Motivation

• The *North Atlantic Oscillation (NAO)* is a leading mode of variability of the Northern Hemisphere and beyond.
• It affects *the atmosphere and oceans* on several time and space scales.
• Its *predictive understanding* could help interannual and decadal-scale climate prediction over and around the North Atlantic basin.
• The *hierarchical modeling* approach allows one to give proper weight to the understanding provided by the models *vs.* their realism, respectively.
• Back-and-forth between *“toy”* (conceptual) and *detailed* (“realistic”) *models*, and between *models* and *data*.

Outline, Tipping Points II

- The **NAO** and the oceans’ wind-driven circulation
- The low-frequency variability of the **double-gyre circulation**
  - bifurcations in a toy model
    ➔ multiple equilibria, periodic and chaotic solutions
  - some intermediate model results
- Atmospheric impacts
  - simple and intermediate models + GCMs
- Some data analysis – atmospheric and oceanic
- Some very promising NAO results
- Conclusions and bibliography
The NAO and the oceans’ wind-driven circulation

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The North Atlantic Oscillation (NAO)

Positive phase

Negative phase

NAO Index

[Graph showing the NAO index from 1860 to 2000]
An example of bifurcations and hierarchical modeling: The oceans’ wind-driven circulation

The mean surface currents are (largely) wind-driven
Kuroshio Extension (KE) Path Changes

Monthly paths from altimeter:

Stable vs. unstable periods

Qiu & Chen (Deep-Sea Res., 2009)
“Limited-contour” analysis for atmospheric low-frequency variability

10-day sequences of subtropical jet paths: blocked vs. zonal flow regimes

Kimoto & Ghil, JAS, 1993a
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Modeling Hierarchy for the Oceans

**Ocean models**

- 0-D: box models – chemistry (BGC), paleo
- 1-D: vertical (mixed layer, thermocline)
- 2-D – meridional plane – THC
  - → also 1.5-D: a little longitude dependence
  - horizontal – wind-driven
  - → also 2.5-D: reduced-gravity models (n.5)
- 3-D: OGCMs - simplified
  - with bells & whistles (“kitchen sink”)

**Coupled 0-A models**

- Idealized (0-D & 1-D): intermediate couple models (ICM)
- Hybrid (HCM) - diagnostic/statistical atmosphere
  - highly resolved ocean
- Coupled GCM (3-D): CGCM
The double-gyre circulation and its low-frequency variability

An “intermediate” model of the mid-latitude, wind-driven ocean circulation:
20-km resolution, about 15 000 variables

Shallow-water model

\[
\begin{align*}
\frac{\partial U}{\partial t} + \nabla \cdot (uU) &= -g' h \frac{\partial h}{\partial x} + fV + \alpha_A \nabla^2 U - RU - \frac{\tau_x}{\rho} \\
\frac{\partial V}{\partial t} + \nabla \cdot (uV) &= -g' h \frac{\partial h}{\partial y} - fU + \alpha_A \nabla^2 V - RV \\
\frac{\partial h}{\partial t} &= -\left( \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \right)
\end{align*}
\]

where

- \( U \dot{e}_x + V \dot{e}_y = hu = h(u \dot{e}_x + v \dot{e}_y) \)
- \( g' \): reduced gravity \( (= g(\rho_2 - \rho)/\rho) \)
- \( A \): viscosity coefficient \( (= 300 \text{ m}^2\text{s}^{-1}) \)
- \( R \): Rayleigh coefficient \( (= 1/200 \text{ day}^{-1}) \)
- \( \tau^x \): wind stress \( = \tau_0 \cos 2\pi/L (\tau_0 = 1 \text{ dyn cm}^{-2} \& L = 2000 \text{ km}) \)

Reduced gravity (1.5 - layer) model

The JJG model’s equilibria

Nonlinear (advection) effects break the (near) symmetry: (perturbed) pitchfork bifurcation?

Subpolar gyre dominates

Subtropical gyre dominates

\[ \text{(Exact) Equilibrium state: } (\alpha_A, \alpha_c) = (1.3, 1.2) \]

- linear case –

- nonlinear case –

\[ h(x, y) \]

2000 km = 20 x

\[ 1000 \text{ km} \]

\[ \text{curl } \tau^x = 0 \]

Multiple equilibria (nonlinear case): \( (\alpha_A, \alpha_c) = (1.3, 0.9) \)

\[ h(\tau=0) = 0 \]

\[ h(\tau=0) = h(x, y) \]

\[ h = \text{ULT} = \text{upper-layer thickness} \]
To capture space-time dependence, meteorologists and oceanographers often use Hovmöller diagrams.
Poor man’s continuation method

Bifurcation diagram

Perturbed pitchfork + Hopf + transition to chaos

Position of Merging Point (km)

α_τ

α_A = 1.3
Interannual variability: relaxation oscillation
Global bifurcations in “intermediate” models

Bifurcation tree in a QG, equivalent-barotropic, high-resolution (10 km) model: pitchfork, mode-merging, Hopf, and homoclinic

Figure 1. Schematic bifurcation diagram of an equivalent-barotropic QG model, plotted in terms of an asymmetry measure $\Delta_\phi$ (see Section 3a further below) vs. wind-stress intensity. The limit cycles are schematically drawn for illustrative purpose and the streamfunction patterns corresponding to the three steady-state branches—subtropical, antisymmetric, and subpolar (from top to
Homoclinic orbit: numerical and analytical

Figure 2. Unfolding of the relaxation oscillations induced by the gyre modes, shown in the plane spanned by the total potential energy of the solution $E_p$ and the difference $\Delta E$ between the subpolar potential energy and the subtropical one (see text for details). The orbits of several limit cycles are

