Rodrigues formula, generating functions, recurrence relations, numerous examples, but special care taken of Hermite, Lagrange and Laguerre polynomials all needed for quantum mechanics].

Hypergeometric series [Gamma function – integral representation and basic properties, Hypergeometric series – definition, convergence, special values of argument, differential equations, possibly extended series, Barnes integral and analytic continuations, special case needed for physics].

**M3M9**  APPLIED FUNCTIONAL ANALYSIS

Professor P. Degond

Term 1

Prerequisites: M3M9 provides a more comprehensive exposition of the basic functional analytic tools needed for Advanced Topics in Partial Differential Equations (M4M8). Elements of topology and integration theory will be provided so that students do not need to have previous training in these subjects.

1) Elements of metric topology
2) Elements of Lebesgue's integration theory.

**NUMERICAL/COMPUTATION**

(M3C High Performance Computing – See later)

**M3N7**  NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

Dr I. Shevchenko

Term 1

An analysis of methods for solving ordinary differential equations. Totally examined by project.


**M3N10**  COMPUTATIONAL PARTIAL DIFFERENTIAL EQUATIONS 1
**Professor J. Mestel**

**Term 2**

The module will introduce a variety of computational approaches for solving partial differential equations, focusing mostly on finite difference methods, but also touching on finite volume and spectral methods. Students will gain experience implementing the methods and writing/modifying short programs in Matlab or other programming language of their choice. Applications will be drawn from problems arising in Mathematical Biology, Fluid Dynamics, etc. At the end of the module, students should be able to solve research-level problems by combining various techniques.

Topics (as time permits).


- Solvers for elliptic problems: direct and iterative solvers, Jacobi and Gauss-Seidel method and convergence analysis; geometric multigrid method.

- Methods for the heat equation: explicit versus implicit schemes; stiffness.

- Techniques for the wave equation: finite-difference solution, characteristic formulation, non-reflecting boundary conditions, one-way wave equations, perfectly matched layers. Lax-Friedrichs, Lax-Wendroff, upwind and semi-Lagrangian advection schemes.

- Domain decomposition for elliptic equations: overlapping alternating Schwarz method and convergence analysis, non-overlapping methods.

**M3SC* SCIENTIFIC COMPUTATION**

**Prof P. Schmid**

**Term 2**

Scientific computing is an important skill for any mathematician. It requires both knowledge of algorithms and proficiency in a scientific programming language. The aim of this module is to expose students from a varied mathematical background to efficient algorithms to solve mathematical problems using computation.

The objectives are that by the end of the module all students should have a good familiarity with the essential elements of the Python programming language, and be able to undertake programming tasks in a range of common areas (see below).

There will be four sub-modules: 1. A PDE-module covering elementary methods for the solution of time-dependent problems. 2. An optimization-module covering discrete and derivative-free algorithms. 3. A pattern-recognition-module covering searching and matching methods. 4. A statistics-module covering, e.g., Monte-Carlo techniques.

Each module will consist of a brief introduction to the underlying algorithm, its implementation in the python programming language, and an application to real-life situations.

**M3N9* COMPUTATIONAL LINEAR ALGEBRA**

**Dr E. Keaveny**

**Term 1**

Examined solely by project. Competence in MATLAB is a prerequisite.
Whether it be statistics, mathematical finance, or applied mathematics, the numerical implementation of many of the theories arising in these fields relies on solving a system of linear equations, and often doing so as quickly as possible to obtain a useful result in a reasonable time. This course explores the different methods used to solve linear systems (as well as perform other linear algebra computations) and has equal emphasis on mathematical analysis and practical applications. A portion of the course will also be devoted to optimisation and how linear algebra routines arise in this context.

