MRes in Mathematical Sciences, Student handbook

Welcome to Imperial College for the start of your MRes. This handbook contains all the key information you will need to understand how the course will be structured over the next 12 months, the key dates for the submission of assessed work, what will be expected of you academically, and who are the important contacts within the Department.

The Programme:

Overview and aims

The MRes is a 12-month course, which provides a high-level training in Mathematical research for students who will most likely go on to pursue a PhD. The course is divided into three components, or *streams,* each represents an economically important area of mathematical research in which the Mathematics Department at Imperial College has established strengths. The programme you will follow has been carefully tailored to equip you with the skills you need to begin engaging with contemporary research problems.

Structure and assessment

You will follow exactly one of the following streams:

- Scientific Computation
- Statistics: Advanced pattern detection and signal analysis
- Mathematical Finance and Stochastics

Each stream consists of the following two assessed element, which will contribute to the eventual total with the weightings indicated below.

1. <u>Taught element</u> (25% of total)
   The assessment for this element will comprise:
   - **Examinations**[1]. The total number of courses you need to take depends on the stream you will follow. In the Mathematical Finance and Stochastics and Scientific Computation streams you must choose and attend three lecture courses from the list of *Core Courses* for your stream. Each course will run in one of the three terms, and assessment will usually be by written examination, but could also be by a project, coursework or an oral examination. In the Statistics stream you must select from the list of Core Courses so that the cumulative length of these courses totals 90 hours. In the Statistics stream all Autumn and Spring term courses are 30 and 15 hours, respectively.

   In addition to the Core Courses, you must attend Optional Courses chosen from the list below. In the Mathematical Finance and Stochastics and Scientific Computation streams you must attend two Optional Courses. In the Statistics stream you must attend a total of 60 hours of Optional Courses. These courses will not be assessed, but they are important in

[1] For the Scientific Computation stream only this will also involve a **Mastery Paper.**

broadening your research outlook. For example, you might want to select them to develop an interest in a new area, or to support the research project you will undertake as part of the research element (see below). More information on the courses titles, their contents and timing can be found below in the detailed sections on the different streams.

2. Research element (75% of total)

   A substantial part of your study programme will be an extended research project. This is a piece of work that runs throughout the year, and will involve you writing a dissertation under the supervision of member of staff. Some students starting the MRes will already have had contact with a potential supervisor, others will not. You should speak to the director of your stream about your interests at the start of the first term; he or she will be able to suggest literature to read and academic members of staff with similar interests to whom you could talk. At the end of the fourth week of term we will expect you to have identified a supervisor.

   The topic will concern an exciting area of current mathematical interest, usually in which your supervisor is working, and in which he/she will provide expert guidance. In the process you will develop proficiency in important aspects of research. You will learn how to read and synthesise mathematical literature, you will gain experience in mathematical and scientific writing, and you will develop your communication skills ideas by giving presentations based on your findings.

   The development of these skills will be supported throughout by a variety of courses, reading groups, lectures and seminars. This training is common to students in all three streams and consists of:

   - A lecture on *Reading Scientific Literature.*
   - The **MRes seminar**. This will be a series of one hour seminars which will run weekly throughout the first two terms. They are specifically targeted at students taking the MRes, and the topics will be chosen to appeal to students in all three streams.
   - Subject-specific reading groups: the director for your particular stream will be able to give you more information about these reading groups.
   - Exercises in scientific writing and scientific presentation.
   - Short courses offered by the Graduate School

   You will be advised on details on timings, venues and procedures for registration throughout the year, and you will be expected to attend. Ask the director for your stream if you are unsure of anything.

   At the start of the first term you will be expected to identify a supervisor to work with on the dissertation. You should speak to the director of your stream who will help advise you on this process. You should then approach potential supervisors to discuss possible topics for the dissertation. At the end of week 4 of the Autumn Term, we will contact you to ask you who your supervisor will be. Your supervisor will recommend an academic paper to read, and by the end of week 9 you will need to write a summary and critique the contents of this paper. The purpose of this first term writing exercise is to both to initiate progress on your research topic, and to identify any weaknesses in your writing skills which need to be addressed. Your report will be assessed by your supervisor and a second marker. It will count 5% towards to total marks available in the research element.

An important staging post for the research project is the Preliminary Report, which you must submit by 20th March. This will be a document, of no more than 4000 words, which consists of a literature review together with a plan for the development of the rest project. The report will be marked and you will make a 20 minute oral presentation on its contents. The final dissertation will be a much more substantial piece of work, with an upper limit of 30,000 words. The deadline for submitting the dissertation is 15th September. You will again make a 20 minute oral presentation, which will be immediately followed by a viva in which you will be asked question by two members of staff who are familiar with your research area. The contribution of these different parts is summarised below.

- **First term writing exercise** (5%): Deadline end of week 9. A written report of around 5 pages summarising an academic paper based on ideas close to the intended project area. If a student shows signs of poor fluency in reading or scientific writing, help will be given at this stage.
- **Preliminary report** (10%): Deadline 20th March. A written report consisting of a literature review and outlook; maximum 4000 words; assessment via written report (90%) and oral presentation (10%). The oral presentation will consist of a public presentation in front of other students and members of the academic staff.
- **Final project** (75%): Deadline 15th September. A written dissertation; maximum 30000 words; assessment via the **written dissertation** (90%) and **oral examination and viva** (10%), which will be an oral presentation and viva with two members of staff shortly after the submission of the dissertation.

Degree classification

The MRes degree will be marked as either: Pass, Merit, Distinction or Fail. The requirements for each of these are described below.