Figure 3. Bifurcation diagram of the highly truncated, four-mode model (5), projected onto the $(A_1 + A_2, A_3)$ plane for $\mu = 1$ and $s = 2$. $P$ stands for pitchfork bifurcation at $\sigma = \sigma_P = 7.61$, while $\sigma = \sigma_{HC} = 10.4299$ at the homoclinic bifurcation. The branches of periodic orbits are replaced by several explicitly computed limit cycles.
The double-gyre circulation: A different rung of the hierarchy

Another “intermediate” model of the double-gyre circulation: slightly different physics, higher resolution – down to 10 km in the horizontal and more layers in the vertical, much larger domain, …

Bo Qiu, U. of Hawaii, pers. commun., 1997
Model-to-model, qualitative comparison

Bo Qui, 2.5-layer QG model, 1997

modeled transport \((h_1u_1+h_2u_2)\) along 143°E
Spectra of (a) kinetic energy of 2.5-layer shallow-water model in North-Atlantic–shaped basin; and (b) Cooperative Ocean-Atmosphere Data Set (COADS) Gulf-Stream axis data.

Figure 7. Comparison between low-frequency variability in an idealized double-gyre model and in observations of the Gulf Stream axis. (a) Spectral results for a 2.5-layer SW model for a basin that approximates the North Atlantic in size and shape, using an idealized wind stress. Maximum
Multi-channel SSA analysis of the UK Met Office monthly mean SSTs for the century-long 1895–1994 interval

Marked similarity with the 7–8-year “gyre mode” of a full hierarchy of ocean models, on the one hand, and with the North Atlantic Oscillation (NAO), on the other: explanation?

Figure 8. Phase composites of the reconstructed 7–8-year SST oscillation. The MSSA window length is 40 year and the contour interval is 0.02°C.
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Atmospheric impact of mid-latitude SST anomalies: A highly contentious issue

- A quasi-geostrophic (QG) atmospheric model in a periodic $\beta$-channel, first barotropic (Feliks et al., JAS, 2004; FGS’04), then baroclinic (FGS’07).
- Marine atmospheric boundary layer (ABL), analytical solution.
- Forcing by idealized oceanic SST front.
Ocean-atmosphere coupling mechanism (II)

Vertical velocity at the top of the marine ABL

The nondimensional $w(H_e)$ is given by

$$w(H_e) = \left[ \gamma \zeta_g - \alpha \nabla^2 T \right],$$

with $\gamma = c_1 \left( f_0 L / U \right) \left( H_e / H_a \right)$ and $\alpha = c_2 \left( g / T_0 U^2 \right) \left( H_e^2 / H_a \right)$, where $H_a$ is the layer depth of the free atmosphere ($\sim 10$ km), and $\zeta_g$ the atmospheric geostrophic vorticity.

Two components: one mechanical, due to the geostrophic flow $\zeta_g$ above the marine ABL and one thermal, induced by the SST front.
Evolutive spectral analysis

30-day oscillation

70-day oscillation
Simulate atmospheric response to SODA data over the Gulf Stream region

- Use SST (~5 m) data from the SODA reanalysis (50 years)
- Use the FGS'07 QG model in periodic $\beta$-channel
  - baroclinic + marine ABL
- Figure shows NAO index:
  - simulated (solid)
  - observed (dashed)
Concluding remarks, II

- Tipping points and bifurcations: do they really help?
  - Yes, if properly understood and carefully applied!
- Can we predict them?
  - Yes, depending on the problem and the data!
Forced 7-year cycle in the FGS’04 model

Slow amplitude modulation of 1°C in the SST front

Low-energy phase

High-energy phase
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Waves vs. Particles: 
A Pathway to Prediction?

Is predicting as hard as it is claimed to be? 
No, it’s actually quite easy: Just flip a coin or roll a die! 
What’s difficult, though, is trusting the prediction. 
That’s where a little understanding of what we’re trying to predict helps!

Based on Ghil & Robertson (2002)

"Waves vs. Particles"
in Atmospheric Low-Frequency Variability

1. Are the regimes but slow phases of the oscillations? 

2. Are the oscillations but instabilities of particular equilibria? 

3. How about both: "chaotic itinerancy" (Itoh & Kimoto, JAS, 1999) 

4. How about neither? Null hypotheses: 
   a) It’s all due to interference of linear waves, e.g., neutrally stable Rossby waves; 

Kimoto & Ghil (JAS, 1993a, b) 
Legras & Ghil (JAS, 1985) 
Lindzen et al. (JAS, 1982)
Some references


Reserve slides
Climate models (atmospheric & coupled): A classification

- **Temporal**
  - stationary, (quasi-)equilibrium
  - transient, climate variability

- **Space**
  - 0-D (dimension 0)
  - 1-D
    - vertical
    - latitudinal
  - 2-D
    - horizontal
    - meridional plane
  - 3-D, GCMs (General Circulation Model)
  - Simple and intermediate 2-D & 3-D models

- **Coupling**
  - Partial
    - unidirectional
    - asynchronous, hybrid
  - Full

**Hierarchy:** back-and-forth between the simplest and the most elaborate model, and between the models and the observational data
Spin-up of atmospheric jet

**SST front:**
\[ L_{oc} = 600 \text{ km}, \]
\[ \Delta T = 3.5 \, ^{\circ}\text{C}, \]
\[ d = 50 \text{ km} \]

**Atmospheric jet**
spins up from
\[ L_a = 2000 \text{ km to} \]
\[ L_a = 4000 \text{ km, much} \]
greater speed and
strong recirculation
Can we, nonlinear people, help?

The uncertainties might be intrinsic, rather than mere “tuning problems.”

If so, maybe stochastic structural stability could help! Might fit in nicely with recent taste for “stochastic parameterizations”

The DDS dream of structural stability (from Abraham & Marsden, 1978)