Topics include:
1. Direct methods: Triangular equations, Gauss elimination, LU-decomposition, conditioning and finite-precision arithmetic, partial and complete pivoting, Cholesky factorisation, band matrices, QR-factorisation.
2. Iterative methods: Richardson, Jacobi, Gauss - Seidel, SOR; block variants; convergence criteria; Chebyshev acceleration.
3. Symmetric eigenvalue problem: power method and variants, Jacobi’s method, Householder reduction to tridiagonal form, eigenvalues of tridiagonal matrices, the QR method

PURE MATHEMATICS

M2PM5 METRIC SPACES AND TOPOLOGY

Prof T. Coates
Term 2

This module extends various concepts from analysis to more general spaces.

Metric spaces. Convergence and continuity. Examples (Euclidean spaces, function spaces; uniform convergence). The open sets in a metric space; equivalent metrics. Convergence and continuity in terms of open sets: topological spaces. Subspaces. Hausdorff spaces. Sequential compactness; compactness via open covers; compact spaces; determination of compact subspaces of \( \mathbb{R}^n \). Completeness in metric spaces. Relationship between compactness and completeness. Connected and path connected spaces; equivalence of these notions for open sets in \( \mathbb{R}^n \). Winding numbers, definition of fundamental group, its computation for the circle. Example: proof of fundamental theorem of algebra.

ANALYSIS

M3P6* PROBABILITY THEORY

Dr I. Krasovsky
Term 2

Prerequisites: Measure and Integration (M3P19, Term 1)

A rigorous approach to the fundamental properties of probability.

M3P7* FUNCTIONAL ANALYSIS

Dr D. Gajic
Term 2

This module brings together ideas of continuity and linear algebra. It concerns vector spaces with a distance, and involves linear maps; the vector spaces are often spaces of functions.


M3P18* FOURIER ANALYSIS AND THEORY OF DISTRIBUTIONS

Prof. M. Ruzhansky
Term 2

Spaces of test functions and distributions, Fourier Transform (discrete and continuous), Bessel’s, Parseval’s Theorems, Laplace transform of a distribution, Solution of classical PDE’s via Fourier transform, Basic Sobolev Inequalities, Sobolev spaces.

M3P19* MEASURE AND INTEGRATION

Dr G. Holzegel
Term 1


M3P60* GEOMETRIC COMPLEX ANALYSIS

Dr D. Cheraghi
Term 1

Complex analysis is the study of the functions of complex numbers. It is employed in a wide range of topics, including dynamical systems, algebraic geometry, number theory, and quantum field theory, to name a few. On the other hand, as the separate real and imaginary parts of any analytic function satisfy the Laplace equation, complex analysis is widely employed in the study of two-dimensional problems in physics such as hydrodynamics, thermodynamics, Ferromagnetism, and percolations.

While you become familiar with basics of functions of a complex variable in the complex analysis course, here we look at the subject from a more geometric viewpoint. We shall look at geometric notions associated with domains in the plane and their boundaries, and how they are transformed under holomorphic mappings. In turn, the behavior of conformal maps is highly dependent on the shape of their domain of definition. Below is a rough guide to the syllabus.

Part 1) Elements of holomorphic mappings: Poincare metric, Schwarz-Pick lemma, Riemann mapping theorem, growth and distortion estimates, normal families, canonical mappings of multiply connected regions.


Part 3) Elements of quasi-conformal mappings and elliptic PDEs,
Beltrami equation, singular integral operators, measurable Riemann mapping theorem.

**GEOMETRY**

**M3P5**  GEOMETRY OF CURVES AND SURFACES

Dr M-A Lawn

Term 1

The main object of this module is to understand what is the curvature of a surface in 3-dimensional space.

Topological surfaces: Definition of an atlas; the prototype definition of a surface; examples. The topology of a surface; the Hausdorff condition; the genuine definition of a surface. Orientability, compactness.

Subdivisions and the Euler characteristic.

Cut-and-paste technique, the classification of compact surfaces. Connected sums of surfaces.

Smooth surfaces: Definition of a smooth atlas, a smooth surface and of smooth maps into and out of smooth surfaces. Surfaces in $\mathbb{R}^3$, tangents, normals and orientability.