In order to be awarded a result of pass, a candidate must achieve at least 50 per cent in each element; in order to be awarded a result of a merit, a candidate must achieve at least 60 per cent in each element; in order to be awarded a result of distinction, a candidate must achieve at least 70 per cert in each element.

Where appropriate, a Board of Examiners may award a result of pass where a candidate has achieved an aggregate mark of 50 per cent or greater across the programme as a whole AND has obtained a mark of 50 per cent or greater in each element with the exception of one element AND has obtained a mark of 40 per cent or greater in this latter element.

Where appropriate, a Board of Examiners may award a result of merit where a candidate has achieved an aggregate mark of 60 per cent or greater across the programme as a whole AND has obtained a mark of 60 per cent or greater in each element with the exception of one element AND has obtained a mark of 50 per cent or greater in this latter element.

Where appropriate, a Board of Examiners may award a result of distinction where a candidate has achieved an aggregate mark of 70 per cent or greater across the programme as a whole AND has obtained a mark of 70 per cent or greater in each element with the except

Plagiarism and Examination Offences

The College takes plagiarism and other matters of academic foul play extremely seriously, and offenders are liable to be punished severely. The College's policy and procedures on cheating and examination offences can be found below:

http://www3.imperial.ac.uk/registry/exams/examoffences

The streams

I. Scientific Computation

Scientific Computing underpins research in modern applied mathematics and has become one of the pillars of scientific exploration in the physical and biological sciences. The aim of this MRes stream is to prepare students for advanced research in this expansive field, especially in the topic areas strongly linked to the current research interests of the Applied Mathematics and Mathematical Physics (AMMP) section. This training encompasses the full spectrum of scientific computing research, including the development and analysis of numerical schemes, their fast and robust implementation, and their usage in addressing outstanding scientific problems. Accordingly, the MRes training consists of not only coursework in computational mathematics, but also in applied topic areas such as fluid dynamics or biomathematics. The specific choice and mixture of computational and applied topic courses should be tailored to each student with the selection depending on student background and the intended topic of the MRes project. As such, the student should consult with the stream Director before choosing courses.

Core courses:

M5N7 Numerical Solution of Ordinary Differential Equations (Dr D Moore) (30 hours, Autumn Term)

An analysis of methods for solving ordinary differential equations. Runge-Kutta, extrapolation and linear multistep methods. Analysis of stability and convergence. Error estimation and automatic step control. Introduction to stiffness. Boundary and eigenvalue problems. Solution by shooting and finite difference methods. Introduction to deferred and defect correction. Additional material: contained within the scope of the projects.

M5N10 Computational Partial Differential Equations I (Prof P Schmid) (30 hours, Spring Term)

This is an introductory course on numerical methods for the solution of partial differential equations (PDEs). Classification of PDEs; transform methods and dispersion relations. Grid functions and their Fourier transforms; aliasing. Discretization of PDEs; semi- and fully-discrete methods; explicit and implicit schemes; implementation. Conservation law approach; finite-volume methods. Consistency, stability and convergence for finite-difference methods of initial value problems; Von Neumann stability. Multistep schemes (e.g. leapfrog type schemes); stability of general multistep schemes. Dispersion and dissipation of numerical schemes. Group velocity and wavepackets in numerical schemes. Numerical solution of systems of hyperbolic PDEs; multilevel schemes; stability and convergence. Numerical methods for parabolic PDEs; Crank-Nicolson methods; convection diffusion equations; consistency, stability and convergence; implementation. Second order in time PDEs; wave equation; Euler-Bernoulli (beam) equation. Parabolic equations in 2D and 3D; finite-volume and finite-difference methods; initial boundary value problems; ADI schemes; implementation of boundary conditions; fast solvers.

M5SC Scientific Computation (Prof D Moore) (30 hours, Spring Term)

The aim of this course is to develop proficiency in a scientific programming language and to solve mathematical problems using computation. The objectives are that by the end of the course all students should be familiar with the essential elements of the C programming language, and be able to undertake substantial programming tasks in C, examples of which are given below. The elements of the language to be covered will include: Running a C program on the computers in Mathematics. Basic C data types and data declarations. Arithmetic with C data types. Simple I/O in C. C control structures. Arrays and matrices. Pointers and dynamic allocation. Structures and linked lists. File handling. Examples of programming tasks to be undertaken: Simple numerical tasks (e.g. recursion, numerical integration, differential equations). Data processing (e.g. sorting algorithms, Fast Fourier Transforms). More computational tasks (e.g. iterative methods for linear and non-linear equations). More data processing (e.g. voting algorithms, triangulation).

M5A2 Fluid Dynamics I (Dr S Mughal) (30 hours, Fall Term)

This course is an introduction to Fluid Dynamics. The continuum hypothesis. Knudsen number. The notion of fluid particle. Kinematics of the flow field. Lagrangian and Eulerian variables. Streamlines and pathlines. Vorticity and circulation. The continuity equation. Streamfunction and calculation of the mass flux in 2D flows. First Helmholtz theorem. Constitutive equation. The Navier-Stokes equations. Couette and Poiseuille flows. The flow between two coaxial cylinders. The flow above an impulsively started plate. Diffusion of a potential vortex. Integrals of motion. Kelvin’s circulation theorem. Potential flows. Bernoulli’s equation. Cauchy-Bernoulli integral for unsteady flows. Two-dimensional flows. Complex potential. Vortex, source, dipole and the flow past a circular cylinder. Adjoint mass. Conformal mapping. Joukovskii transformation. Flows past aerofoils. Lift force. The theory of separated flows. Kirchhoff and Chaplygin Models.

