DEPARTMENT OF MATHEMATICS

GUIDE TO MODULES

SECOND YEAR (BSc/MSci)
2016-2017

Notes and syllabus details on modules for students in their Second Year

For degree codings:

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NOTE that GG14, GG41, IG11 and GI43 MATHEMATICS AND COMPUTER SCIENCE are administered by the Department of Computing.

Prof David Evans
Director of Undergraduate Studies

May 2016

TO BE READ IN CONJUNCTION WITH THE UNDERGRADUATE HANDBOOK.
This information WILL be subject to alteration. Updated programmes can be viewed online at: https://www.imperial.ac.uk/natural-sciences/departments/mathematics/study/students/undergraduate/programme-information/
SECOND YEAR OVERVIEW

The programme of study in the Second Year extends and enhances major themes that feature in the First Year and takes place over three terms – Term 1 (also known as Autumn Term), Term 2 (also known as Spring Term) and Term 3 (also known as Summer Term).

The programme has ten examined mathematics modules that have up to 30 lectures or their equivalent. Seven of the modules are compulsory and students must then choose one module from the remaining three module options. (G1EB students do not have this choice and must instead take the module M2T Communicating Mathematics module which is not available to other students until the Third Year.) There is also the (compulsory) assessed M2R Group Project in Term 3, which involves written, oral and collaborative elements.

For the modules M2AA1, M2AA2, M2AA3, M2PM1, M2PM2, M2PM3, M2S1 the Department normally expects to run supporting problems classes. For other modules this provision is expected to be within the timetabled lectures – the normal expectation is that there should be a ‘lecture/class’ balance of about 5/1. The identification of particular class times within the timetabled periods is normally at the discretion of the lecturer, in consultation with the class and as appropriate for the module material.

The choice of 2nd Year option should not crucially restrict the availability of options in later years. It should be regarded as an opportunity to ‘taste’ special interests. This is especially true for the BSc degree codings G102, G125, G1F3, G1G3, G1GH, GG31, but apart from those on the G1EB coding, all students have a free choice. Note that one of the M2 optional modules can also feature as part of the Third Year choice if not taken in the Second Year.

**Computation:** There is no specifically computational module in the 2nd year. However, all other modules will be encouraged to set one assessed exercise which has a computational element.

**It is normally required that students pass ALL programme elements (including the M2R Group Project) in order to proceed into the Third Year.** Note also that it is normally required that students pass ALL programme elements in order to graduate. However, the College may condone a narrow fail in the Final Year of study in order to permit a student to graduate, but this cannot be guaranteed.

**MSci Degrees:** The first two years of study on the BSc and MSci degree codings are the same. Those students who were originally registered on the MSci coding and those who have transferred onto it are normally required to maintain a good level of performance in mathematics (at Upper Second Class level or better) in order to remain on the coding in the Third Year. Students on the MSci coding are also required to make satisfactory progress in the Third Year to remain on the Four Year programme.

**G103:** The primary criterion for eligibility to remain on G103 is to achieve a year total of at least 600 in Second Year. Students who score 600+ in Second Year, 580+ in Third Year and pass all their Third Year modules, have the automatic right to continue on to the Fourth Year of the MSci degree. Anyone scoring less than 580 in their Third Year, or who fails a module, does not have this right and may be graduated with a BSc at the Department’s discretion (which is exercised only rarely).

Those who score less than 600 in their Second Year may be allowed to remain on G103 at the Senior Tutor’s discretion but will have conditions set for their Third Year performance that take precedence over the rule above.

For further information on year totals, please see pages 4-5.

**G104:** Students registered for G104 Mathematics with a Year in Europe spend their Third Year (of four) studying mathematics modules/project material at another European institution. At the end of Second Year, students must be in a position to take advantage of the Third Year of the course, both mathematically and linguistically.

**Note:** Whilst G104 students must pass the language examinations at the end of First and Second Year in order to stay on G104, language examination results do not directly contribute to mathematics degree Honours marks.
ADVICE ON THE CHOICE OF OPTIONS

Students are advised to read these notes carefully and to discuss their programme with their Personal Tutor.

It is anticipated that lecturers will give advice on suitable books at the start of each module. Students should contact the proposed lecturers if they desire any further details about module content in order to make their choice of course options. Students should also feel free to seek advice from Year Level Tutors and the Senior Tutor, the Undergraduate Liaison Officer and the Director of Undergraduate Studies. Information may also be available online and in the College Library.