The first fundamental form, lengths and areas, isometries.

The second fundamental form, principal curvatures and directions.

The definition of a geodesic, existence and uniqueness, geodesics and co-ordinates.

Gaussian curvature, definition and geometric interpretation, Gauss curvature is intrinsic, surfaces with constant Gauss curvature.

The Gauss-Bonnet theorem.

(Not examinable and in brief) Abstract Riemannian surfaces, metrics.

**M3P20**  GEOMETRY 1: ALGEBRAIC CURVES

Prof. M. Haskins

Term 1

Plane algebraic curves including inflection points, singular and non-singular points, rational parametrisation, Weierstrass form and the Group Law on non-singular cubics. Abstract complex manifolds of dimension 1 (Riemann surfaces); elliptic curves as quotients of $\mathbb{C}$ by a lattice. Elliptic integrals and Abel’s theorem.

**M3P21**  GEOMETRY 2: ALGEBRAIC TOPOLOGY

Prof. M. Haskins

Term 2

Homotopies of maps and spaces. Fundamental group. Covering spaces, Van Kampen (only sketch of proof).

Homology: singular and simplicial (following Hatcher’s notion of Delta-complex). Mayer-Vietoris (sketch proof) and long exact sequence of a pair. Calculations on topological surfaces. Brouwer fixed point theorem.

**M3P23**  COMPUTATIONAL ALGEBRA AND GEOMETRY (Not running this year)

**Prerequisites:** Algebra 2. Some (useful) overlap with Algebra 3.


**ALGEBRA AND DISCRETE MATHEMATICS**

**M3P8**  ALGEBRA 3
Dr T. Schedler
Term 1

Rings, integral domains, unique factorization domains.
Modules, ideals homomorphisms, quotient rings, submodules quotient modules.
Fields, maximal ideals, prime ideals, principal ideal domains.
Euclidean domains, rings of polynomials, Gauss’s lemma, Eisenstein’s criterion.
Field extensions.
Noetherian rings and Hilbert’s basis theorem.
Dual vector space, tensor algebra and Hom.
Basics of homological algebra, complexes and exact sequences.

M3P10* GROUP THEORY

Prof A. Ivanov
Term 1

An introduction to some of the more advanced topics in the theory of groups.

Composition series, Jordan-Hölder theorem, Sylow’s theorems, nilpotent and soluble groups.
Permutation groups. Types of simple groups.

M3P11* GALOIS THEORY

Professor A. Corti
Term 1

The formula for the solution to a quadratic equation is well-known. There are similar formulae for cubic and quartic equations, but no formula is possible for quintics. The module explains why this happens.

Irreducible polynomials. Field extensions, degrees and the tower law. Extending isomorphisms.
Normal field extensions, splitting fields, separable extensions. The theorem of the primitive Element.
Groups of automorphisms, fixed fields. The fundamental theorem of Galois theory.
The solubility of polynomials of degree at most 4. The insolvability of quintic equations.

M3P12* GROUP REPRESENTATION THEORY

Dr. R. Bellovin
Term 2

Representations of groups: definitions and basic properties. Maschke’s theorem, Schur’s lemma.
Representations of abelian groups. Tensor products of representations.
The character of a group representation. Class functions. Character tables and orthogonality relations.

M3P17* ALGEBRAIC COMBINATORICS

Professor M.W. Liebeck
Term 2

An introduction to a variety of combinatorial techniques that have wide applications to other areas of mathematics.

**NUMBER THEORY**

**M3P14**  NUMBER THEORY

Dr D. Helm

Term 1

The module is concerned with properties of natural numbers, and in particular of prime numbers, which can be proved by elementary methods.


**M3P15**  ALGEBRAIC NUMBER THEORY

Dr A. Pal

Term 2

An introduction to algebraic number theory, with emphasis on quadratic fields. In such fields the familiar unique factorisation enjoyed by the integers may fail, but the extent of the failure is measured by the class group. The following topics will be treated with an emphasis on quadratic fields \( \mathbb{Q}(\sqrt{d}) \).

