DEPARTMENT OF MATHEMATICS

GUIDE TO OPTIONAL MODULES

FOURTH/FINAL YEAR (MSci)
2017-2018

Notes and syllabus details on Fourth Year modules for students in their Fourth/Final Year

For degree codings:

<table>
<thead>
<tr>
<th>Degree Code</th>
<th>Programme Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>G103</td>
<td>MATHEMATICS (BSc, MSci)</td>
</tr>
<tr>
<td>G104</td>
<td>MATHEMATICS WITH A YEAR IN EUROPE (MSci)</td>
</tr>
<tr>
<td>G1EM</td>
<td>MATHEMATICS WITH EDUCATION (MSci)</td>
</tr>
</tbody>
</table>

NOTE that GG41, IG11 and GI43 MATHEMATICS AND COMPUTER SCIENCE are administered by the Department of Computing.

Professor David Evans
Director of Undergraduate Studies

June 2017

TO BE READ IN CONJUNCTION WITH THE UNDERGRADUATE HANDBOOK.

This information WILL be subject to alteration. Updated programmes can be viewed on the MathsCentral Blackboard site and online at:
https://www.imperial.ac.uk/natural-sciences/departments/mathematics/study/students/undergraduate/programme-information/
# FOURTH YEAR OVERVIEW

## ADVICE ON THE CHOICE OF OPTIONS

## M4R PROJECT

## NON-MATHEMATICS MODULES

## GRADUATION

## MARKS, YEAR TOTALS AND YEAR WEIGHTINGS

## ECTS

## MODULE ASSESSMENT AND EXAMINATIONS

## FOURTH YEAR MODULE LIST

## FOURTH YEAR MATHEMATICS SYLLABUSES:

- **APPLIED MATHEMATICS/MATHEMATICAL PHYSICS/NUMERICAL ANALYSIS**
- **PURE MATHEMATICS**
- **STATISTICS**
- **OTHER “NON-MATHEMATICAL” MATHEMATICS MODULES**

## CLCC/BUSINESS SCHOOL

## IMPERIAL HORIZONS
FOURTH YEAR OVERVIEW

The MSci Fourth Year is available to those on the G103, G104 and G1EM codings who perform to a satisfactory standard in their Third Year, here or elsewhere in Europe. There is considerable overlap with the taught postgraduate MSc programmes in Pure and Applied Mathematics, but the MSci is a separate degree.

The MSci programme is designed to provide a breadth and depth in mathematics to a level of attainment broadly equivalent to that of an MSc degree and takes place over three terms – Term 1 (also known as Autumn Term), Term 2 (also known as Spring Term) and Term 3 (also known as Summer Term).

Students choose six lectured modules from those made available to them in the Department and from certain modules elsewhere. Students also take the compulsory M4R project, which is equivalent to two lecture modules. Students on the Mathematics with Education G1EM coding take Education modules in the first term. They then take 3 Mathematics modules in the 2nd term, and produce a half-length M4R project.

Most, but not all, of the M4 modules are also available in M3 form. Fourth Year examinations normally consist of 5 questions and are 2.5 hours long, whereas the corresponding exams for 3rd year students (if any) contain 4 questions in 2 hours. Students may not take an M4 module if they have already taken the M3 version.

Lecturing will take place during Term 1 and Term 2 with three hours per week, which usually includes some classes. The normal expectation is that there should be a 'lecture'/class' balance of about 5/1. The identification of particular class times within the timetabled periods is at the discretion of the lecturer, in consultation with the class and as appropriate for the module material.

ADVICE ON THE CHOICE OF OPTIONS

Students are advised to read these notes carefully and to discuss their option selections with their Personal Tutor. An `Options Fair’ will take place after exams in the Summer Term, where staff will answer questions on all available options.

It is anticipated that lecturers will give advice on suitable books at the start of each module. Students should contact the proposed lecturers if they desire any further details about module content in order to make their choice of course options. Students should also feel free to seek advice from Year Level Tutors and the Senior Tutor, and the Director of Undergraduate Studies.

Course option choices should be registered on the designated website between the 12th June and the 1st of July, 2017. This is predominantly to help us timetable the modules to minimise clashes between student selections. Students will not be committed to taking those modules for which they initially register until the completion of their examination entry at the beginning of Term 2. HOWEVER, timetabling of modules will be determined by the course option choices and there is an increased likelihood that your chosen modules will clash if you do not register for them by 1 July. Note that students do become committed to the completion of certain modules examined only by project at some stage during the module, as will be made clear by the lecturer.

M4R PROJECT

M4R ADVANCED RESEARCH PROJECT IN MATHEMATICS

Compulsory

Supervised by Various Academic Staff
Co-ordinator: Dr J. Britnell
(Terms 1, 2 & 3)

A fundamental part of the MSci degree is a substantial compulsory project equivalent to two lecture modules. The main aim of this module is to give a deep understanding of a particular area/topic by means of a supervised
project in some area of mathematics. The project may be theoretical and/or computational and the area/topic for each student is chosen in consultation with the Department.

The project provides an excellent ‘apprenticeship in research’ and is therefore of particular value to students who are considering postgraduate study leading to a PhD.

Arrangements for this project will be set in motion after the Third Year examinations. **Students should approach potential supervisors in an area of interest before the end of their Third Year** and some preparatory work should be performed over the vacation between the Third and Fourth Years. Work on the project should continue throughout all three terms of the Fourth Year and submitted shortly after the Fourth Year examinations.

**G104:** For those on a Maths with a year in Europe coding, the third year is spent abroad at another university. G104 students should ideally negotiate with possible M4R supervisors by e-mail during their abroad, but this is not always possible. On return to Imperial, students take the regular Year 4 MSci programme (with the additional option of M4T.) On the rare occasion that a G104 student performs very poorly in their year away they may, at the discretion of the Senior Tutor, be transferred to the BSc G100 Mathematics Degree and take M3 subjects in their Final Year.

**G1EM:** For the Maths with Education MSci, the project is only half the M4R length, and takes place only in the second term. During the first term students take Education modules rather than Mathematics ones.

**NON-MATHEMATICS MODULES**

MSc students may take one Centre for Co-Curricular Studies/Business School option in their Fourth Year from the approved list below.

**No more than two of these 'External' options may be taken as part of a student's degree.**

In addition, the department offers a few options, which are deemed to be “non-Mathematical”. These may be taken as an alternative to a Centre for Co-Curricular Studies/Business School option. However, as they are Department of Mathematics modules, their ECTS value is 8 (rather than 6).

For 2017-18, these options are:

- M3C High Performance Computing
- M3H History of Mathematics
- M3B Mathematics of Business
- M3T Communicating Mathematics (only for G104 students)

Subject to the Department's approval, students may take a mathematical module given outside the Department, e.g. in the Department of Physics. Students must obtain permission from the Director of Undergraduate Studies if they wish to consider such an option.

**GRADUATION**

**Students graduating will receive an MSci degree that explicitly incorporates a BSc.**

It is normally required that MSci students pass all course components in order to graduate. However, the College may compensate a narrowly failed module in the Final Year of study. The Examination Board may also graduate students who have one or more badly failed module, provided the overall average mark is high enough.

The total of marks for examinations, assessed coursework, progress tests, assignments and projects, with the
appropriate year weightings, is calculated and recommendations are made to the Examiners’ Meeting (normally held at the end of June) for consideration by the Academic Staff and External Examiners. Degrees are formally decided at this meeting.

Students at graduation may be awarded Honours degrees classified as follows: First, Second (upper and lower divisions) and Third, with a good Final Year and project being viewed favourably by the External Examiners for borderline cases.

Rarely, circumstances may require the Department to graduate an MSci student with a BSc.

Further information on degree classes can be found in the Scheme for the Award of Honours online at:

https://www.imperial.ac.uk/natural-sciences/departments/mathematics/study/students/undergraduate/programme-information/

In general, applications for postponement of consideration for Honours will NOT be granted by the Department except in special cases, such as absence through illness.

Information about Commemoration (Graduation) ceremonies can be found online at:

http://www3.imperial.ac.uk/graduation

MARKS, YEAR TOTALS AND YEAR WEIGHTINGS

What follows is a brief summary – more details of these topics can be found online at:

https://www.imperial.ac.uk/natural-sciences/departments/mathematics/study/students/undergraduate/programme-information/

(information for 2017-2018 will be updated over the summer of 2017).

Within the Department each total module assessment is rescaled so that overall performances in different modules may be compared. From 2017-18, all marks will be computed on the College scale, rather than the Mathematics scale used in previous years. The rescaling onto the scale 0 – 100 marks is such that 50 then corresponds to the lowest Pass Honours mark for a Masters level module and 70 corresponds to the lowest First Class performance.

Marks from the modules taken in the fourth year are combined into a year total expressed as a percentage.

Further information can found in the Scheme for the Award of Honours.

For the four year MSci codings G103, G104, G1EM, the year weightings are 1 : 3 : 4 : 5.

(For G104 students who first enrol in the Department from 2017-18 onwards, the year weightings will be 1 : 3 : 3 : 5.)

The differences in year weighting reflect the increasing level of mathematical complexity.

ECTS

To comply with the European ‘Bologna Process’, degree programmes are required to be rated via the ECTS (European Credit Transfer System) – which is based notionally on hour counts for elements within the degree.

As in Third Year, each Fourth Year mathematics module, including M3B, M3T, M3H, M3C and other mathematical optional modules, has an ECTS value of 8 except for M4R which has an ECTS value of 16. Centre for Co-Curricular Studies/Business School modules have an ECTS value of 6. Each Second Year mathematics module has an ECTS value of 7 with M2R having an ECTS value of 5. First Year mathematics modules have an ECTS value of 6.5 except for M1R which has an ECTS value of 4.5 and M1C which has an ECTS value of 4. Language modules, taken by G104 Mathematics with a Year in Europe students, have an ECTS value of 6.
MSci students who wish to increase their ECTS counts from roughly 240 to 270 must undertake additional study over the summer vacations of their Second and Third Years. Contact the Director of Undergraduate Studies for further information.