M5A10 Fluid Dynamics II (Prof A Ruban) (30 hours, Spring Term)

A continuation of M5A2. Dynamic and Geometric Similarity of fluid flows. Reynolds Number and Strouhal Number. Fluid Flows at Low Values of The Reynolds Number: Stokes equations. Stokes flow past a sphere. Stokes flow past a circular cylinder. Stokes paradox. Large Reynolds Number Flows: the notion of singular perturbations. Method of matched asymptotic expansions. Prandtl?s boundary-layer equations. Prandtl’s hierarchical concept. Displacement thickness of the boundary layer and its influence on the flow outside the boundary layer. Self-Similar Solutions of the Boundary-Layer Equations: Blasius solution for the boundary layer on a flat plate surface. Falkner-Skan solutions for the flow past a wedge. Schlichting’s jet solution. Tollmien’s far field solution. Viscous drag of a body. Shear layers. Prandtl transposition theorem. Triple-Deck Theory: The notion of boundary-layer separation. Formulation of the triple-deck equations for a flow past a corner. Solution of the linearised problem (small corner angle case).

M5A30 Hydrodynamic Stability (Prof P Hall) (30 hours, Spring Term)

This course extends basic fluid mechanics into study of hydrodynamic stability. Rayleigh-Taylor and Kelvin-Helmholtz instabilities. Jet break-up. Inviscid instability: Rayleigh's equation. Viscous instability: Orr-Sommerfeld equation. Squire's theorem. Circular flows, Taylor vortices. The e^n method for transition prediction. Numerical methods for eigenvalue problems.

M5A28 Introduction to Geophysical Fluid Dynamics (Dr P Berloff) (30 hours, Fall Term)

Representation of fluid flows. Governing equations (continuity of mass, material tracer, momentum equations, equation of state, thermodynamic equation, spherical coordinates), Geostrophic dynamics (shallow-water model, potential vorticity conservation law, rotation-dominated scales, thinlayer framework, Boussinesq approximation, Rossby number, geostrophic balance, hydrostatic balance, geostrophic density anomaly, ageostrophic flow, vorticity equation), Quasigeostrophic theory (two-layer isopycnal model, two-layer quasigeostrophic model, potential vorticity conservation, continuous stratification, buoyancy frequency, boundary conditions) Ekman layers (non-QG near-boundary effects, Ekman pumping), Rossby waves (phenomenon overview, mechanism, energy equation, mean-flow effect, two-layer and continuously stratified waves), Linear instabilities (phenomenon overview, barotropic and baroclinic instabilities, necessary conditions, Eady model, Phillips model, mechanism of baroclinic instability, energetics), Nonlinear dynamics and wave-mean flow interactions (closure problem, parameterization of unresolved eddies, transformed Eulerian mean, homogeneous and stationary 3D turbulence without rotation, 2D homogeneous turbulence, model solutions, effects of rotation and stratification, homogeneous turbulent diffusion), Ageostrophicmotions (linearized shallow-watermodel, Poincare and Kelvin waves,equatorial waves, ENSO delayed oscillator, geostrophic adjustment, Stokes drift)

M5A32 Vortex Dynamics (Prof D Crowdy) (30 hours, Spring Term)

This course will focus on the mathematical study of the dynamics in an ideal fluid in two and three dimensions. Fundamental properties of vorticity. Helmholtz Laws and Kelvin's circulation theorem. Singular distributions of vorticity; Biot-Savart law. Dynamics of line vortices in 2d and other geometries. Dynamics of 2d vortex patches, contour dynamics. Axisymmetric vortex rings. Dynamics of vortex filaments. Stability problems. Miscellaneous topics (effects of viscosity, applications to turbulence, applications in aerodynamics).

M5A42 Applied Stochastic Processes (Dr M Ottobre) (30 hours, Fall Term)

This is an introduction to stochastic processes in continuous time with particular emphasis to Markov processes. Elements of probability theory. Stochastic processes: basic definitions, examples. Stationary processes. the Karhunen-Loeve expansion. Continuous time Markov processes, basic definitions, examples. Diffusion processes: the generator, backward Kolmogorov and Fokker-Planck (forward Kolmogorov) equations. Methods of solution for the Fokker-Planck equation. Mean first passage time problems and applications. Stochastic differential equations (SDEs), basic definitions and examples. Numerical methods for SDEs. Stochastic resonance, Brownian motors.

M5A21 Mathematical Biology I: Molecular Topology and Stereochemistry (Dr D Buck) (30 hours, Fall Term)

Introductory Molecular Biology: DNA, including linked and knotted; Proteins that change DNA Topology: recombinases, topoisomerases, transposases; Open Questions (and why the answers matter). Introductory Knot (and Tangle) Theory: Definition of **S**3 and manifolds; Definitions of Knots; Equivalence: invariants, including **Lk**; Transforming knots to knots; Crossing change, Generalised crossing change; Tangles. Applications of Knot Invariants in Molecular Biology: **Lk** = **Tw** +**Wr**;

Colouring and the Transposome; Arf invariant and Chirality. Applications of Knot Operations: Band surgery: Recombination; Crossing change: Topoisomerase II; Tangle surgery: Recombination. Open knots: Topological and Biological Proteins as knotted open chains; Characterising `knot types' of open chains. Topological Stereochemistry: Graphs and Knots; Flavours of chirality; Planarity; Flavours of stereoisomers; Molecular Models and Mobius Ladders; Symmetries of Embedded and Molecular Graphs

M5A25 Mathematical Biology of the Cell (Dr V Shahrezaei) (30 hours, Spring Term)

The aim is to provide an introduction to physical and systems biology of the cell. A variety of current research topics where quantitative and theoretical approaches have been used successfully in cell biology is discussed. Cell biology by the numbers. Physical biology of the cell: energy and entropy. Systems biology of the cell: biochemical networks. Metabolic, signally, gene-regulatory networks. Biological modules and network motifs. Stochastic dynamics of biochemical networks. Spatial dynamics of biochemical networks. Robustness, sensitivity and specificity.