Field extensions, minimum polynomial, algebraic numbers, conjugates and discriminants, Gaussian integers, algebraic integers, integral basis, quadratic fields, cyclotomic fields, norm of an algebraic number, existence of factorisation. Factorisation in \( \mathbb{Q}(\sqrt{d}) \). Ideals, Z-basis, maximal ideals, prime ideals, unique factorisation theorem of ideals and consequences, relationship between factorisation of numbers and of ideals, norm of an ideal. Ideal classes, finiteness of class number, computations of class number.

**M3P16**  ANALYTIC NUMBER THEORY (not running in 2016-17)

Abel and Euler summation. Euclid and Euler's proofs that there are infinitely many primes. Von Mangoldt's function, Chebychev's Inequalities, Merten's theorem. Dirichlet series, generating functions. The Riemann zeta function. Analytic continuation, and non-vanishing on the 1-line. The Prime Number Theorem (PNT) and its relatives.

**STATISTICS**

**M2S2**  STATISTICAL MODELLING 1
Dr B. Calderhead  
Term 2  

Traditional concepts of statistical inference, including maximum likelihood, hypothesis testing and interval estimation are developed and then applied to the linear model, which arises in many practical situations.

Maximum likelihood estimation, likelihood ratio tests and their properties, confidence intervals.  

Linear models - including non-full rank models: estimation, confidence intervals and hypothesis testing. The analysis of variance.

M3S1*  STATISTICAL THEORY (Previously Statistical Theory I)  
Dr R. Drikvandi  
Term 2  

This module deals with the criteria and the theoretical results necessary to develop and evaluate optimum statistical procedures in hypothesis testing, point and interval estimation.

Theories of estimation and hypothesis testing, including sufficiency, completeness, exponential families, minimum variance unbiased estimators, Cramér-Rao lower bound, maximum likelihood estimation, Rao-Blackwell and Neyman-Pearson results, and likelihood ratio tests as well as elementary decision theory and Bayesian estimation.

M3S2*  STATISTICAL MODELLING 2  
Dr D-H. Lau  
Term 2  

Prerequisites: This module leads on from the linear models covered in M2S2 and Probability and Statistics 2 covered in M2S1.

The Generalised Linear Model is introduced from a theoretical and practical viewpoint and various aspects are explained.  


The R statistical package will be used to expose how the different models can be applied on example data.

M3S4*  APPLIED PROBABILITY  
Dr A. Veraart  
Term 1  

This module aims to give students an understanding of the basics of stochastic processes. The theory of different kinds of processes will be described, and will be illustrated by applications in several areas. The groundwork will be laid for further deep work, especially in such areas as genetics, finance, industrial applications, and medicine.

A fundamental aim in statistics is classifying things. For example, this occurs in medicine, where the aim is to assign people to diseases, speech recognition, where the aim is to assign words to meanings, banking, where the aim is to assign customers to risk classes, and in a vast number of other areas. This module provides a modern view of methods for performing such so-called pattern recognition tasks. Totally examined by projects.

Discriminant analysis, generalised linear models, nearest neighbour methods, recursive partitioning methods, trees, ensemble classifiers, and other tools. The areas of pre-processing, feature selection and classification performance assessment will also be explored, and an introduction to unsupervised pattern recognition methods will be given.

An introduction to the analysis of time series (series of observations, usually evolving in time) is given, which gives weight to both the time domain and frequency domain viewpoints. Important structural features (e.g. reversibility) are discussed, and useful computational algorithms and approaches are introduced. The module is self-contained.


Computational techniques have become an important element of modern statistics (for example for testing new estimation methods and with notable applications in biology and finance). The aim of this module is to provide an up-to-date view of such simulation methods, covering areas from basic random variate generation to Monte Carlo methodology. The implementation of stochastic simulation algorithms will be carried out in R, a language that is widely used for statistical computing and well suited to scientific programming.