Details can be viewed online at: http://www.imperial.ac.uk/natural-sciences/departments/mathematics/study/students/undergraduate/programme-information/

MODULE ASSESSMENT AND EXAMINATIONS

Most M4 modules are examined by one written examination of 2.5 hours in length. Written examinations for M3 modules are 2 hours in length.

Some of the modules may have an assessed coursework/progress test element, limited in most cases to 10% of overall module assessment. Some modules have a more substantial coursework component (for example, 25 percent) and others are assessed entirely by coursework. Details can be found in the tables below. Precise details of the number and nature of coursework assignments will be provided at the start of each module.

The modules M4A29, M4A50, M4N7, M4N9, M4N10, M3C and M4SC are examined solely by projects.

The module M4PA48 is examined by coursework, oral exam and an in-class test.

The module M4R is examined by a research project; an oral element forms part of the assessment.

The modules M3B, M3H are examined by a 2 hour examination.

See module description for assessment of the M4S18++ modules.

The module M3T is examined by a journal of teaching activity, teacher's assessment, oral presentation, and end of module report.

Note: Students who take modules which are wholly assessed by project will be deemed to be officially registered on the module through the submission of a specified number of pieces of assessed work for that module. Thus, once a certain point is reached in these modules, a student will be committed to completing it. In contrast, students only become committed to modules with summer examinations when they enter for the examinations in February.

Students who do not obtain Passes in examinations at the first attempt may be expected to attend resit examinations the following May/June (NOT normally in September) spending a year not in attendance. Two resit attempts are normally available to students. However, the Examinations Board has the power to compensate not-too-serious fails in final year modules and permit graduation. Note that it is very rare for a 4th year student to fail any module, because of the high selection standards for the MSci.

Note: Resits may not be offered for modules assessed solely by project.

Resit examinations are for Pass credit only – a maximum mark of the pass mark (50 percent for Masters level modules) will be credited. Once a Pass is achieved, no further attempts are permitted.

FOURTH YEAR MODULE LIST

Note that not all of the individual modules listed below are offered every session and the Department reserves the right to cancel a particular module if, for example, the number of students attending that module does not make it viable. Similarly, some modules are occasionally run as ‘Reading/Seminar Courses’.

Modules marked below with a * are also available in M3 form for Third Year undergraduates students (who
typically take a shorter examination). When a module is offered it is usually, but not always, available in both forms. **No student may take both the M3 and M4 forms of a module.** In the rare event that the M4 version of a module is not available, the Department may permit one M3 module to be taken.

M3B, M3H and M3C are also available to Fourth Year students but function like a Centre for Co-Curricular Studies/Business School option, except that their ECTS value is 8. The module M3T may only be taken in year 4 by returning G104 students.

All M3 and M4 modules except M4R are equally weighted and are worth 8 ECTS points unless otherwise specified. The M4R project is double-weighted and is worth 16 ECTS points.

In the tables below:

Column on % Exam – this indicates a standard closed-book written exam, unless otherwise indicated.

Column on % CW – this indicates any coursework that is completed for the module. This may include in-class tests, projects, or problem sets to be turned in.

The groupings of modules below have been organised to indicate some natural affinities and connections.

### APPLIED MATHEMATICS/MATHEMATICAL PHYSICS/NUMERICAL ANALYSIS

#### FLUIDS

<table>
<thead>
<tr>
<th>Module Codes</th>
<th>Module Titles</th>
<th>Terms</th>
<th>Lecturer</th>
<th>% exam</th>
<th>% CW</th>
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<tbody>
<tr>
<td>M4A2*</td>
<td>Fluid Dynamics 1</td>
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<td>M4A10*</td>
<td>Fluid Dynamics 2</td>
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<td>Professor P. Hall</td>
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<tr>
<td>M4A28*</td>
<td>Introduction to Geophysical Fluid Dynamics</td>
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<td>Dr P. Berloff</td>
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<td>M4A30</td>
<td>Hydrodynamic Stability</td>
<td>2</td>
<td>Dr M.S. Mughal</td>
<td>90</td>
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<tr>
<td>M4A32</td>
<td>Vortex Dynamics</td>
<td>2</td>
<td>Professor D. Crowdy</td>
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<tr>
<td>M4M7*</td>
<td>Asymptotic Analysis</td>
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<td>Professor X. Wu</td>
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<tr>
<td>M4PA48*</td>
<td>Dynamics of Games</td>
<td>1</td>
<td>Professor D. Turaev</td>
<td>40 (oral)</td>
<td>60</td>
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<tr>
<td>M4PA23*</td>
<td>Dynamical Systems</td>
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<td>Dr M. Rasmussen</td>
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<tr>
<td>M4PA24*</td>
<td>Bifurcation Theory</td>
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<td>M4PA36</td>
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<td>Professor J. Lamb</td>
<td>40 (oral)</td>
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<td>M4PA16*</td>
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<td>M4PA34*</td>
<td>Dynamics, Symmetry and Integrability</td>
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<td>Professor D. Holm</td>
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<tr>
<td>M4PA50*</td>
<td>Introduction to Riemann Surfaces and Conformal Dynamics</td>
<td>2</td>
<td>Dr F. Bianchi</td>
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#### FINANCE

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<tr>
<td>M4F22*</td>
<td>Mathematical Finance: An Introduction to Option Pricing</td>
<td>1</td>
<td>Professor N.H. Bingham</td>
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#### BIOLOGY

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<td>Mathematical Biology</td>
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<td>Dr N. Jones</td>
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<tr>
<td>M4A50*</td>
<td>Methods for Data Science</td>
<td>1</td>
<td>Dr C. Colijn</td>
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#### MATHEMATICAL PHYSICS

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<tr>
<td>M4A4*</td>
<td>Mathematical Physics 1: Quantum Mechanics</td>
<td>1</td>
<td>Dr E-M Graefe</td>
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<tr>
<td>M4A6*</td>
<td>Special Relativity and Electromagnetism</td>
<td>1</td>
<td>Dr G. Pruessner</td>
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<td>M4A29*</td>
<td>Theory of Complex Systems</td>
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<td>Professor H. Jensen</td>
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<tr>
<td>M4A52*</td>
<td>Quantum Mechanics II</td>
<td>2</td>
<td>Dr R. Barnett</td>
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### APPLIED PDEs, NUMERICAL ANALYSIS and COMPUTATION

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<td>Professor G Pavliot</td>
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<td>M4M3*</td>
<td>Introduction to Partial Differential Equations</td>
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<td>M4M11*</td>
<td>Function Spaces and Applications</td>
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<td>Professor Pierre Degond</td>
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<td>M4M12*</td>
<td>Advanced Topics in Partial Differential equations</td>
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<td>Professor P. Degond</td>
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<td>M4A47</td>
<td>Finite Elements: Numerical Analysis and Implementation</td>
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### PURE MATHEMATICS

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### ANALYSIS

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<td>M4P18*</td>
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<td>M4P62</td>
<td>Random Matrices</td>
<td>2</td>
<td>Dr I. Krasovsky</td>
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### GEOMETRY

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<td>Geometry 1: Algebraic Curves</td>
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<td>Riemannian Geometry</td>
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<td>Dr M. Taylor</td>
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<td>M4P52</td>
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<td>Dr G. F. Fernandes da Silva</td>
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<td>Dr S. A. Filippini</td>
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### ALGEBRA AND DISCRETE MATHEMATICS

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<td>Professor A. Skorobogatov</td>
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<td>Dr R. Kurinczuk</td>
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### PROJECT (Compulsory)

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OTHER MATHEMATICAL OPTIONS

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<td>Professor E.J. McCoy, Dr L.V. White</td>
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<td>M3C</td>
<td>High Performance Computing</td>
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<td>Professor N.H. Bingham</td>
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FOURTH YEAR MATHEMATICS SYLLABUSES

Most modules running in 2017-2018 will also be available in 2018-2019, although there can be no absolute guarantees.

APPLIED MATHEMATICS/MATHEMATICAL PHYSICS/NUMERICAL ANALYSIS

FLUIDS

M4A2* FLUID DYNAMICS 1

Professor A. Ruban
Term 1

This module is an introduction to the Fluid Dynamics. It will be followed by Fluid Dynamics 2 in Term 2.

Fluid Dynamics deals with the motion of liquids and gases. Being a subdivision of Continuum Mechanics the fluid dynamics does not deal with individual molecules. Instead an ‘averaged’ motion of the medium is of interest. Fluid dynamics is aimed at predicting the velocity, pressure and temperature fields in flows past rigid bodies. A theoretician achieves this goal by solving the governing Navier-Stokes equations. In this module a derivation of the Navier-Stokes equations will be presented, followed by description of various techniques to simplify and solve the equation with the purpose of describing the motion of fluids at different conditions.

Aims of this module:
To introduce students to fundamental concepts and notions used in fluid dynamics. To demonstrate how the governing equations of fluid motion are deduced, paying attention to the restriction on their applicability to real flows. Then a class of exact solutions to the Navier-Stokes equations will be presented. This will follow by a discussion of possible simplifications of the Navier-Stokes equations. The main attention will be a wide class of flows that may be treated as inviscid. To this category belong, for example, aerodynamic flows. Students will be introduced to theoretical methods to calculate inviscid flows past aerofoils and other aerodynamic bodies. They will be shown how the lift force produced by an aircraft wing may be calculated.