Course option choices must be registered on the designated website between the 8th June and the 1st of July, 2016. You will not be committed to taking those modules until the completion of your examination entry at the beginning of Term 2.

SECOND YEAR COURSE STRUCTURE

Term 1: ALL 4 COMPULSORY

- **MULTIVARIABLE CALCULUS**
  - M2AA2
  - Honours Mark - 100
  - ECTS Value - 7

- **REAL ANALYSIS**
  - M2PM1
  - Honours Mark - 100
  - ECTS Value - 7

- **ALGEBRA 2**
  - M2PM2
  - Honours Mark - 100
  - ECTS Value - 7

- **PROBABILITY AND STATISTICS 2**
  - M2S1
  - Honours Mark - 100
  - ECTS Value - 7

Term 2: 3 COMPULSORY, 1 OPTIONAL

- **INTRODUCTION TO NUMERICAL ANALYSIS**
  - M2AA3
  - Honours Mark - 100
  - ECTS Value - 7

- **COMPLEX ANALYSIS**
  - M2PM3
  - Honours Mark - 100
  - ECTS Value - 7

- **DIFFERENTIAL EQUATIONS**
  - M2AA1
  - Honours Mark - 100
  - ECTS Value - 7

- **COMMUNICATING MATHEMATICS**
  - M2T
  - for and only for G1EB
  - Honours Mark - 100
  - ECTS Value - 7

**OPTIONS (not G1EB)**

- **ONE FROM**
  - **NON-LINEAR WAVES**
    - M2AM
    - Honours Mark - 100
    - ECTS Value - 7

- **METRIC SPACES AND TOPOLOGY**
  - M2PM5
  - Honours Mark - 100
  - ECTS Value - 7

Term 3: REVISION CLASSES/EXAMINATIONS

- **GROUP PROJECT**
  - M2R
  - Honours Mark - 50
  - ECTS Value - 5
IMPERIAL HORIZONS

The College has created the ‘Imperial Horizons’ programme to broaden students’ education and enhance their career prospects. This programme is open to all undergraduate students.

The Department of Mathematics always endeavours to avoid timetabling Mathematics modules during the times allocated for Horizons modules.

Note that modules on this programme do not contribute to degree Honours marks but they do have an ECTS value of 6.

Further information about the ‘Horizon’ programme can be viewed online at: http://www3.imperial.ac.uk/horizons

EXAMINATIONS

In May/June there is one written examination in each of M2AM, M2AA1, M2AA2, M2AA3, M2PM1, M2PM2, M2PM3, M2PM5, M2S1, M2S2. The modules generally have an assessed coursework/progress test element which is limited to ~10% of the overall assessment in each case.

Students who do not obtain Passes in examinations at the first attempt will be expected to attend resit examinations where appropriate. September resits are available to Second Year students followed by resit opportunities the following May/June. Two resit attempts are normally available to students.

Note: Resits may not be offered for modules assessed solely by project.

Resit examinations are for Pass credit only – a maximum mark of 30 will be credited. Once a Pass is achieved, no further attempts are permitted.

Students who have not achieved the required Passes by the beginning of the new academic year are required by College to spend a year out of attendance. During this time they are not considered College students. This may create a number of issues and hold visa implications.

M2R GROUP PROJECT AND MINI-CONFERENCE

After the examinations in Term 3 there is the M2R Group Project, which counts in the assessment of Honours and must be passed for progression into the Third Year. The M2R Group Project has oral and collaborative elements and it counts for Honours marks up to the equivalent of half of one of the lectured modules.

Students are asked to select a broad subject area in which they wish to do a project and are allocated an appropriate group and a staff supervisor. A written project must be produced, for which all group members usually get the same mark. All groups then present their work orally at a mini-conference towards the end of term. An individual oral mark is then added to the written mark.

There is normally no resit opportunity within the session for the M2R Group Project

PROGRESSION TO THE THIRD YEAR

It is normally required that students pass ALL programme elements (including the M2R Group Project) in order to proceed into the Third Year.
HONOURS MARKS, YEAR TOTALS AND YEAR WEIGHTINGS

What follows is a brief summary – more details of these topics can be found in the Scheme for the Award of Honours online at:
https://www.imperial.ac.uk/natural-sciences/departments/mathematics/study/students/undergraduate/programme-information/

Within the Department each total module assessment is rescaled so that overall performances in different cases may be compared. The rescaling onto the scale 0 – 100 Honours marks is such that 30 then corresponds to the lowest Pass Honours mark and 75 corresponds to the lowest First Class performance. [These are not percentages!]