Simple probabilistic and mathematical tools are used to study the theory of games where two opponents are in conflict, including the celebrated Prisoners’ Dilemma problem. Utilities, based on (apparently) reasonable axioms are introduced, leading to a study of decision theory.

M3S14* SURVIVAL MODELS AND ACTUARIAL APPLICATIONS

Prof A. Gandy
Term 2

Survival models are fundamental to actuarial work, as well as being a key concept in medical statistics. This module will introduce the ideas, placing particular emphasis on actuarial applications.

Explain concepts of survival models, right and left censored and randomly censored data. Introduce life table data and expectation of life.
Graduation and testing crude and smoothed estimates for consistency.

M3S16* CREDIT SCORING

Dr A. Bellotti
Term 1

Prerequisites: Statistical Modelling 1 (M2S2) with some dependency on Statistical Modelling 2 (M3S2) and Applied Probability (M3S4).

Introduction and background: Aims and objectives of scoring, legislative and commercial aspects. Consumer credit data: characteristics, transformations, data quality, transaction types, challenges. Notions of statistical scorecards.
The R statistical package will be used to explore credit scoring models on example data.

M3S17* QUANTITATIVE METHODS IN RETAIL FINANCE

Dr A. Bellotti
Term 2

Prerequisites: Essential - Credit Scoring 1 (M3/4S16). Useful – Applied Probability (M3S4).

Asset correlation and dynamic random effects models. 
Stress testing: concepts and Monte Carlo simulation approaches

PROJECT

M3R RESEARCH PROJECT IN MATHEMATICS
Available only to Final Year BSc students

Supervised by Various Academic Staff
Co-ordinator: Dr J. Britnell
(Terms 2 & 3)

The main aim of this module is to give a deep understanding of a mathematical area/topic by means of a supervised project in applied mathematics, mathematical physics, pure mathematics, numerical analysis or statistics. The project may be theoretical and/or computational and the area/topic for each student is chosen in consultation with the Department.

The module provides an excellent ‘apprenticeship in research’ and is therefore particularly strongly recommended for BSc students who are considering postgraduate study leading to MSc/MPhil/PhD.

There is an oral presentation as part of the module assessment.

There will be a meeting in mid-November for students interested in the M3R and a required Research Skills workshop at the start of Term 2.

OTHER “NON-MATHEMATICAL” MATHEMATICS MODULES

The modules M3E/M3H/M3B/M3C/M3T may each be taken as an alternative to a Centre for Co-Curricular Studies/Business School option. However, as Department of Mathematics modules, their ECTS value is 8.

M3E ECONOMETRIC THEORY AND METHODS
Dr H. Battey
Term 1

Econometric Theory and Methods aims to help students understand the econometric techniques that are widely employed in modern economic applications and research. It presents an advanced treatment of econometric principals for cross-sectional and panel (or longitudinal) data sets.

Linear regression model. Finite sample properties of ordinary least squares (OLS) and generalized leased squares (GLS) estimators. Testing linear hypotheses.
Review of convergence concepts, limit theorem and central limit theorems.
Asymptotic properties, asymptotic distribution theory and hypothesis testing for linear least squares estimators. Asymptotic properties and hypothesis testing for instrumental variable (IV) and linear generalised method of moments (GMM) estimators. Panel data.
Stochastic orders of magnitude and delta method.
Estimation for non-linear models. Consistency and asymptotic normality.
Asymptotic distribution theory for the Likelihood ratio test, Wald test and Langrange Multiplier test in the context of maximum likelihood and GMM.

M3H History of Mathematics

Prof N.H. Bingham
Term 2
The aim of this module is to give an overview of the development of mathematics from ancient to modern times.

The ancient world.
Prehistory. Egypt; Mesopotamia. The Greeks: from Pythagoras through Euclid and Archimedes to Apollonius.