Content:
Exact Solutions of the Navier-Stokes Equations: Couette and Poiseuille flows. The flow between two coaxial cylinders. The flow above an impulsively started plate. Diffusion of a potential vortex.
Fluid Dynamics 2

Professor P. Hall
Term 2

Prerequisites: Fluid Dynamics 2 is a continuation of the module Fluid Dynamics 1 given in Term 1.

In Fluid Dynamics 1 the main attention was with exact solutions of the Navier-Stokes equations governing viscous fluid motion. The exact solutions are only possible in a limited number of situations when the shape of the body is rather simple. A traditional way of dealing with more realistic shapes (like aircraft wings) is to seek possible simplifications in the Navier-Stokes formulation. We shall start with the case when the internal viscosity of the fluid is very large, and the Navier-Stokes equations may be substituted by the Stokes equations. The latter are linear and allow for simple solutions in various situations. Then we shall consider the opposite limit of very small viscosity, which is characteristic, for example, of aerodynamic flows. In this cast the analysis of the flow past a rigid body (say, an aircraft wing) requires Prandtl's boundary-layer equations to be solved. These equations are parabolic, and in many situations may be reduced to ordinary differential equations. Solving the Prandtl equations allows us to calculate the viscous drag experienced by the bodies. The final part of the module will be devoted to the theory of separation of the boundary layer, known as Triple-Deck theory.

Aims of the module:
To introduce the students to various aspects of Viscous Fluid Dynamics, and to demonstrate the power (and beauty) of modern mathematical methods employed when analysing fluid flows. This includes the Method of Matched Asymptotic Expansions, which was put forward by Prandtl for the purpose of mathematical description of flows with small viscosity. Now this method is used in all branches of applied mathematics.

Content:
Dynamic and Geometric Similarity of fluid flows. Reynolds Number and Strouhal Number.
Large Reynolds Number Flows: the notion of singular perturbations. Method of matched asymptotic expansions. Prandtl’s boundary-layer equations. Prandtl’s hierarchical concept. Displacement thickness of the boundary layer and its influence on the flow outside the boundary layer.
Triple-Deck Theory: The notion of boundary-layer separation. Formulation of the triple-deck equations for a flow past a corner. Solution of the linearised problem (small corner angle case).

Introduction to Geophysical Fluid Dynamics

Dr P. Berloff
Term 2

This is an advanced-level fluid-dynamics course with geophysical flavours. The lectures target upper-level undergraduate and graduate students interested in the mathematics of planet Earth, and in the variety of motions and phenomena occurring in planetary atmospheres and oceans. The lectures are a mix of theory and applications. Take a look at the lecture notes to get some idea of the material:
http://wwwf.imperial.ac.uk/~pberloff/gfd lectures.html

Main topics
• Introduction and basics;
• Governing equations (continuity of mass, material tracer, momentum equations, equation of state, thermodynamic equation, spherical coordinates, basic approximations);
• Geostrophic dynamics (shallow-water model, potential vorticity conservation law, Rossby number expansion, geostrophic and hydrostatic balances, ageostrophic continuity, vorticity equation);
• Quasigeostrophic theory (two-layer model, potential vorticity conservation, continuous stratification, planetary geostrophy);
• **Ekman layers** (boundary-layer analysis, Ekman pumping);
• **Rossby waves** (general properties of waves, physical mechanism, energetics, reflections, mean-flow effect, two layer and continuously stratified models);
• **Hydrodynamic instabilities** (barotropic and baroclinic instabilities, necessary conditions, physical mechanisms, energy conversions, Eady and Phillips models);
• **Ageostrophic motions** (linearized shallow-water model, Poincare and Kelvin waves, equatorial waves, ENSO “delayed oscillator”, geostrophic adjustment, deep-water and stratified gravity waves);
• **Transport phenomena** (Stokes drift, turbulent diffusion);
• **Nonlinear dynamics and wave-mean flow interactions** (closure problem and eddy parameterization, triad interactions, Reynolds decomposition, integrals of motion, enstrophy equations, classical 3D turbulence, 2D turbulence, transformed Eulerian mean, Eliassen-Palm flux).

Suggested textbooks: *Introduction to geophysical fluid dynamics* (Cushman-Roisin); *Atmospheric and oceanic fluid dynamics* (Vallis); *Geophysical fluid dynamics* (Pedlosky); *Fundamentals of geophysical fluid dynamics* (McWilliams).

Prerequisites: Introductory fluid mechanics.

**M4A30 HYDRODYNAMIC STABILITY**

**Dr M.S. Mughal**

**Term 2**

*(New syllabus for 2017-18)*

This course is an introduction to the basic concepts and techniques of modern hydrodynamic stability theory. Many physical systems can become unstable, in that small disturbances to the basic state can amplify and significantly alter the initial state. The course introduces the basic theoretical concepts and analysis methods required to understand and predict hydrodynamic instabilities in a variety of fluid flows ranging from thin layers heated from below, to flow between rotating cylinders and boundary layers. The general concepts apply in many disparate fields a few of which are: oceanography, meteorology, physics, astrophysics, aerodynamics, combustion, laminar to turbulent transition, sand dune formation, river meandering.

Classical as well as more modern ideas and techniques will be covered to give students a broad background in the field. Some prior experience of the mathematical modelling of fluid problems (from e.g. M2AM or M3A2) is desirable.

Numerical methods for eigenvalue problems will also be covered and students expected to complete an assessed numerical assignment (involving MATLAB).

Topics covered will be a selection from the following list:

Basic concepts of linear stability theory, normal modes, dispersion relations, marginal stability, temporal/spatial instability.

Rayleigh-Benard instability. Formulation of the linearised stability problem. Boundary conditions for free-free, rigid-free and rigid-rigid problems. Some exact solutions and discussion of marginal stability properties and cell patterns.

Interfacial instabilities: Rayleigh-Taylor, Kelvin Helmholtz instabilities. Jet break up and capillary instabilities.


Linear and nonlinear Parabolised Stability Equations and Receptivity modelling.

Linear instability versus nonlinear instability. Weakly nonlinear theory, derivation of Ginzburg-Landau and Stuart-Landau equations. Local bifurcation theory.

**Recommended Background Reading:**
Charru, Hydrodynamic Instabilities, Cambridge Univ. Press

Drazin, Introduction to Hydrodynamic Stability, Cambridge Univ. Press

Huerre & Rossi, Hydrodynamic Instabilities in Open Flows, Cambridge Univ. Press

Schmid & Henningson, Stability and Transition in Shear Flows, Springer

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**M4A32 VORTEX DYNAMICS**

**Professor D. Crowdy**
**Term 2**

*Prerequisites:* A knowledge of basic applied mathematical methods is the only prerequisite. A basic knowledge of inviscid fluid dynamics (e.g. M3/4A2) is desirable but not required.

The module will focus on the mathematical study of the dynamics of vorticity in an ideal fluid in two and three dimensions. The module will be pitched in such a way that it will be of interest both to fluid dynamicists and as an application of various techniques in dynamical systems theory.

Fundamental properties of vorticity.
Helmholtz Laws and Kelvin's circulation theorem. Singular distributions of vorticity; Biot-Savart law.
Dynamics of line vortices in 2d and other geometries; dynamics of 2d vortex patches, contour dynamics.
Axisymmetric vortex rings. Dynamics of vortex filaments.
Stability problems.
Miscellaneous topics (effects of viscosity, applications to turbulence, applications in aerodynamics).

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**M4M7* ASYMPTOTIC ANALYSIS**

**Professor X. Wu**
**Term 1**


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**DYNAMICS**

**M4PA48* DYNAMICS OF GAMES**

**Prof D. Turaev**
**Term 1**

Recently there has been quite a lot of interest in modeling learning through studying the dynamics of games. The settings to which these models may be applied is wide-ranging, from ecology and sociology to business, as actively pursued by companies like Google. Examples include

(i) optimization of strategies of populations in ecology and biology
(ii) strategies of people in a competitive environment, like online auctions or (financial) markets.
(iii) learning models used by technology companies
This module is aimed at discussing a number of dynamical models in which learning evolves over time, and which have a game theoretic background. The module will take a dynamical systems perspective. Topics will include replicator dynamics and best response dynamics.

**M4PA23\* DYNAMICAL SYSTEMS**

**Dr M. Rasmussen**  
**Term 1**

The theory of Dynamical Systems is an important area of mathematics which aims at describing objects whose state changes over time. For instance, the solar system comprising the sun and all planets is a dynamical system, and dynamical systems can be found in many other areas such as finance, physics, biology and social sciences. This course provides a rigorous treatment of the foundations of discrete-time dynamical systems, which includes the following subjects:

- Periodic orbits
- Topological and symbolic dynamics
- Chaos theory
- Invariant manifolds
- Statistical properties of dynamical systems

**M4PA36 ERGODIC THEORY**

**Professor J. Lamb**  
**Term 2**

Ergodic theory has strong links to analysis, probability theory, (random and deterministic) dynamical systems, number theory, differential and difference equations and can be motivated from many different angles and applications. In contrast to topological dynamics, Ergodic theory focusses on a probabilistic description of dynamical systems, and hence, a proper background of probability and measure theory is required to understand even the basic material in ergodic theory. For this reason, the first part of the course will concentrate on a self-contained review of the required background; this can take up to three weeks and might be skipped if not necessary. The second part of the course will focus on selected topics in ergodic theory. The course will be organised as a reading course; there will weekly meetings, where selected material will be presented and discussed within the group; this will guide the independent study. The students will do a project in the second part of the course, which should be submitted by the end of the term, so that the project does not come into conflict with the exams. The project will count towards 60% of the mark. There will also be a thirty-minute regular oral exam, which consists of two parts, each of which will contribute 20% to the mark. The first part of the regular oral exam will concern a discussion about the project: the student will have five minutes time to explain the project, after which there will questions related to the project (up to ten minutes). The second half of the exam will consist of questions about the material of the course.