Note that Registry report individual module performances on transcripts using the uniform College Scale. On this scale a Pass mark is reported as 40 and the lowest First Class mark is 70. From 2014-2015 onwards, year average marks will also be reported on transcripts.

Information on the Mathematics and College scales can be viewed in the Course Handbook available online at the above link.

The total Second Year Honours mark available is 850, made up as 8x100 lecture modules together with 1x50 (M2R Group Project)

Note: For uniformity, the total Honours marks for each year are scaled out of 1000 and are known as a year total. Students are informed of their year total on Blackboard.

For three year BSc codings, the year weightings are 1 : 3 : 5.

For the four year MSci codings the year weightings are 1 : 3 : 4 : 5.

These differences in year weighting reflect the increasing level of mathematical complexity.

ECTS

To comply with the European ‘Bologna Process’, degree programmes are required to be rated via the European Credit Transfer System (ECTS) – which is based notionally on hour counts for elements in the degree.

Each Second Year mathematics module has an ECTS value of 7 with M2R having an ECTS value of 5. First Year mathematics modules have an ECTS value of 6.5 except for M1R which has an ECTS value of 4.5 and M1C which has an ECTS value of 4. Language modules, taken by G104 Mathematics with a Year in Europe students, have an ECTS value of 6.

ECTS ratings of Department of Mathematics courses and degrees can be viewed online at:
http://www.imperial.ac.uk/natural-sciences/departments/mathematics/study/students/undergraduate/programme-information/

MSci students who wish to increase their ECTS counts from roughly 240 to 270 must undertake additional study over the summer vacations of their Second and Third Years. Contact the Director of Undergraduate Studies for further information.
SECOND YEAR MATHEMATICS SYLLABUSES

COMPULSORY MODULES

M2AA1 DIFFERENTIAL EQUATIONS

Prof S. van Strien
Term 2

Differential equations play a key role in both pure and applied mathematics and also in many applications in physics, engineering, biology, economics, etc. Usually it is impossible to solve differential equations explicitly. In this course it is shown when solutions exist, and information is obtained about these solutions without necessarily having an explicit expression for these solutions.

Topics:

Contraction theorem: a general method in analysis for proving that solutions exist

Existence and uniqueness of solutions of ODEs

Linear systems

Power series solutions

Boundary value problems and Sturm-Liouville problems

Nonlinear theory

Dynamical Systems

M2AA2 MULTIVARIABLE CALCULUS

Dr A. Walton
Term 1

The calculus of functions of several independent variables is developed in this module, together with an introduction to partial differential equations. The importance of these ideas is emphasized by the inclusion of a number of applications in Science and Engineering.

• Fourier series: orthonormal systems, periodic functions, even and odd functions, fullrange series, the Gibbs phenomenon, Parseval’s theorem, half-range series, integration and differentiation of Fourier series, exponential form.
• Fourier transforms: exponential, cosine and sine transforms, elementary properties, convolution theorem, energy theorem, Dirac delta function.
• Partial differential equations: the wave equation: method of separation of variables, use of Fourier transforms, D’Alembert’s solution; the heat equation: solution using same methods
as above; Laplace and Poisson equation: types of boundary conditions, uniqueness of solution, point sources, Green's functions, method of images.

M2AA3 INTRODUCTION TO NUMERICAL ANALYSIS

Dr R. Nurnberg
Term 2

An important applied analysis module leading also to an introduction to some of the standard algorithms in numerical analysis.

Orthogonality: Inner/outer products on \( \mathbb{R}^n \), review linear dependence/independence, orthogonal/orthonormal vectors, classical Gram-Schmidt, orthogonal matrices, Givens rotations, QR factorization, Cauchy-Schwartz inequality.

Gradients/Hessians: Taylor series for \( f: \mathbb{R}^n \rightarrow \mathbb{R} \), classification of stationary points, positive definite matrices, generalised inner products on \( \mathbb{R}^n \), Cholesky factorization of symmetric positive definite matrices.

Least Squares Problems: Normal equations and QR approach for overdetermined systems \( Ax=b \), inner products on a general vector space, least squares problems, Gram matrices, Hilbert matrix.

Orthogonal Polynomials: Monic orthogonal polynomials, three term recurrence relationship, best approximation by polynomials in inner product induced norms, Chebyshev and Legendre polynomials. Fourier series.

Polynomial Interpolation: Lagrange, Newton and canonical forms of interpolating polynomial, existence and uniqueness, divided differences, error analysis, Runge phenomenon.