The Middle Ages.
The Arabs. India and China.
The beginnings of mathematics in Europe; the Renaissance; Copernicus and Galileo.

The modern world.
The Scientific Revolution; Newton, Leibniz and their followers.

The Enlightenment. Academies and universities.
The 18th C.; the Bernoullis, Euler, Lagrange. The Napoleonic period: Laplace, Gauss.
The 19th C.: Cauchy, Riemann, Cantor; Poincare and Hilbert.
The 20th C., up to 1950: the development of modern algebra, analysis, probability, statistics, applied mathematics.

M3B The Mathematics of Business and Economics

Dr J.S. Martin
Term 2

This module aims to:
Give a broad mathematical introduction to both microeconomics and macroeconomics
Consider the motivations and optimal behaviours of both firms and consumers in the marketplace, and show how this leads to the widely observed laws of supply and demand
Look at the interaction of firms and consumers in markets of varying levels of competition and further consider the roles of both, as well as the government, in a macroeconomic setting

Syllabus:
Theory of the firm
Profit maximisation for a competitive firm. Cost minimisation. Geometry of costs. Profit maximisation for a non-competitive firm.
Theory of the consumer
Consumer preferences and utility maximisation. The Slutsky equation.
Levels of competition in a market
Macroeconomic theory
Circular flow of income. Aggregate supply & demand. The multiplier effect.

M3C Introduction to High Performance Scientific Computing

Dr P. Ray
Term 1

High-performance computing centres on the solution of large-scale problems that require substantial computational power. This will be a practical module that introduces a range of powerful tools that can be used to efficiently solve such problems. By the end of the module, which will be examined by projects, students will be prepared to tackle research problems using the tools of modern high-performance scientific computing in an informed, effective, and efficient manner.

Contents:
Getting started: working with UNIX at the command line
Software version control with git and Bitbucket
Programming and scientific computing with Python
Modular programming with modern Fortran, using scientific libraries, interfacing Python and Fortran
OpenMP (with Fortran) for parallel programming of shared-memory computers
MPI (with Fortran) for programming on distributed-memory machines such as clusters
Cloud computing
Good programming practice: planning, unit testing, debugging, validation (to be integrated with the above topics and the programming assignments.)

M3T COMMUNICATING MATHEMATICS

Professor E.J. McCoy/Dr L.V. White
(Terms 2 & 3)

This module will give students the opportunity to observe and assist with teaching of Mathematics in local schools. Entry to the module is by interview in the preceding June and numbers will be limited. It is required for anyone on the Mathematics with Education degree coding.

For those selected there will follow a one day training course in presentation skills and other aspects of teaching. Students will be assigned to a school where they will spend ten half days in Term 2, under the supervision of a teacher. Assessment will be based on a portfolio of activities in the school, a special project, evaluation by the school teacher and an oral presentation.

CENTRE FOR LANGUAGE, CULTURE AND COMMUNICATION/BUSINESS SCHOOL

Students may consider broadening their study programme by taking advantage of the CLCC/Business School provision.

Note that Centre for Co-Curricular Studies modules extend throughout Terms 1 and 2 and some modules may be examined in January. Taking the HSCS3006 Humanities Project normally also requires explicit permission from the Centre for Co-Curricular Studies.