The core content of the course is given as follows:

2. Invariant measures and Krylov–Bogolubov Theorem,
3. Poincaré recurrence,
4. Ergodic theorems (such as Birkhoff Ergodic Theorem, Maximal Ergodic Theorem),
5. Decay of correlations,
6. Detailed discussion of examples (such as circle maps, maps with critical points, hyperbolic toral automorphisms, Bernoulli shifts),
7. Ergodicity via Fourier series
8. Mixing,
9. Markov chains and ergodicity/mixing of Markov measures,
M4PA24* BIFURCATION THEORY

Dr D. Turaev  
Term 2

This module serves as an introduction to bifurcation theory, concerning the study of how the behaviour of dynamical systems (ODEs, maps) changes when parameters are varied.

The following topics will be covered:

1) Bifurcations on a line and on a plane.
2) Centre manifold theorem; local bifurcations of equilibrium states.
3) Local bifurcations of periodic orbits – folds and cusps.
4) Homoclinic loops: cases with simple dynamics, Shilnikov chaos, Lorenz attractor.
5) Saddle-node bifurcations: destruction of a torus, intermittency, blue-sky catastrophe.
6) Routes to chaos and homoclinic tangency.

M4PA16* GEOMETRIC MECHANICS

Professor D. Holm  
Term 1

This module on geometric mechanics starts with Fermat's principle, that light rays follow geodesics determined from a least action variational principle. It then treats subsequent developments in mechanics by Newton, Euler, Lagrange, Hamilton, Lie, Poincaré, Noether, and Cartan, who all dealt with geometric optics.

The module will explicitly illustrate the following concepts of geometric mechanics:
* Configuration space, variational principles, Euler-Lagrange equations, geodesic curves,
* Legendre transformation, phase space, Hamilton’s canonical equations,
* Poisson brackets, Hamiltonian vector fields, symplectic transformations,
* Lie group symmetries, conservation laws, Lie algebras and their dual spaces,
* Divergence free vector fields, momentum maps and coadjoint motion.

All of these concepts from geometric mechanics will be illustrated with examples, first for Fermat’s principle and then again for three primary examples in classical mechanics: (1) motion on the sphere, (2) the rigid body and (3) pairs of n : m resonant oscillators.


M4PA34* DYNAMICS, SYMMETRY AND INTEGRABILITY

Professor D. Holm  
Term 2

The following topics will be covered:

* Introduction to smooth manifolds as configuration spaces for dynamics.
* Transformations of smooth manifolds as flows of smooth vector fields.
* Introduction to differential forms, wedge products and Lie derivatives.
* Adjoint and coadjoint actions of matrix Lie groups and matrix Lie algebras
* Action principles on matrix Lie algebras, their corresponding Euler-Poincaré ordinary differential equations and the Lie-Poisson Hamiltonian formulations of these equations.
* EPDiff: the Euler-Poincaré partial differential equation for smooth vector fields acting on smooth manifolds
* The Hamiltonian formulation of EPDiff: Its momentum maps and soliton solutions
* Integrability of EPDiff: Its bi-Hamiltonian structure, Lax pair and isospectral problem, as well as the relationships of these features to the corresponding properties of KdV.
**M4PA50**  INTRODUCTION TO RIEMANN SURFACES AND CONFORMAL DYNAMICS

Dr F. Bianchi  
Term 2

This elementary course starts with introducing surfaces that come from special group actions (Fuchsian / Kleinian groups). It turns out that on such surfaces one can develop a beautiful and powerful theory of iterations of conformal maps, related to the famous Julia and Mandelbrot sets. In this theory many parts of modern mathematics come together: geometry, analysis and combinatorics.


Syllabus:

Part 1: Discrete groups, complex Mobius transformations, Riemann surfaces, hyperbolic metrics, fundamental domains.


Recommended texts:

1) Kleinian Groups by Berbard Maskit,
2) The Geometry of Discrete Groups by Alan F. Beardon,
3) Dynamics in one complex variable by John Milnor,
4) Riemann surfaces, dynamics and geometry, lecture notes by Curtis McMullen.

**FINANCE**

**M4F22** MATHEMATICAL FINANCE: AN INTRODUCTION TO OPTION PRICING

Professor N.H. Bingham  
Term 1

*Prerequisites:* Differential Equations (M2AA1), Multivariable Calculus (M2AA2), Real Analysis (M2PM1) and Probability and Statistics 2 (M2S1).

The mathematical modeling of derivatives securities, initiated by the Louis Bachelier in 1900 and developed by Black, Scholes and Merton in the 1970s, focuses on the pricing and hedging of options, futures and other derivatives, using a probabilistic representation of market uncertainty. This module is a mathematical introduction to this theory, which uses a wide array of tools from stochastic analysis, which are covered in the module in a self-contained manner: Brownian motion, stochastic integration, Ito calculus and parabolic partial differential equations.

Outline:

Filtrations and information. Conditional expectation.
Brownian motion. Simulation of Brownian motion.
Gaussian properties Markov property, martingale property.
Relation with the heat equation. Feynman-Kac formula. Quadratic variation.
Bachelier's model. The Black-Scholes model.
The Ito stochastic integral: definition, properties. Ito processes.
Arbitrage strategies. Arbitrage-free markets.
The Ito formula. Applications of the Ito formula.
Stochastic exponentials. Lévy’s theorem.
Options and derivative securities. Call and put options.
Sensitivity analysis of an option: Delta, Gamma and Theta.
Arbitrage pricing of derivative securities: a one period example.
Dynamic hedging of options.
The Black-Scholes partial differential equation. Relation with heat equation.
The Black-Scholes formula.
Dynamic hedging in presence of uncertain volatility. Gamma exposure.

**BIOLOGY**

**M4A49** MATHEMATICAL BIOLOGY

Dr N. Jones
Term 1

The aim of the module is to describe the application of mathematical models to biological phenomena. A variety of contexts in human biology and diseases are considered, as well as problems typical of particular organisms and environments.

The syllabus includes topics from:

2. Epidemiology - the spread of plagues.
3. Reaction-Diffusion models: Turing mechanism for pattern formation. How the leopard got his spots (and sometimes stripes).
5. Mass transport; Taylor dispersion.
7. Other particular problems from biology.

**M4A50** METHODS FOR DATA SCIENCE

Dr C. Colijn
Term 1

This course is in two halves: machine learning and complex networks. We will begin with an introduction to the R language and to visualisation and exploratory data analysis. We will describe the mathematical challenges and ideas in learning from data. We will introduce unsupervised and supervised learning through theory and through application of commonly used methods (such as principle components analysis, k-nearest neighbours, support vector machines and others). Moving to complex networks, we will introduce key concepts of graph theory and discuss model graphs used to describe social and biological phenomena (including Erdos-Renyi graphs, small-world and scale-free networks). We will define basic metrics to characterise data-derived networks, and illustrate how networks can be a useful way to interpret data.
Quantum mechanics is one of the most successful theories in modern physics and has an exceptionally beautiful underlying mathematical structure. It provides the basis for many areas of contemporary physics, including atomic and molecular, condensed matter, high-energy particle physics, quantum information theory, and quantum cosmology, and has led to countless technological applications.

This module aims to provide an introduction to quantum phenomena and their mathematical description. Quantum theory combines tools and concepts from various areas of mathematics and physics, such as classical mechanics, linear algebra, probability theory, numerical methods, analysis and even geometry. However, most of the concepts are basic, and little background knowledge is required before we can put them to practical use.

Core topics: Hamiltonian dynamics; Schrödinger equation and wave functions; stationary states of one-dimensional systems; mathematical foundations of quantum mechanics; quantum dynamics; angular momentum

Additional optional topics may include: Approximation techniques; explicitly time-dependent systems; geometric phases; numerical techniques; many-particle systems; cold atoms; entanglement and quantum information.
This module will provide the basic foundation in terms of concepts and mathematical methodology needed to analyse and model complex systems.

1) Simple functional integration: to discuss the emergent vortex solutions in terms extremal configurations for the partition integral of the 2D XY model.
2) Record statistics and record dynamics: to discuss the statistics of intermittent slowly decelerating dynamics as observed in models of evolution and many other complex systems. Relations to extreme value statistics.
3) Branching processes: to present a mean field discussion of avalanche dynamics in models of complex systems such as the sand pile, forest fires and more recent models of fusions of banks.
4) The Kuramoto transition to synchronisation as an example of collective cooperative dynamical behaviour of potential relevance to brain dynamics.
5) Intermittency in low (non-linear maps) and high dimensional systems (e.g. Tangled Nature model) and relation to renormalisation theory (low dim.) and mean field stability analysis (high dim).

Assessment: Two mini projects.

M4A52* QUANTUM MECHANICS II
Dr R. Barnett
Term 2

Quantum mechanics is one of the most successful theories in modern physics and has an exceptionally beautiful underlying mathematical structure. It provides the basis for many areas of contemporary physics, including atomic and molecular, condensed matter, high-energy particle physics, quantum information theory, and quantum cosmology, and has led to countless technological applications. Quantum theory combines tools and concepts from various areas of mathematics and physics, such as classical mechanics, linear algebra, probability theory, numerical methods, analysis and even geometry. However, most of the concepts are basic, and little background knowledge is required before we can put them to practical use.

This module is intended to be a second course in quantum mechanics and will build on topics covered in Quantum Mechanics I.

In addition to the material below, this level 7 (Masters) version of the module will have additional extension material for self-study. This will require a deeper understanding of the subject than the corresponding level 6 (Bachelors) module.