M5M3 Introduction to Partial Differential Equations (Prof J Carillo) (30 hours, Fall Term)

Introduction: Basic Concepts: PDEs, linearity, superposition principle; Boundary and Initial value problems. Gauss Theorem: gradient, divergence and rotational; Main actors: continuity, heat or diffusion, Poisson-Laplace, and the wave equations.
Method of Characteristics: Linear and Quasilinear first order PDEs in two independent variables. Wellposedness for the Cauchy problem. The linear transport equation. Upwinding scheme for the discretization of the advection equation. A brief introduction to conservation laws: The traffic equation and the Burgers equation. Singularities.
Diffusion: Derivation of the heat equation. The boundary value problem: separation of variables. Fourier Series. Explicit Euler scheme for the 1d heat equation: stability. The Cauchy problem for the heat equation: Poissons Formula. Uniqueness by maximum principle.
Waves: The 1D wave equation. D'Alembert Formula. The boundary value problem by Fourier Series. Explicit finite difference scheme for the 1d wave equation: stability. 2D and 3D waves. Casuality and Energy conservation: Huygens principle.
Stationary States: Laplace-Poisson: Greens functions: Newtonian potentials. Dirichlet and Neumann problems. Harmonic functions. Uniqueness: mean property and maximum principles.

M5M7 Asymptotic Analysis (Prof X Wu) (30 hours, Fall Term)

In many applications, the mathematical problems formulated involve integrals and/or differential equations which consist of large or small parameters. This course will introduce a range of techniques which allow us to evaluate such integrals, or to obtain approximate solutions to such equations, analytically. The usefulness of the techniques will be demonstrated by examples drawn from different topics in applied sciences.
Syllabus: Asymptotic series and expansions. Estimate of integrals: Laplace's method, Watson's lemma, stationary phase and steepest descent methods. Singular perturbations, matched asymptotic expansions and the matching principle - boundary and internal layers. Multiple scale method and averaging method. Differential equations with a large parameter: the WKBJ method, turning point problems, ray tracing, caustics.

II. Statistics: Advanced pattern detection and signal analysis

The Statistics Section at Imperial has an international reputation for conducting methodological and applied statistical research at the highest level. Particular areas of current activity include statistical genetics and biostatistics, statistical methods in retail financial services, time series, core statistical methodology, classification and data mining, and cyber security, with many interactions and overlaps between these areas of research. The Statistics stream of the MRes degree programme provides a platform for gaining the necessary core research skills in preparation for PhD research in these areas. The training covers three key areas: understanding fundamental statistical theory, gaining hands-on experience in statistical computing, and exposure to a diverse range of application areas which each have open and constantly evolving statistical research problems.

Below is a list of courses available from Imperial's MSc Statistics programme, which may be taken as core or optional courses. It might also be possible to sit courses run by other departments/institutions, with the agreement of your supervisor and the stream Director; this might be appropriate, for example, if your research project is of an applied nature and requires advanced level of domain expertise.

Core and optional courses:

Students attend lecture courses throughout the year and must register for assessment in a total of 90 hours of Core Courses which will be chosen from the following list. Assessment for the Core Courses will either be by examination or project work or a combination of both, depending on the course.

Autumn term (all 30 hours)

Probability for Statistics (Prof N Bingham)

Fundamentals of Statistical Inference (Prof A Young)

Applied Statistics (Dr N Kantas)

Computational Statistics (Dr D Mortlock)

Spring term (all 15 hours)

Advanced Statistical Theory (Prof A Young)
Bayesian Statistics (Prof D van Dyk)
Non-parametric Smoothing and Wavelets (Dr B Missoui)
Multivariate Analysis (Dr E Cohen)
Graphical Models (Prof A Walden)
Machine Learning (Dr B Calderhead)
Statistics for Extreme Events (Dr A Veraart)
Financial Econometrics (Dr A Veraart)
Pricing and Hedging in Financial Markets (Dr M Pakkanen)
Statistics in Retail Finance (Dr B Missaoui)
Medical Statistics (Dr L Bottolo)
Official Statistics (Dr N Kantas)

Summer term: None

In addition, every student on the course will attend 60 hours of Optional Courses. These may be courses from the above list not already submitted for examination, or any Basic or Advanced courses offered by the London Taught Course Centre.

Detail of course contents.

M5MS01 Probability for Statistics (Prof N Bingham) (30 hours, Autumn Term)

Review of axiomatic probability theory: probability spaces, distributions and their characteristics [including generating functions], conditional distributions. Asymptotic theorems and convergence. Convergence modes and stochastic orders, convergence of transformations, laws of large numbers, central limit theorem, martingales. Multivariate normal distribution. Gaussian processes. Markov chains. Markov processes, classification of chains, stationary distributions, continuous-time Markov chains.