<table>
<thead>
<tr>
<th>Module Codes</th>
<th>Module Titles</th>
<th>Terms</th>
<th>Honours Marks</th>
<th>ECTS Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>HGC31</td>
<td>Lessons from History</td>
<td>1 + 2</td>
<td>100</td>
<td>6</td>
</tr>
<tr>
<td>HGC32</td>
<td>Global Challenges Independent Project</td>
<td>1 + 2</td>
<td>100</td>
<td>6</td>
</tr>
<tr>
<td>HSCS3001</td>
<td>Advanced Creative Writing</td>
<td>1 + 2</td>
<td>100</td>
<td>6</td>
</tr>
<tr>
<td>HSCS3002</td>
<td>History of Science, Technology and Industry</td>
<td>1 + 2</td>
<td>100</td>
<td>6</td>
</tr>
<tr>
<td>HSCS3003</td>
<td>Philosophy of Mind</td>
<td>1 + 2</td>
<td>100</td>
<td>6</td>
</tr>
<tr>
<td>HSCS3004</td>
<td>Contemporary Philosophy</td>
<td>1 + 2</td>
<td>100</td>
<td>6</td>
</tr>
<tr>
<td>HSCS3005</td>
<td>Science, Politics and Human Identity</td>
<td>1 + 2</td>
<td>100</td>
<td>6</td>
</tr>
<tr>
<td>HSCS3006</td>
<td>Humanities Project</td>
<td>1 + 2</td>
<td>100</td>
<td>6</td>
</tr>
<tr>
<td>HSCS3007</td>
<td>Conflict, Crime and Justice</td>
<td>1 + 2</td>
<td>100</td>
<td>6</td>
</tr>
<tr>
<td>HSCS3008</td>
<td>Visual Culture, Knowledge and Power</td>
<td>1 + 2</td>
<td>100</td>
<td>6</td>
</tr>
<tr>
<td>HSCS3009</td>
<td>Music and Western Civilization</td>
<td>1 + 2</td>
<td>100</td>
<td>6</td>
</tr>
<tr>
<td>HSCS2007</td>
<td>Music Technology</td>
<td>1 + 2</td>
<td>100</td>
<td>6</td>
</tr>
<tr>
<td>BS0808</td>
<td>Finance and Financial Management</td>
<td>2</td>
<td>100</td>
<td>6</td>
</tr>
<tr>
<td>BS0820</td>
<td>Managing Innovation</td>
<td>1</td>
<td>100</td>
<td>6</td>
</tr>
</tbody>
</table>

Third Year BSc students are permitted to take up to two options from this list. MSci students may take one option, at most, in each of their Third and Fourth Years.

Syllabus and timetabling information can be viewed online at:
Note that places in CLCC and Business School modules are normally limited and registration should be done separately via the Centre for Co-Curricular Studies and Business School websites.

Note that a change in degree code registration can lead to your registration for a BPES code being revoked. This is an unfortunate side-effect of how the Business School runs things. Save a screenshot of your registration to help in any dispute.

Subject to the Department’s approval, in addition to the Centre for Co-Curricular Studies/Business School options, students may take a Mathematical module given outside the Department, e.g. in the Department of Physics. Students are advised to discuss this with the Director of Undergraduate Studies if they wish to consider such an option.

IMPERIAL HORIZONS

The College has created the ‘Imperial Horizons’ programme to broaden students’ education and enhance their career prospects. This programme is open to all undergraduate students.

The Department of Mathematics always endeavours to avoid timetabling Mathematics modules during the times allocated for Horizons modules.

Note that modules on this programme (except for the ones listed separately above as approved modules for 3rd year students) do not contribute to degree Honours marks but they do have an ECTS value of 6.

Further information about the ‘Horizons’ programme can be found at: http://www.imperial.ac.uk/horizons

BSC DEGREE COURSE CODING REQUIREMENTS

All modules within the Department are registered for G100, G103 Mathematics. To qualify for any degree a student must satisfy the overall College requirements.

As well as the regular G100 degree, the department offers several specialist degree codings. To qualify for the BSc codings G102, G125, G1F3, G1G3, G1GH, GG31, a suitable number of modules must eventually be passed from subsets of the general list as follows:

<table>
<thead>
<tr>
<th>Degree</th>
<th>Required Modules</th>
</tr>
</thead>
<tbody>
<tr>
<td>G102</td>
<td>4 from Mathematics with Mathematical Computation M3N3, M3N4, M3N7, M3N9, M3N10, M3SC, M3R, M3C.</td>
</tr>
<tr>
<td>G125</td>
<td>6 from Mathematics (Pure Mathematics) M2PM5, M3P5, M3P6, M3P7, M3P8, M3P10, M3P11, M3P12, M3P14, M3P15, M3P17, M3P18, M3P19, M3P20, M3P21, M3P22, M3P23, M3P60, M3PA50, M3R.</td>
</tr>
<tr>
<td>G1F3</td>
<td>6 from Mathematics with Applied Mathematics/Mathematical Physics M2AM, M3A2, M3A4, M3A6, M3A7, M3A10, M3PA16, M3F22, M3A28, M3A29, M3A49, M3PA34, M3M6, M3M7, M3M9, M3PA23, M3PA24, M3PA48, M3SC, M3R.</td>
</tr>
<tr>
<td>G1G3</td>
<td>6 from Mathematics with Statistics M2S2, M3S1, M3S2, M3S4, M3S8, M3S9, M3S10, M3S11, M3S12, M3S14, M3S15, M3S16, M3S17, M3R.</td>
</tr>
<tr>
<td>G1GH</td>
<td>6 from Mathematics with Statistics for Finance M2S2, M3F22, M3S1, M3S2 M3S4, M3S7, M3S8, M3S9, M3S11, M3S14, M3S16, M3S17, M3P17, M3P22, M3SC, M3R.</td>
</tr>
<tr>
<td>GG31</td>
<td>6 from Mathematics, Optimisation and Statistics M2S2, M3S1, M3S2, M3S4, M3S8, M3S9, M3S11, M3S14, M3S16, M3S17, M3P17, M3P22, M3SC, M3R.</td>
</tr>
</tbody>
</table>
It is generally possible to swap between the above BSc codings, subject to the stated requirements, at a fairly late stage.

As part of the continuing review of the undergraduate programme of study, amendments to this list can be expected, including changes in module numbering. **Not all of the individual modules listed are offered every session. The above are the normal requirements – the Department has the discretion to modify them.**

**FOUR YEAR (MSCI) DEGREES: G103/G104/G1EM**

Students are normally required to maintain a good level of performance in Mathematics (at Upper Second Class level or better) in order to remain on this coding in their Third and Final Years – see page 3.

Note that the Third and Fourth Year syllabuses substantially overlap. If a module may be attended by both 3rd and 4th Year students then the 4th year students typically take an extended (2.5 hour) examination. (This replaces the Mastery Examination, which operated before 2015.)

**THE M4R PROJECT:**

A fundamental part of the G103/G104 MSci degree is a substantial compulsory project (M4R) equivalent to two lecture modules. The main aim of this module is to give a deep understanding of a particular area/topic by means of a supervised project in some area of mathematics. The project may be theoretical and/or computational and the area/topic for each student is chosen in consultation with the Department.

Arrangements for this project will be set in motion after the Third Year examinations. **Students should approach potential supervisors in an area of interest before the end of their Third Year** and some preparatory work should be performed over the vacation between the Third and Fourth Years. Work on the project should continue throughout all three terms of the Fourth Year and submitted shortly after the Fourth Year examinations.

**G104:** For those on a Maths with a year in Europe coding, the third year is spent abroad at another university. G104 students should ideally negotiate with possible M4R supervisors by e-mail during their abroad, but this is not always possible. On return to Imperial, students take the regular Year 4 MSci programme (with the additional option of M4T.) On the rare occasion that a G104 student performs very poorly in their year away they may, at the discretion of the Senior Tutor, be transferred to the BSc G100 Mathematics degree or the BSc G101 Mathematics with a Year in Europe degree and take M3 subjects in their Final Year.

**G1EM:** For the Maths with Education MSci, the project is only half length the M4R length, and takes place only in the second term. During the first term students take Education modules rather than Mathematics ones.

For more complete details of the Fourth Year programme, the relevant documentation can be viewed online at: [https://www.imperial.ac.uk/natural-sciences/departments/mathematics/study/students/undergraduate/programme-information/](https://www.imperial.ac.uk/natural-sciences/departments/mathematics/study/students/undergraduate/programme-information/)