Core topics: Quantum mechanics in three spatial dimensions and the hydrogen atom, the Heisenberg picture, perturbation theory, addition of spin, adiabatic processes and the geometric phase, Floquet-Bloch theory, second quantization and introduction to many-particle systems, Fermi and Bose statistics, quantum magnetism. Additional topics may include WKB theory and the Feynman path integral.

M4M6* METHODS OF MATHEMATICAL PHYSICS
Dr S. Olver
Term 1

Complex integration [revision]
Wiener-Hopf technique continued [principal part integral – definition and examples, analytic properties of fns defined via a Cauchy-type integral on finite smooth contours, investigation of limits – Plemelj formulae, Hilbert problem (possibly limited to inversion formula possibly extended to the general case), log kernels].
Orthogonal polynomials [polynomials solutions to 2nd order differential equations, orthogonality, Gramm-Schmidt process, Rodrigues formula, generating functions, recurrence relations, numerous examples, but special care taken of Hermite, Lagrange and Laguerre polynomials all needed for quantum mechanics].
Hypergeometric series [Gamma function – integral representation and basic properties, Hypergeometric series – definition, convergence, special values of argument, differential equations, possibly extended series, Barnes integral and analytic continuations, special case needed for physics].

**APPLIED PDEs, NUMERICAL METHODS and COMPUTATION**

**M4M3+ INTRODUCTION TO PARTIAL DIFFERENTIAL EQUATIONS**

Professor J. Carrillo  
Term 1

1. Basic concepts: PDEs, linearity, superposition principle. Boundary and Initial value problems.  
3. Linear and Quasilinear first order PDEs in two independent variables. Well-posedness for the Cauchy problem. The linear transport equation. Upwinding scheme for the discretization of the advection equation.  

**M4M11+ FUNCTION SPACES AND APPLICATIONS (Replaces M9)**

Professor P. Degond  
Term 1

The purpose of this course is to introduce the basic function spaces and to train the student into the basic methodologies needed to undertake the analysis of Partial Differential Equations and to prepare them for the course ‘Advanced topics in Partial Differential Equations’ where this framework will be applied. The course is designed as a stand-alone course. No background in topology or measure theory is needed as these concepts will be reviewed at the beginning of the course.

The course will span the basic aspects of modern functional spaces: integration theory, Banach spaces, spaces of differentiable functions and of integrable functions, convolution and regularization, compactness and Hilbert spaces. The concepts of Distributions, Fourier transforms and Sobolev spaces will be taught in the follow-up course “Advanced topics in Partial Differential Equations” as they are tightly connected to the resolution of elliptic PDE's and the material taught in the present course is already significant.

In addition to the material below, this Masters version of the module will have additional extension material for self-study. This will require a deeper understanding of the subject than the corresponding M3 module.

The syllabus of the course is as follows:  
1) Elements of metric topology  
2) Elements of Lebesgue’s integration theory.  

M4M12* ADVANCED TOPICS IN PARTIAL DIFFERENTIAL EQUATIONS (Replaces M8)

Professor P. Degond
Term 2

This course develops the analysis of boundary value problems for elliptic and parabolic PDE’s using the variational approach. It is a follow-up of ‘Function spaces and applications’ but is open to other students as well provided they have sufficient command of analysis. An introductory Partial Differential Equation course is not needed either, although certainly useful.

The course consists of three parts. The first part (divided in two chapters) develops further tools needed for the study of boundary value problem, namely distributions and Sobolev spaces. The following two parts are devoted to elliptic and parabolic equations on bounded domains. They present the variational approach and spectral theory of elliptic operators as well as their use in the existence theory for parabolic problems.

Extra reading about elliptic equations on the whole space case will be given. The aim of the course is to expose the students some important aspects of Partial Differential Equation theory, aspects that will be most useful to those who will further work with Partial Differential Equations be it on the Theoretical side or on the Numerical one.

The syllabus of the module is as follows:


M4A51 STOCHASTIC DIFFERENTIAL EQUATIONS (Replaces A42)

Professor G. Pavliotis
Term 1

This is a basic introductory course on the theory and applications of stochastic differential equations. The goal will be to present the basic theory of SDEs and, time permitting, to also present some specific applications such as stochastic optimal control, applications of SDEs to Partial differential equations etc.

The contents of the module are:

1) Modelling with SDEs, overview of applications.
2) Background on probability theory and the theory of stochastic processes.
3) Ito’s theory of stochastic integration and Ito’s formula.
4) Stochastic differential equations, basic theory including existence and uniqueness of solutions.
5) Ergodic properties of SDEs.
6) Connection between SDEs and the forward and backward Kolmogorov equations.
7) Applications including stochastic optimal control, Feynman-Kac formulas, etc.

(M3C HIGH PERFORMANCE COMPUTING – See later)

M4A44 COMPUTATIONAL STOCHASTIC PROCESSES

Dr S. Gomes
Term 2

Prerequisites: Some knowledge of stochastic processes, ODEs, PDEs, linear algebra, scientific computing, numerical analysis will be useful. Knowledge of Matlab or any other programming language.

Numerical methods for stochastic differential equations, weak and strong convergence, stability, numerical simulation of ergodic SDEs.
Statistical inference for diffusion processes, maximum likelihood, method moments. Markov Chain Monte Carlo, sampling from probability distributions

Applications: computational statistical mechanics, molecular dynamics

M4A47 FINITE ELEMENTS: NUMERICAL ANALYSIS AND IMPLEMENTATION.

Dr C Cotter and Dr D Ham
Term 2

Finite element methods form a flexible class of techniques for numerical solution of PDEs that are both accurate and efficient.

The finite element method is a core mathematical technique underpinning much of the development of simulation science. Applications are as diverse as the structural mechanics of buildings, the weather forecast, and pricing financial instruments. Finite element methods have a powerful mathematical abstraction based on the language of function spaces, inner products, norms and operators.

This module aims to develop a deep understanding of the finite element method by spanning both its analysis and implementation. In the analysis part of the module you will employ the mathematical abstractions of the finite element method to analyse the existence, stability, and accuracy of numerical solutions to PDEs. At the same time, in the implementation part of the module you will combine these abstractions with modern software engineering tools to create and understand a computer implementation of the finite element method.

Syllabus:

• Basic concepts: Weak formulation of boundary value problems, Ritz-Galerkin approximation, error estimates, piecewise polynomial spaces, local estimates.
• Efficient construction of finite element spaces in one dimension, 1D quadrature, global assembly of mass matrix and Laplace matrix.
• Construction of a finite element space: Ciarlet’s finite element, various element types, finite element
interpolants.

- Construction of local bases for finite elements, efficient local assembly.
- Sobolev Spaces: generalised derivatives, Sobolev norms and spaces, Sobolev's inequality.
- Numerical quadrature on simplices. Employing the pullback to integrate on a reference element.
- Computational meshes: meshes as graphs of topological entities. Discrete function spaces on meshes, local and global numbering.
- Global assembly for Poisson equation, implementation of boundary conditions. General approach for nonlinear elliptic PDEs.
- Variational problems: Poisson's equation, variational approximation of Poisson's equation, elliptic regularity estimates, general second-order elliptic operators and their variational approximation.
- Residual form, the Gâteaux derivative and techniques for nonlinear problems.

The course is assessed 50% by examination and 50% by coursework (implementation exercise in Python).

M4N7* NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

Dr I. Shevchenko
Term 1

An analysis of methods for solving ordinary differential equations. Totally examined by project.

Error estimation and automatic step control. Introduction to stiffness.
Boundary and eigenvalue problems. Solution by shooting and finite difference methods.
Introduction to deferred and defect correction.

M4N10* COMPUTATIONAL PARTIAL DIFFERENTIAL EQUATIONS

Professor J. Mestel
Term 2

The module will introduce a variety of computational approaches for solving partial differential equations, focusing mostly on finite difference methods, but also touching on finite volume and spectral methods. Students will gain experience implementing the methods and writing/modifying short programs in Matlab or other programming language of their choice. Applications will be drawn from problems arising in Mathematical Biology, Fluid Dynamics, etc. At the end of the module, students should be able to solve research-level problems by combining various techniques. Assessment will be by projects, probably 3 in total. The first project will only count for 10% and will be returned quickly with comments, before students become committed to completing the module. Typically, the projects will build upon each other, so that by the end of the module a research level problem may be tackled. Matlab codes will be provided to illustrate similar problems and techniques, but these will require modification before they can be applied to the projects. The use of any reasonable computer language is permitted.

Topics (as time permits).


- Solvers for elliptic problems: direct and iterative solvers, Jacobi and Gauss-Seidel method and convergence analysis; geometric multigrid method.

- Methods for the heat equation: explicit versus implicit schemes; stiffness.
- Techniques for the wave equation: finite-difference solution, characteristic formulation, non-reflecting boundary conditions, one-way wave equations, perfectly matched layers. Lax-Friedrichs, Lax-Wendroff, upwind and semi-Lagrangian advection schemes.

- Domain decomposition for elliptic equations: overlapping alternating Schwarz method and convergence analysis, non-overlapping methods.

**M4SC* SCIENTIFIC COMPUTATION**

**TBA**

**Term 2**

Scientific computing is an important skill for any mathematician. It requires both knowledge of algorithms and proficiency in a scientific programming language. The aim of this module is to expose students from a varied mathematical background to efficient algorithms to solve mathematical problems using computation.

The objectives are that by the end of the module all students should have a good familiarity with the essential elements of the Python programming language, and be able to undertake programming tasks in a range of common areas (see below).

There will be four sub-modules: 1. A PDE-module covering elementary methods for the solution of time-dependent problems. 2. An optimization-module covering discrete and derivative-free algorithms. 3. A pattern-recognition-module covering searching and matching methods. 4. A statistics-module covering, e.g., Monte-Carlo techniques.

Each module will consist of a brief introduction to the underlying algorithm, its implementation in the python programming language, and an application to real-life situations.