M5MS02 Fundamentals of Statistical Inference (Prof A Young) (30 hours, Autumn Term)

Approaches to inference: Bayesian, Fisherian, frequentist.
Decision theory: risk, criteria for a decision rule, minimax and Bayes rules, finite decision problems.
Bayesian methods: fundamental elements, choice of prior, general form of Bayes rules. Empirical Bayes, hierarchical modelling. Predictive distributions, shrinkage and James-Stein estimation.
Data reduction and special models. Exponential families, transformation models. Sufficiency and completeness. Conditionality and ancillarity.
Key elements of frequentist theory. Hypothesis testing: Neyman-Pearson, uniformly most powerful tests, two-sided tests, conditional inference and similarity. Optimal point estimation. Confidence sets.
Introduction to likelihood theory. Asymptotic properties of maximum likelihood estimators, testing procedures. Multiparameter problems.

M5MS03 Applied Statistics (Dr N Kantas) (30 hours, Autumn Term)

Statistical Models and modelling illustrated with real examples.
Data pre-processing.
Simple and multiple linear regression. Model diagnostics and iterative modelling. Handling messy data, such as missing values. Sparsity and Lasso.
Experimental Design.
Generalised linear models: logistic, log-linear.
Basic linear time series models, e.g. ARMA.
Multi-level models and repeated measures.
Classification and discrimination.

M5MS04 Computational Statistics (Dr D Mortlock) (30 hours, Autumn Term)

Statistical Computing: R programming: data structures, programming constructs, object system, graphics. Numerical methods: root finding, numerical integration, optimisation methods such as EM-type algorithms.
Simulation: generating random variates, Monte Carlo integration.

Simulation approaches in inference: randomisation and permutation procedures, bootstrap, MCMC, Sequential Monte Carlo/particle filtering.

M5MS05 Advanced Statistical Theory (Prof A Young) (15 hours, Spring Term)

This course aims to give an introduction to key developments in contemporary statistical theory, building on ideas developed in the core course Fundamentals of Statistical Inference. Reasons for wishing to extend the techniques discussed in that course are several. Optimal procedures of inference, as described, say, by Neyman-Pearson theory, may only be tractable in unrealistically simple statistical models. Distributional approximations, such as those provided by asymptotic likelihood theory, may be judged to be inadequate, especially when confronted with small data samples (as often arise in various fields, such as particle physics and in examination of operational loss in financial systems). It may be desirable to develop general purpose inference methods, such as those given by likelihood theory, to explicitly incorporate ideas of appropriate conditioning. In many settings, such as bioinformatics, we are confronted with the need to simultaneously test many hypotheses. More generally, we may be confronted with problems where the dimensionality of the parameter of the model increases with sample size, rather than remaining fixed.
We consider here a number of topics motivated by such considerations. Focus will be on developments in likelihood-based inference, but we will give consideration too to: problems of multiple testing, objective Bayes methods, bootstrap alternatives to analytic distributional approximation, and introduce too more theoretical notions involved in high-dimensional inference.

M5MS06 Bayesian Statistics (Prof D van Dyk) (15 hours, Spring Term)

Scientific inquiry is an iterative process of integrating and accumulating information. Investigators assess the current state of knowledge regarding the issue of interest, gather new data to address remaining questions, and then update and refine their understanding to incorporate both new and old data. Bayesian inference provides a logical, quantitative framework for this process. This framework is based fundamentally on the familiar theorem from basic probability theory known as Bayes' Theorem.
Although Bayesian statistical methods have long been of theoretical interest, the relatively recent advent of sophisticated computational methods such as Markov chain Monte Carlo has catapulted Bayesian methods to centre stage. We can now routinely fit Bayesian models that are custom made to describe the idiosyncratic complexities of particular data streams in a multitude of scientific, technological, and policy settings. The ability to easily develop customized statistical techniques has revolutionized our ability to handle complex data. Bayesian methods now play an important role in the analysis of data from Marketing and sales, online activity, security camera streams, transportation, climate and weather, medical records, bioinformatics, and a myriad of other human activities and physical processes. In this course we will develop tools for designing, fitting, validating, and comparing the highly structured Bayesian models that are so quickly transforming how scientists, researchers, and statisticians approach their data.

M5MS07 Non-parametric Smoothing and Wavelets (Dr B Missoui) (15 hours, Spring Term)

Kernel estimators: window width, adaptive kernel estimators.
Roughness penalties: Cubic splines; Spline smoothing, Reinsch algorithm; alternative penalties: lasso, ridge regression.

Basis function approach: B-spines, wavelets: discrete wavelet transform; wavelet filters; the maximal overlap discrete wavelet transform; wavelet variance, wavelet shrinkage, thresholding.
Generic Model choice: AIC, BIC, cross-validation.

M5MS08 Multivariate Analysis (Dr E Cohen) (15 hours, Spring Term)

As the name indicates, multivariate analysis comprises a set of techniques dedicated to the analysis of data sets with more than one outcome variable. A situation that is ubiquitous across all areas of science. Multiple uses of univariate statistical analysis is insufficient in this settings where interdependency between the multiple random variables are of influence and interest. In this course we look at some of the key ideas associated with multivariate analysis. Topics covered include a comprehensive introduction to standard multivariate notations, the multivariate normal distribution, selected likelihood ratio testing, conditional distributions, the Wishart distribution and multiple and partial correlations. We will then finish with a brief look at the complex valued representations in multivariate analysis and look at the elegancy this representation achieves. As an interesting case study we will look at the use of complex representation in exploring oceanic current rotations.

M5MS09 Graphical Models (Prof A Walden) (15 hours, Spring Term)

Probabilistic graphical models encode the relationships between a set of random variables, in a manner that relies on networks and graph-theoretic intuitions. Primarily, they encode conditional independence assumptions, whereby A is statistically independent of B conditional on the value of C. Just as conditional probability is one of the pillars of modern probability, conditional independence is critical in statistical modelling. It underlies model specification, and allows us to infer, elicit, and understand correlation structures between unobserved variables, given the value s of variables we already know. This course will entail a variety of material, including discrete mathematics (graph theory), statistical modelling, algorithms and computational aspects, as well as applications, involving real data and actual applications. We will also touch upon abstract questions, such as the difference between causality and correlation.

M5MS10 Machine Learning (Dr B Calderhead) (15 hours, Spring Term)

Machine Learning is readily becoming one of the most important areas of general practice, research and development within statistics and computer science. This growth can be partly explained by the increase in the quantity and diversity of measurements we are able to make of the world. A particularly fascinating example arises from the wave of new biological technologies that preceded the sequencing of the human genome. We are now able to measure the detailed molecular state of an organism in ways that would have been hard to imagine only a short time ago. Machine learning techniques have been heavily involved in the distillation of useful structure from this data, and have been critical for many scientific discoveries done in the last few years. Many other areas and application domains, from social networks analysis to algorithmic trading, benefit from machine learning methods, which are routinely used for the detection of patterns and anomalies in large quantities of data.

M5MS11 Statistics for Extreme Events (Dr A Veraart) (15 hours, Spring Term)

This course introduces extreme value theory. We focus on statistical methods for extreme events and study applications in insurance and finance. The main topics are as follows:

Extreme value theory: Fluctuations of maxima; fluctuations of upper order statistics;
Statistical Methods: Probability and quantile plots; mean excess function; Gumbel's method of exceedances; parameter estimation for the generalised extreme value distribution; estimating under maximum domain of attraction conditions; fitting excess over a threshold.

M5MS12 Financial Econometrics (Dr A Veraart) (15 hours, Spring Term)

Financial econometrics is an interdisciplinary area focusing on a wide range of quantitative problems arising from finance. This course gives an introduction to the field and presents some of the key statistical techniques needed to deal with both low and high frequency financial data. Main topics of the course are:
Discrete time framework: ARCH, GARCH models and their estimation;
Continuous time framework: Brownian motion, stochastic integration and stochastic differential equations, Itô's formula, stochastic volatility, realised quadratic variat ion and its asymptotic properties, Lévy processes, testing for jumps, volatility estimation in the presence of market microstructure effects.

M5MS13 Pricing and Hedging in Financial Markets (Dr M Pakkanen) (15 hours, Spring Term)

The fundamentals of no-arbitrage theory and risk neutral valuation of contingent claims in the setting of the trinomial model will be explained. The most commonly traded contingent claims in the financial markets (vanilla and forward starting options, barrier and volatility derivatives, American options) will be described in detail and their pricing discussed in the context of trinomial models.

M5MS15 Statistics in Retail Finance (Dr B Missaoui) (15 hours, Spring Term)

Retail finance is a business sector that has used mathematical and statistical methods successfully for credit risk analysis and operational decision making for over 50 years. In this course we will cover credit scores and their use in credit application decision making. We will introduce the segmented logistic regression model as a standard model of credit scorecard development and consider industry standard methods for assessing the performance of these models such as the receiver-operating characteristics (ROC) curve and Gini coefficient.
Issues that are very specific to retail finance, such as sample selection bias and reject inference will be covered in some depth, along with methods for fraud detection. Behavioural models of credit usage using survival and Markov transition models will be studied. These can be used as the basis of calculations of expected profitability. This leads naturally to portfolio level models where estimates of profit or loss are required across portfolios of financial products. In particular, Value-at-Risk (VaR) and expected shortfall estimates are used for capital requirements calculations, and derive the Merton/Vasicek formula which forms the basis of capital requirements calculations in the international Basel Accord on banking. Finally, we consider statistical approaches to stress testing, using simulation based on portfolio-level credit risk models.

M5MS17 Medical Statistics (Dr L Bottolo) (15 hours, Spring Term)

The objective of the course is to provide a broad range of statistical techniques to analyse biomedical data that are produced by pharmaceutical companies, research units and the NHS. Besides a general introduction to linear, generalised linear models and survival analysis, the course will focus on clinical trials (study design, randomisation, sample size and power, covariates and subgroups adjustment) to examine the effect of treatments on the disease process over time and longitudinal data analysis from the perspective of clinical trials. The statistical theory and the derivation and estimation of model parameters will be illustrated as well as the application of longitudinal models on real case studies drawn from biomedical and health sciences. The analysis of the real examples will be performed using standard statistical software. At the end of the course, students will be able to plan basic clinical trials, analyse longitudinal data and interpret the results.

The course will cover the following models and topics:
- Introduction to linear/generalised linear models and survival analysis
- Introduction to clinical trials
- Treatment allocation, monitoring and effect estimation
- Introduction to longitudinal data and repeat measures
- General and generalised linear model for longitudinal data
- Random and mixed-effects models

M5MS18 Official Statistics (Dr N Kantas) (15 hours, Spring Term)

"Statistics are the mirror through which we view society" (David Hand). How can the well-being of a nation be measured? Has the UK been faring better than Europe during the financial crisis? Were the policies of our government successful? Sound policy making must rely on evidence, and the task of gathering such evidence reliably across a multitude of individuals, social groups, businesses and types of activities, is monumental. This is an exciting time for official statistics: the raw data are increasingly becoming available for public scrutiny, and recent developments are redefining what constitutes "well-being", "progress", and how they can be measured.