**M4N9* COMPUTATIONAL LINEAR ALGEBRA**

**Dr E. Keaveny**

**Term 1**

Examined solely by project. Competence in MATLAB is a prerequisite.

Whether it be statistics, mathematical finance, or applied mathematics, the numerical implementation of many of the theories arising in these fields relies on solving a system of linear equations, and often doing so as quickly as possible to obtain a useful result in a reasonable time. This course explores the different methods used to solve linear systems (as well as perform other linear algebra computations) and has equal emphasis on mathematical analysis and practical applications.

Topics include:
1. Direct methods: Triangular and banded matrices, Gauss elimination, LU-decomposition, conditioning and finite-precision arithmetic, pivoting, Cholesky factorisation, QR factorisation.
2. Symmetric eigenvalue problem: power method and variants, Jacobi's method, Householder reduction to tridiagonal form, eigenvalues of tridiagonal matrices, the QR method
3. Iterative methods:
   (a) Classic iterative methods: Richardson, Jacobi, Gauss - Seidel, SOR
   (b) Krylov subspace methods: Lanczos method and Arnoldi iteration, conjugate gradient method, GMRES, preconditioning.

**PURE MATHEMATICS**

**PURE MATHEMATICS STUDY GROUP**
Various lecturers  
Terms 1 and 2

This is a non-examined, not-for-credit module. It will consist of a mixture of independent study and discussion groups, together with lectures delivered by students or staff. The choice of topics will complement that available in taught modules and will be determined by students in discussion with and under the guidance of a member of staff.

ANALYSIS

M4P6* PROBABILITY THEORY
Prof B. Zegarlinski  
Term 2

Prerequisites: Measure and Integration (M3/4P19, Term 1)

A rigorous approach to the fundamental properties of probability.


M4P7* FUNCTIONAL ANALYSIS
Prof B. Zegarlinski  
Term 2

This module brings together ideas of continuity and linear algebra. It concerns vector spaces with a distance, and involves linear maps; the vector spaces are often spaces of functions.


M4P18* FOURIER ANALYSIS AND THEORY OF DISTRIBUTIONS
Prof. M. Ruzhansky  
Term 2

Spaces of test functions and distributions, Fourier Transform (discrete and continuous), Bessel’s, Parseval’s Theorems, Laplace transform of a distribution, Solution of classical PDE’s via Fourier transform, Basic Sobolev Inequalities, Sobolev spaces.

M4P19* MEASURE AND INTEGRATION
Prof M. Ruzhansky  
Term 1

M4P60* GEOMETRIC COMPLEX ANALYSIS

Dr D. Cheraghi
Term 2

Complex analysis is the study of the functions of complex numbers. It is employed in a wide range of topics, including dynamical systems, algebraic geometry, number theory, and quantum field theory, to name a few. On the other hand, as the separate real and imaginary parts of any analytic function satisfy the Laplace equation, complex analysis is widely employed in the study of two-dimensional problems in physics such as hydrodynamics, thermodynamics, Ferromagnetism, and percolations.

While you become familiar with basics of functions of a complex variable in the complex analysis course, here we look at the subject from a more geometric viewpoint. We shall look at geometric notions associated with domains in the plane and their boundaries, and how they are transformed under holomorphic mappings. In turn, the behavior of conformal maps is highly dependent on the shape of their domain of definition. Below is a rough guide to the syllabus.

Part 1) Elements of holomorphic mappings: Poincare metric, Schwarz-Pick lemma, Riemann mapping theorem, growth and distortion estimates, normal families, canonical mappings of multiply connected regions.


Part 3) Elements of quasi-conformal mappings and elliptic PDEs, Beltrami equation, singular integral operators, measurable Riemann mapping theorem.

M4P41 ANALYTIC METHODS IN PARTIAL DIFFERENTIAL EQUATIONS

Dr G. Holzegel
Term 2

The main object of this module is to introduce several fundamental techniques of analysis for the study of partial differential equations.

The topic will include Fourier analysis, distributions, differential operators, pseudo-differential operators. There will be a review of Sobolev spaces, embedding theorems, potentials. We will apply it to study L2 properties, almost orthogonality, and the regularity of wave (hyperbolic) equations as well as elliptic and parabolic equations.

M4P67 Stochastic Calculus with Applications to Nonlinear Filtering (replaces P47)

Professor D. Crisan
Term 2

Prerequisites: Ordinary differential equations, partial differential equations, real analysis, probability theory.

The course offers a bespoke introduction to stochastic calculus required to cover the classical theoretical results of nonlinear filtering as well as some modern numerical methods for solving the filtering problem. The first part of the course will equip the students with the necessary knowledge (e.g., Ito Calculus, Stochastic Integration by Parts, Girsanov's theorem) and skills (solving linear stochastic differential equation, analysing continuous martingales, etc) to handle a variety of applications. The focus will be on the use of stochastic calculus to the theory and numerical solution of nonlinear filtering.

1. Martingales on Continuous Time (Doob Meyer decomposition, L_p bounds, Brownian motion, exponential martingales, semi-martingales, local martingales, Novikov's condition)
2. Stochastic Calculus (Itô's isometry, chain rule, integration by parts)
3. Stochastic Differential Equations (well posedness, linear SDEs, the Ornstein-Uhlenbeck process, Girsanov's Theorem)
4. Stochastic Filtering (definition, mathematical model for the signal process and the observation process)
5. The Filtering Equations (well-posedness, the innovation process, the Kalman-Bucy filter)

M4P62  RANDOM MATRICES

Dr I. Krasovsky
Term 2

Prerequisite: Measure and Integration; Corequisite: Probability.


Literature:

The course is an introduction to the theory of random matrices. Generally, a random matrix is a finite dimensional matrix whose elements are random variables. The theory aims to describe properties of the spectrum of such matrices in the limit when their dimension is large. Foundations of the theory were laid in 1950-60's, but it remains a very active research area. We will discuss the basics of the theory and some of the recent results.

Random matrices were invented as a model to describe large systems of particles whose precise law of interaction is unknown and can be considered random. They have therefore applications in several branches of physics. Random matrices also have intriguing conjectural connections to Riemann's zeta-function, these will be mentioned in the course.

GEOMETRY

M4P5*  GEOMETRY OF CURVES AND SURFACES

Dr S. Sivek
Term 1

The main object of this module is to understand what is the curvature of a surface in 3-dimensional space.

Topological surfaces: Definition of an atlas; the prototype definition of a surface; examples. The topology of a surface; the Hausdorff condition, the genuine definition of a surface. Orientability, compactness. Subdivisions and the Euler characteristic. Cut-and-paste technique, the classification of compact surfaces. Connected sums of surfaces. Smooth surfaces: Definition of a smooth atlas, a smooth surface and of smooth maps into and out of smooth surfaces. Surfaces in $\mathbb{R}^3$, tangents, normals and orientability. The first fundamental form, lengths and areas, isometries. The second fundamental form, principal curvatures and directions. The definition of a geodesic, existence and uniqueness, geodesics and co-ordinates. Gaussian curvature, definition and geometric interpretation, Gauss curvature is intrinsic, surfaces with constant Gauss curvature. The Gauss-Bonnet theorem. (Not examinable and in brief) Abstract Riemannian surfaces, metrics.
Mean curvature and minimal surfaces, including the definition of mean curvature, its geometric interpretation, the definition of minimal surfaces and some examples.

**M4P20* GEOMETRY 1: ALGEBRAIC CURVES**

Dr C. Manolache  
Term 1

Plane algebraic curves; Projective spaces; Projective curves; Smooth cubics and the group structure; Intersection of projective curves.  
Genus of a curve (Riemann surfaces); Meromorphic differentials and Abel's theorem.

**M4P21* GEOMETRY 2: ALGEBRAIC TOPOLOGY**

Dr J. Nicaise  
Term 2

Homotopies of maps and spaces. Fundamental group. Covering spaces, Van Kampen (only sketch of proof).  
Homology: singular and simplicial (following Hatcher's notion of Delta-complex). Mayer-Vietoris (sketch proof) and long exact sequence of a pair. Calculations on topological surfaces. Brouwer fixed point theorem.

**M4P33 ALGEBRAIC GEOMETRY**

Dr M. Orr  
Term 2

Pre-requisites: M4P55 Commutative Algebra

Algebraic geometry is the study of the space of solutions to polynomial equations in several variables. In this course, you will learn to use algebraic and geometric ideas together, studying some of the basic concepts from both perspectives and applying them to numerous examples.

Affine varieties, projective varieties. The Nullstellensatz.  
Regular and rational maps between varieties. Completeness of projective varieties.  
Dimension. Regular and singular points.  
Examples of algebraic varieties.

**M4P51 RIEMANNIAN GEOMETRY**

Dr M. Taylor  
Term 2

*Prerequisites: Geometry of Curves and Surfaces (M4/4P5) and Manifolds (M4P52).*

The main aim of this module is to understand geodesics and curvature and the relationship between them. Using these ideas we will show how local geometric conditions can lead to global topological constraints.


**M4P52 MANIFOLDS**

Dr G. F. Fernandes da Silva  
Term 1
Smooth manifolds, quotients, smooth maps, submanifolds, rank of a smooth map, tangent spaces, vector fields, vector bundles, differential forms, the exterior derivative, orientations, integration on manifolds (with boundary) and Stokes’ Theorem. This module focuses on foundations as well as examples.

M4P54  DIFFERENTIAL TOPOLOGY

Dr S.A. Filippini
Term 2

Prerequisites: Fundamental group and covering spaces from Algebraic Topology (M4P21) and vector fields and differential forms, derivatives and pull-backs of smooth maps, exterior differentiation and integration from Manifolds (M4P52).