III. Mathematical Finance and Stochastics

This stream provides intensive and focused training for research in Mathematical Finance, and the closely related discipline of Stochastic Analysis. Mathematical Finance is a subject area with a long history stretching back centuries. But in recent decades research in the area has witnessed an explosion in research activity, which is largely driven by demand from the global finance industry. Many exciting problems emerge in the field all the time, and the subject is an important crossing point for mathematicians from different backgrounds giving them the opportunity to collaborate together. The modern foundation of the theory can be traced back to the seminal 1973 paper of Black and Scholes. Their key observation, that certain types of financial risk can *always* be perfectly hedged, underpins today’s options pricing industry. It is one of the finest examples of mathematics applied to the field of commerce.

The department offers expertise in many leading-edge issues in the area including implied volatility, Levy processes, stochastic optimal control, and the theory of backward stochastic differential equations. As well as topics which have been given impetus after the 2008 crisis, including systemic risk, risk measures, and the challenges posed by financial regulation.

A key mathematical tool is the discipline of Stochastic Analysis, which constitutes a second focus for the stream. The theory has been developed into a coherent body of knowledge over the last seventy years or so, and is now an essential toolkit for anyone who aims to model and understand randomly-evolving real-world phenomena. An example is Ito’s theory of Stochastic Calculus, which gives a precise mathematical theory of integration with respect to random, and possibly highly irregular, paths such as the sample paths of Brownian motion. The so-called stochastic differential equations based on this calculus provide a fundamental modelling framework for random systems in finance, engineering, computer science and biology.

Stochastic Analysis is the cornerstone of most applicable research, and the department offers world-leading expertise in a range of subjects from stochastic filtering, functional Ito calculus, the theory of rough paths, to stochastic simulation methods.

Core courses:

Taught element

Students attend lecture courses throughout the year and must register for assessment in three Core Courses which will be chosen from the following list.

Autumn term:

MF0: Stochastic integrals: an introduction to Itô calculus (Prof. R. Cont)

Course objectives:

This course is a PhD-level introduction to the theory of stochastic integration and the *Ito calculus*, a calculus applicable to functions of stochastic processes with irregular paths, which has many applications in finance, engineering and physics. The course shall focus on the mathematical foundations of stochastic calculus. We shall develop the theory in the setting of *semimartingales*, which covers most examples of stochastic processes of interest in applications - including jump processes and diffusion processes.

Stochastic processes (Dr. T. Cass)

This course gives an introduction to probability theory and measure theory and introduces stochastic processes and the basic tools from stochastic analysis to provide the mathematical foundations for option pricing theory. It includes an intermediate introduction to axiomatic probability theory and measure theory, explaining notions like probability spaces, measures, measurable functions, integration with respect to measures, convergence concepts for random variables, joint distributions, independence and conditional expectations. It studies stochastic processes in discrete and continuous time; mainly the random walk, Brownian motion, and their properties. These in turn involve notions like the quadratic variation, the reflection principle, the Markov property and the martingale property. We will cover the stochastic Ito integral, the Ito formula, and their mathematical applications; for example, stochastic differential equations and some references to partial differential equations

Spring term:

Advanced methods in derivative pricing (Dr. H. Zheng)

This course explores the mathematical aspects of derivative security valuation and hedging based on the 'no-arbitrage' principle. Starting with the Black-Scholes paradigm and construction of perfectly-replicating portfolios, the connection between absence of arbitrage and the existence of martingale pricing measures is explored in detail. Applications to 'exotic' options involving such features as path-dependency, multi-currency payoffs, multi-factor models and early exercise options are covered.

Advanced topics in stochastic analysis (Dr. T. Cass)

The course will present a view of contemporary view of problems in stochastic analysis by adopting the approach of T. Lyons' theory of rough paths. We begin by looking at some elementary concepts: controlled differential equations the definition of the signature and its algebraic properties. And then move to more advanced topics: spaces of rough paths, weakly geometric and geometric, controlled rough paths, rough integration and the universal limit theorem. We give a treatment of stochastic processes as rough paths, including enhanced Brownian motion and its properties, and explore rough integration compared to Ito/Stratonovich integration. Applications to classical results including the Wong-Zakai theorem and the Stroock-Varadhan support theorem will be considered as time allows. Novel application in stochastic simulation and mathematical finance: the use of the expected signature and Gaussian processes as rough paths will also be covered.

Advanced topics in volatility modelling (Dr. A. Jacquier)

This course focuses on the characterisation and the properties of the different concepts of volatility in mathematical finance, namely implied, local, stochastic and historical volatilities. These will serve as the framework to answer the following questions: given a set of observed (European) option prices, (i) are there arbitrage opportunities and (ii) which model (with continuous paths) should one choose to model stock prices?

Algorithmic trading and machine learning (Dr. G. Di Graziano and Dr. S. Ramaswamy)

The aim of the course is to present a series of cutting-edge topics in the area of “Algorithmic trading” in a unified and systematic fashion. For each of the problems presented, we try to emphasize both the mathematical theory as well as industry applications. The course consists of two main parts: 1) Optimal Execution Problems and 2) Machine Learning in Finance. Optimal execution techniques are particularly relevant for market makers and quantitative brokers whereas machine learning is often used by hedge fund and prop desks to generate trading signals. However machine learning algorithms can be also applied as part of optimal execution tools, for example in order to choose order types or speed of execution. The basic optimal execution problem consists of an agent (e.g. a bank or a broker) who needs to buy or sell a pre-specified number of units of a given asset within a fixed time frame (e.g. an hour, a day, etc). Assuming that the purchase or sale of the asset will have an impact on its price, what is the execution policy which minimizes market impact? Having decided on the execution schedule, what type of order (market or limit order) is better to submit? The first problem can be formulated as a trade-off between the expected execution cost and the price risk due to exogenous factors. We shall solve the optimization problem for different types of

- Price dynamics (ABM vs GBM, with drift or without drift);
- Market impact type (temporary, transient, permanent);
- Exogenous Risk functions (variance, VaR).

Machine learning techniques are becoming increasingly popular in the financial industry. They are typically used to help predict asset price patterns, volatility regimes, etc. The course starts by formalizing the concept of "learning" and providing an overview of various learning techniques. The subsequent lectures analyze in detail some of the most popular machine learning algorithms such as neutral networks and support vector machines. We then introduce various smoothing tools (kernel regression, wavelets, HHTs) which have historically been developed for signal processing applications but have found their way into finance over the last few years. Those methods can be used as stand alone or jointly with other learning algorithms, e.g. SVM. Finally, we shall analyze issues related to model selection and how to combine different models to improve the learning outcome. Trading applications using real market data will be presented during the course

Dynamic portfolio theory (Dr. H. Zheng)

This is an introductory course on dynamic portfolio theory. The objective is to cover the basic mathematical methods for solving DPT problems. We will discuss Merton's optimal investment problem, utility maximization in complete and incomplete markets, stochastic control, dynamic programming principle, HJB equation, classical solution, verification theorem, viscosity solution, convex duality, martingale representation, dual stochastic control, Markov modulated model, etc. We will also discuss many applications, including utility indifference pricing, wealth maximization, optimal liquidation, turnpike property, mean-variance portfolio with constraints, quadratic hedging, etc.

Lévy processes: Theory and Applications (Dr. M. Pistorius)

In this course we present an introduction to the theory of Levy processes, a fundamental class of continuous time stochastic processes, which includes the Poisson process, the Wiener process and the stable process and which is encountered in many financial modelling applications. We start by considering jump-diffusions and develop the corresponding stochastic calculus for this class of stochastic processes. By way of illustration, a number of financial applications are presented. We then move on to infinitely divisible distributions, the Levy-Khintchine formula, Levy-Ito decomposition and discuss the path-wise construction and simulation of paths of general Levy processes. When time permits we cover elements of fluctuation theory and Markov process theory.

Simulation methods for finance (Dr J-F. Chassagneux)

This course is an introduction to simulation methods in finance and more generally to probabilistic numerical methods for PDEs. It starts with discussion of random number generators, statistical tests and moves on to cover numerical schemes for solving Stochastic Differential Equations: the Euler, Milstein and certain higher-order schemes. Properties of weak and strong convergence, consistency and numerical stability are established. It then discusses variance reduction techniques and estimation of sensitivities. The course will be concluded by studying a numerical method for American Options and non-linear PDEs, if time permits.

Forward-Backward stochastic differential equations and aplications (Dr J-F. Chassagneux)

In this course, we will first present the main results concerning Stochastic Differential Equations (strong and weak existence, uniqueness, regularity). We will then introduce the class of Backward Stochastic Differential Equations. In a Markovian setting, we will discuss the link with nonlinear parabolic PDE proving a nonlinear Feynmann-Kac formula. We will motivate the course by different examples, some coming from mathematical finance. If time permits, we will discuss the basics of the numerical approximation of those equations.

Summer term:

Advanced topics in mathematical finance: Nonlinear valuation under credit gap risk, initial and variation margins and funding costs (Prof. D. Brigo)

The market for financial products and derivatives reached an outstanding notional size of 708 USD Trillions in 2011, amounting to ten times the planet gross domestic product. Even discounting double counting, derivatives appear to be an important part of the world economy and have played a key role in the onset of the financial crisis in 2007. After briefly reviewing the Nobel-awarded option pricing paradigm by Black Scholes and Merton, hinting at precursors such as Bachelier and DeFinetti, we explain how the self-financing condition and Ito's formula lead to the Black Scholes Partial Differential Equation (PDE) for basic option payoffs. We hint at the Feynman Kac theorem that allows to interpret the Black Scholes PDE solution as the expected value under a risk neutral probability of the discounted future cash flows, and explain how no arbitrage theory followed. Following this quick introduction, we describe the changes triggered by post 2007 events. We re-discuss the valuation theory assumptions and introduce valuation under counterparty credit risk, collateral posting, initial and variation margins, and funding costs. We explain model dependence induced by credit effect, hybrid features, contagion, payout uncertainty, and nonlinear effects due to replacement closeout at default and possibly asymmetric borrowing and lending rates in the margin interest and in the funding strategy for the hedge of the relevant portfolio. Nonlinearity manifests itself in the valuation equations taking the form of semi-linear PDEs or Backward SDEs. We discuss existence and uniqueness of solutions for these equations. We also present a high level analysis of the consequences of nonlinearities, both from the point of view of methodology and from an operational angle. Finally, we connect these development to interest rate theory under multiple discount curves, thus building a consistent valuation framework encompassing most post-2007 effects.

In addition, every student on the course will attend two Optional Courses. These may be courses from the above list not already submitted for examination any courses offered by the

London Graduate School in Mathematics and Finance,

or the following courses

M5P7 **Functional analysis** (Dr G. Holzegel)

M3M3 **Introduction to partial differential equations** (Prof J.A. Carrillo de la Plata),

or any Core Course which you do no submit for examination.

Important Contacts:

• Postgraduate Tutor: Dr John Gibbons

• Overall Course Director MRes in Mathematical Sciences: Dr Thomas Cass

• Director for the Scientific Computation stream: Dr Eric Keaveney

• Director for the Statistics stream: Dr Nicholas Heard

• Director for the Finance and Stochastics stream: Dr Thomas Cass

• Course Administrator: Ms Anna Lisowska (a.lisowska@imperial.ac.uk, Office: 652 Huxley; Tel: 020 7594 2843)