Differential topology is concerned with the topology of smooth manifolds. The first part of the module deals with de Rham cohomology, a form of cohomology defined in terms of differential forms. We will prove the Mayer-Vietoris exact sequence, Künneth formula and Poincaré duality in this context, and discuss degrees of maps between manifolds. The second part of the module introduces singular homology and cohomology, the relation to de Rham cohomology via de Rham’s theorem, and the general form of Poincaré duality. Time permitting, there will also be a brief introduction to Morse theory.

M4P57  COMPLEX MANIFOLDS

Dr Spicer
Term 2

Prerequisite: Manifolds (M4P52). Some useful overlap with Differential Topology (M4P54).

Complex and almost complex manifolds, integrability. Examples such as the Hopf manifold, projective space, projective varieties. Hermitian metrics, Chern connection. Various equivalent formulations of the Kaehler condition. Hodge decomposition for Kaehler manifolds. Line bundles and Kodaira embedding. Statement of GAGA. Basic Kodaira-Spencer deformation theory.

ALGEBRA AND DISCRETE MATHEMATICS

M4P8*  ALGEBRA 3

Dr A. Skorobogatov
Term 1


M4P10* GROUP THEORY

Prof A. Ivanov
Term 1
An introduction to some of the more advanced topics in the theory of groups.

Composition series, Jordan-Hölder theorem, Sylow’s theorems, nilpotent and soluble groups. Permutation groups. Types of simple groups.

**M4P11* GALOIS THEORY**

**Professor A. Corti**
**Term 2**

The formula for the solution to a quadratic equation is well-known. There are similar formulae for cubic and quartic equations but no formula is possible for quintics. The module explains why this happens.


**M4P12* GROUP REPRESENTATION THEORY**

**Dr T. Schedler**
**Term 2**

Representations of groups: definitions and basic properties. Maschke’s theorem, Schur’s lemma. Representations of abelian groups. Tensor products of representations.

The character of a group representation. Class functions. Character tables and orthogonality relations.


Representations of quivers.

**M4P17* ALGEBRAIC COMBINATORICS**

**Professor M.W. Liebeck**
**Term 1**

An introduction to a variety of combinatorial techniques that have wide applications to other areas of mathematics.


Strongly regular graphs: examples, basic theory, and relationship with codes and designs.

The Mathieu group and their relationship with codes and strongly regular graphs.

**M4P46 LIE ALGEBRAS**

**Dr A. Pal**
**Term 2**


**M4P55 COMMUTATIVE ALGEBRA**
Dr J-S. Koskivirta  
Term 1


M4P61 INFINITE GROUPS  
Dr J. Britnell  
Term 1

Free groups. Group presentations, Tietze transformations, the word problem. Residually finite groups. Cayley graphs, actions on graphs, the Nielsen–Schreier Theorem. Free products, the Table-Tennis Lemma, amalgams. HNN extensions, the Higman Embedding Theorem, the Novikov–Boone Theorem. Geometry of groups, hyperbolic groups.

M4P63 ALGEBRA IV  
Prof. A. Skorobogatov  
Term 1

This course is a selection of topics in advanced algebra. It will be useful for the students who want to specialise in algebra, number theory, geometry or topology.

Co-requisites: Algebra 3 (M3P8) and Galois Theory (M3P11). Group Theory (M3P10) and Group Representations (M3P12) will be useful but are not obligatory.

Projective, injective and flat modules.

Modules over principal ideal domains.

Abelian categories, resolutions and derived functors.

Group homology and cohomology.

M4P65* MATHEMATICAL LOGIC  
Prof D. M. Evans  
Term 1

The module is concerned with some of the foundational issues of mathematics. In propositional and predicate logic, we analyse the way in which we reason formally about mathematical structures. In set theory, we will look at the ZFC axioms and use these to develop the notion of cardinality. These topics have applications to other areas of mathematics: formal logic has applications via model theory and ZFC provides an essential toolkit for handling infinite objects.

In addition to the material below, this M4 version of the module will have additional extension material for self-study. This will require a deeper understanding of the subject than the corresponding M3 module.

Propositional logic: Formulas and logical validity; a formal system; soundness and completeness.

Predicate logic: First-order languages and structures; satisfaction and truth of formulas; the formal system; Goedel’s completeness theorem; the compactness theorem; the Loewenheim-Skolem theorem.

Set theory: The axioms of ZF set theory; ordinals; cardinality; the Axiom of Choice.

NUMBER THEORY
M4P14* NUMBER THEORY

Prof T. Gee
Term 1

The module is concerned with properties of natural numbers, and in particular of prime numbers, which can be proved by elementary methods.
Arithmetic functions, multiplicative functions, perfect numbers, Möbius inversion, Dirichlet Convolution.
Primitive roots, Gauss's theorem, indices.
Quadratic residues, Euler's criterion, Gauss's lemma, law of quadratic reciprocity, Jacobi symbol.
Sums of squares. Distribution of quadratic residues and non-residues.
Irrationality, Liouville's theorem, construction of a transcendental number.
Diophantine equations. Pell's equation, Thue's Theorem, Mordell's equation.

M4P15* ALGEBRAIC NUMBER THEORY

Dr A. Pal
Term 2

An introduction to algebraic number theory, with emphasis on quadratic fields. In such fields the familiar unique factorisation enjoyed by the integers may fail, but the extent of the failure is measured by the class group.
The following topics will be treated with an emphasis on quadratic fields $\mathbb{Q}(\sqrt{d})$.
Field extensions, minimum polynomial, algebraic numbers, conjugates and discriminants, Gaussian integers, algebraic integers, integral basis, quadratic fields, cyclotomic fields, norm of an algebraic number, existence of factorisation.
Factorisation in $\mathbb{Q}(\sqrt{d})$: Ideals, $\mathbb{Z}$-basis, maximal ideals, prime ideals, unique factorisation theorem of ideals and consequences, relationship between factorisation of numbers and of ideals, norm of an ideal. Ideal classes, finiteness of class number, computations of class number.
Fractional ideals, Minkowski’s theorem on linear forms, Ramification, characterisation of units of cyclotomic fields, a special case of Fermat’s last theorem.

M4P32 NUMBER THEORY: ELLIPTIC CURVES

Prof T. Gee
Term 1


M4P58 MODULAR FORMS

Dr R. Kurinczuk
Term 1

Non-examinable: relation to elliptic curves. Fermat's Last Theorem.
STATISTICS

M4S1* STATISTICAL THEORY

TBA
Term 2

This module deals with the criteria and the theoretical results necessary to develop and evaluate optimum statistical procedures in hypothesis testing, point and interval estimation.

Theories of estimation and hypothesis testing, including sufficiency, completeness, exponential families, minimum variance unbiased estimators, Cramér-Rao lower bound, maximum likelihood estimation, Rao-Blackwell and Neyman-Pearson results, and likelihood ratio tests as well as elementary decision theory and Bayesian estimation.

M4S2* STATISTICAL MODELLING 2

TBA
Term 2

Prerequisites: This module leads on from the linear models covered in M2S2 and Probability and Statistics 2 covered in M2S1.

The Generalised Linear Model is introduced from a theoretical and practical viewpoint and various aspects are explained.


The R statistical package will be used to expose how the different models can be applied on example data.

M4S4* APPLIED PROBABILITY

Dr A. Veraart
Term 1

This module aims to give students an understanding of the basics of stochastic processes. The theory of different kinds of processes will be described, and will be illustrated by applications in several areas. The groundwork will be laid for further deep work, especially in such areas as genetics, finance, industrial applications, and medicine.


M4S8* TIME SERIES

32
An introduction to the analysis of time series (series of observations, usually evolving in time) is given, which gives weight to both the time domain and frequency domain viewpoints. Important structural features (e.g. reversibility) are discussed, and useful computational algorithms and approaches are introduced. The module is self-contained.


M4S9* STOCHASTIC SIMULATION

TBA

Term 1

Prerequisites: Material from M2S1 would form a firm foundation.

Computational techniques have become an important element of modern statistics (for example for testing new estimation methods and with notable applications in biology and finance). The aim of this module is to provide an up-to-date view of such simulation methods, covering areas from basic random variate generation to Monte Carlo methodology. The implementation of stochastic simulation algorithms will be carried out in R, a language that is widely used for statistical computing and well suited to scientific programming.


M4S11* GAMES, RISKS AND DECISIONS

Dr L.V. White

Term 1

Simple probabilistic and mathematical tools are used to study the theory of games where two opponents are in conflict, including the celebrated Prisoners' Dilemma problem. Utilities, based on (apparently) reasonable axioms are introduced, leading to a study of decision theory.


n-person cooperative games, coalitions and characteristic functions, imputations, the core of a game, Shapley values.

M4S14* SURVIVAL MODELS AND ACTUARIAL APPLICATIONS

Prof A. Gandy

Term 2

Survival models are fundamental to actuarial work, as well as being a key concept in medical statistics. This module will introduce the ideas, placing particular emphasis on actuarial applications.

Explain concepts of survival models, right and left censored and randomly censored data. Introduce life table data and expectation of life.

M4S16* CREDIT SCORING
Dr A. Bellotti
Term 1

Prerequisites: Statistical Modelling 1 (M2S2) with some dependency on Statistical Modelling 2 (M3S2).

This course introduces the fundamentals of credit scoring and predictive analytics. We cover the aims and objectives of scoring, along with legislative and commercial aspects. We consider issues regarding consumer credit card: characteristics, transformations, data quality and transaction types. The concept of a statistical scorecard is introduced and models developed using logistic regression, Naive Bayes and decision tree methods. Application and behavioural model types and characteristics, including segmented models are explored. Basic methods of model selection, estimation and testing are considered, along with issues of selection bias and reject inference. Probability of default (PD) models are introduced, along with probability calibration and cost-based measures for model assessment. The R statistical package will be used to explore credit scoring models on example data.

M4S17* QUANTITATIVE METHODS IN RETAIL FINANCE
Dr A. Bellotti
Term 2

Prerequisites: Essential - Credit Scoring 1 (M3/4S16). Useful – Statistical Modelling 2 (M3S2).

This course explores advanced and new methods in retail finance, dealing with statistical modelling and optimization problems. Core topics will be: behavioural models, profitability, fraud detection and regulatory requirements. Specific topic areas are:- Survival models for credit scoring to determine time to default and include time varying information. Roll-rate and Markov transition models to determine patterns of missed payments. Mover-Stayer models of behaviour. Profit estimation: concepts and use of behavioural models. Setting optimal credit limits. Fraud detection Neural networks for fraud detection. Back-propagation and gradient descent methods. Cost analysis of AUC and the H measure. Expected Loss, PD, EAD and LGD models (using beta regression, Tobit and classification tree structures). Regulation and portfolio-level analysis. Capital requirements. One-factor Merton-type model. Asset correlation and mixed effects panel models. The R statistical package will be used to explore a topic from the course based on a retail finance data set.

M4S18++ TOPICS IN ADVANCED STATISTICS
Dr E. Cohen, Dr D. Mortlock, Prof A. Walden, Dr B. Calderhead
Term 2

This is a demanding module comprising a choice of two from four already existing half-modules from the MSc in Statistics, one from pool A and one from pool B.

Note for example that the choice A1 & B1 will be assessed by 2 x 90 minute exams, and A2 & B1 by a 90 minute exam (B1) and continuous assessment (A2).

Pool A:
M4S18A1: Multivariate Analysis (Dr Cohen) [90 minute exam]

M4S18A2: Machine Learning (Dr Calderhead) [Continuous assessment through coursework]
Introduction to statistical pattern recognition and machine learning. Methods for feature extraction, dimensionality reduction, data clustering and pattern classification. State-of-art approaches such as support vector machines and ensemble learning methods. Real-world applications to real data sets.

Pool B:
M4S18B1: Graphical Models (Prof Walden) [90 minute exam]
Graphical modelling for both (a) a vector of random variables, and (b) vector-valued time series. Conditional independence. Dependence structure and graphical representation. Markov properties. Conditional independence graphs. Decomposable models. Graphical Gaussian models. Model selection. Directed acyclic graphs (DAGs), Bayesian networks. Graphical modelling of time series (model selection, Kullback-Leibler approach). *(Some prior knowledge of time series analysis would be helpful for part (b), the last section.)*

M4S18B2: Bayesian Data Analysis (Dr Mortlock) [90 minute exam + Coursework]
Probability (definitions and interpretations); Bayes’s theorem and the law of total probability; parameter estimation; multi-level models; model comparison; hypothesis testing; information theory; the maximum entropy principle; and experimental design.

OTHER “NON-MATHEMATICAL” MATHEMATICS MODULES

The modules M3H/M3B/M3C/M3T may each be taken as an alternative to a Centre for Co-Curricular Studies/Business School option. However, as Department of Mathematics modules, their ECTS value is 8.

M3H HISTORY OF MATHEMATICS

Prof N.H. Bingham
Term 2
The aim of this module is to give an overview of the development of mathematics from ancient to modern times.

The ancient world.
Prehistory. Egypt; Mesopotamia. The Greeks: from Pythagoras through Euclid and Archimedes to Apollonius.
The Middle Ages.
The Arabs. India and China.
The beginnings of mathematics in Europe; the Renaissance; Copernicus and Galileo.
The modern world.
The Scientific Revolution; Newton, Leibniz and their followers.
The Enlightenment. Academies and universities.
The 18th C.; the Bernoullis, Euler, Lagrange. The Napoleonic period: Laplace, Gauss.
The 19th C.: Cauchy, Riemann, Cantor; Poincaré and Hilbert.
The 20th C., up to 1950: the development of modern algebra, analysis, probability, statistics, applied mathematics.

M3B THE MATHEMATICS OF BUSINESS AND ECONOMICS

TBA
Term 2

This module aims to:
Give a broad mathematical introduction to both microeconomics and macroeconomics
Consider the motivations and optimal behaviours of both firms and consumers in the marketplace, and show how this leads to the widely observed laws of supply and demand
Look at the interaction of firms and consumers in markets of varying levels of competition and further consider the roles of both, as well as the government, in a macroeconomic setting

Syllabus:
Theory of the firm
Profit maximisation for a competitive firm. Cost minimisation. Geometry of costs. Profit maximisation for a non-competitive firm.
Theory of the consumer
Consumer preferences and utility maximisation. The Slutsky equation.
Levels of competition in a market
Macroeconomic theory
Circular flow of income. Aggregate supply & demand. The multiplier effect.

M3C INTRODUCTION TO HIGH PERFORMANCE SCIENTIFIC COMPUTING

Dr P. Ray
Term 1

High-performance computing centres on the solution of large-scale problems that require substantial computational power. This will be a practical module that introduces a range of powerful tools that can be used to efficiently solve such problems. By the end of the module, which will be examined by projects, students will be prepared to tackle research problems using the tools of modern high-performance scientific computing in an informed, effective, and efficient manner.

Contents:
Getting started: working with UNIX at the command line
Software version control with git and Bitbucket
Programming and scientific computing with Python
Modular programming with modern Fortran, using scientific libraries, interfacing Python and Fortran
OpenMP (with Fortran) for parallel programming of shared-memory computers
MPI (with Fortran) for programming on distributed-memory machines such as clusters
Cloud computing
Good programming practice: planning, unit testing, debugging, validation (to be integrated with the above topics and the programming assignments.)

M3T Communicating Mathematics

Professor E.J. McCoy, Dr L.V. White
(Terms 2 & 3)
(Note: only G104 students who have already registered for it may take this module in their 4th year)

This module will give students the opportunity to observe and assist with teaching of Mathematics in local schools. Entry to the module is by interview in the preceding June and numbers will be limited. It is required for anyone on the Mathematics with Education degree coding.

For those selected there will follow a one day training course in presentation skills and other aspects of teaching. Students will be assigned to a school where they will spend ten half days in Term 2, under the supervision of a teacher. Assessment will be based on a portfolio of activities in the school, a special project, evaluation by the
school teacher and an oral presentation.

CENTRE FOR LANGUAGE, CULTURE AND COMMUNICATION/BUSINESS SCHOOL

Students may consider broadening their study programme by taking advantage of the CLCC/Business School provision.

Note that Centre for Co-Curricular Studies modules extend throughout Terms 1 and 2 and some modules may be examined in January. Taking the HSCS3006 Humanities Project normally also requires explicit permission from the Centre for Co-Curricular Studies.

<table>
<thead>
<tr>
<th>Module Codes</th>
<th>Module Titles</th>
<th>Terms</th>
<th>ECTS Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>HGC31</td>
<td>Lessons from History</td>
<td>1+2</td>
<td>6</td>
</tr>
<tr>
<td>HGC32</td>
<td>Global Challenges Independent Project</td>
<td>1+2</td>
<td>6</td>
</tr>
<tr>
<td>HSCS3001</td>
<td>Advanced Creative Writing</td>
<td>1+2</td>
<td>6</td>
</tr>
<tr>
<td>HSCS3002</td>
<td>History of Science, Technology and Industry</td>
<td>1+2</td>
<td>6</td>
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<tr>
<td>HSCS3003</td>
<td>Philosophy of Mind</td>
<td>1+2</td>
<td>6</td>
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<tr>
<td>HSCS3004</td>
<td>Contemporary Philosophy</td>
<td>1+2</td>
<td>6</td>
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<tr>
<td>HSCS3006</td>
<td>Humanities Project</td>
<td>1+2</td>
<td>6</td>
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<tr>
<td>HSCS3007</td>
<td>Conflict, Crime and Justice</td>
<td>1+2</td>
<td>6</td>
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<tr>
<td>HSCS3008</td>
<td>Visual Culture, Knowledge and Power</td>
<td>1+2</td>
<td>6</td>
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<tr>
<td>HSCS3009</td>
<td>Music and Western Civilization</td>
<td>1+2</td>
<td>6</td>
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<tr>
<td>HSCS2007</td>
<td>Music Technology</td>
<td>1+2</td>
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<tr>
<td>BS0808</td>
<td>Finance and Financial Management</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>BS0820</td>
<td>Managing Innovation</td>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

MSci students may take at most one option from the collection of these modules and M3B/C/E/H/T in each of their Third and Fourth Years.

Syllabus and timetabling information can be viewed online at:

**CLCC:** [http://www.imperial.ac.uk/horizons](http://www.imperial.ac.uk/horizons)


Note that places in CLCC and Business School modules are normally limited and registration should be done separately via the Centre for Co-Curricular Studies and Business School websites.

Note that a change in degree code registration can lead to your registration for a BPES code being revoked. This is an unfortunate side-effect of how the Business School runs things. Save a screenshot of your registration to help in any dispute.

Subject to the Department’s approval, in addition to the Centre for Co-Curricular Studies/Business School options, students may take a Mathematical module given outside the Department, e.g. in the Department of Physics. Students are advised to discuss this with the Director of Undergraduate Studies if they wish to consider such an option.

IMPERIAL HORIZONS

The College has created the ‘Imperial Horizons’ programme to broaden students’ education and enhance their career prospects. This programme is open to all undergraduate students.

The Department of Mathematics always endeavours to avoid timetabling Mathematics modules during the times allocated for Horizons modules.

Note that modules on this programme (except for the ones listed separately above as approved modules
for 4th year students) do not contribute to degree Honours marks but they do have an ECTS value of 6.

Further information about the 'Horizons' programme can be found at: http://www.imperial.ac.uk/horizons