Mini-Max Approximation: Best approximation by polynomials in the uniform sense, Chebyshev equioscillation theorem, optimal interpolation points – zeros of Chebyshev polynomial.

Gaussian Quadrature: Optimal sampling points – zeros of corresponding polynomial.

Visualisation through Matlab commands and graphics.

M2PM1 REAL ANALYSIS

Dr C. Warnick
Term 1

The aim of this module is to provide a rigorous foundation for the differential and integral calculus.

Review of differentiation of functions in one variable: Rolle’s theorem, mean value theorem, Taylor’s theorem.

Uniform continuity and convergence: definitions and applications.

Integration in one dimension: the Darboux integral; the fundamental theorem of calculus; interchanging limits; applications.

Differentiation of functions from \( \mathbb{R}^n \) to \( \mathbb{R}^m \): definition, directional derivatives, Schwartz theorem; inverse function theorem.

Multi-variable integration: the Darboux integral on \( \mathbb{R}^n \); Green’s theorem in the plane.

M2PM2 ALGEBRA 2

Dr J. Britnell
Term 1

A continuation from Algebra I in Groups and Linear Algebra.

Groups: Homomorphisms, isomorphisms, normal subgroups, first isomorphism theorem. Fundamental theorem of abelian groups, groups of small order. Group actions, Burnside’s lemma, applications.


M2PM3 COMPLEX ANALYSIS
Prof A. Laptev  
Term 2

*This module concerns continuity, differentiation and integration of complex valued functions, with applications to real integration.*

The complex numbers; algebraic and metric structure. Sequences and series of complex numbers. Power series. Exponential, Trigonometric and Logarithmic functions. Complex functions; continuity, differentiability, behaviour on compact sets.

Functions defined by power series. Line integrals, theorem of the primitive, the ML inequality. Cauchy’s Theorem for a triangle, a star domain; computation of real integrals. Local theory for analytic functions; uniform convergence of Power Series, Cauchy’s Integral Formula, Taylor’s Theorem, Cauchy Estimates. Liouville’s Theorem.


Classification of Singularities. The Residue Theorem; more real integrals!

**M2S1  PROBABILITY AND STATISTICS 2**

Prof D. A. van Dyk  
Term 1

*This module extends the ideas met in M1S and introduces the distributional results needed for the study of statistical inference and applied probability.*


**OPTIONAL MODULES**

**M2AM  NON-LINEAR WAVES**

Prof D. Papageorgiou  
Term 2

*This module considers the dynamics of a continuous medium or fluid. One dimensional flows and waves are considered in detail to model gas dynamics and water waves as well as models of traffic flow. Shock formation and propagation in single or two by two systems of conservation laws are developed and solutions constructed for different problems. The course concludes with the theory of water waves including progressing and standing waves.*

The continuum hypothesis and fluid particles.
Eulerian and Lagrangian descriptions of ID fluid motion.
Fluid acceleration, material derivative.
Simple problems in finding position, given velocity and vice versa.
Conservation of mass, equation of continuity.
Pressure and gravitational forces.
Conservation of momentum – Euler’s equation.
Non-linear solutions leading to kinematic wave equation and general implicit solution.
Wave steepening. The Burgers equation.
Method of characteristics.

**M2PM5  METRIC SPACES AND TOPOLOGY**

Prof T. Coates
Term 2

*This module extends various concepts from analysis to more general spaces.*

Metric spaces. Convergence and continuity. Examples (Euclidean spaces, function spaces; uniform convergence). The open sets in a metric space; equivalent metrics. Convergence and continuity in terms of open sets: topological spaces. Subspaces. Hausdorff spaces. Sequential compactness; compactness via open covers; compact spaces; determination of compact subspaces of \( \mathbb{R}^n \). Completeness in metric spaces. Relationship between compactness and completeness. Connected and path connected spaces; equivalence of these notions for open sets in \( \mathbb{R}^n \). Winding numbers, definition of fundamental group, its computation for the circle. Example: proof of fundamental theorem of algebra.

**M2S2  STATISTICAL MODELLING 1**

Dr B. Calderhead
Term 2

*Traditional concepts of statistical inference, including maximum likelihood, hypothesis testing and interval estimation are developed and then applied to the linear model that arises in many practical situations.*

Maximum likelihood estimation, likelihood ratio tests and their properties, confidence intervals. Linear models - including non-full rank models: estimation, confidence intervals and hypothesis testing. The analysis of variance.

**DEGREE COURSE CODING REQUIREMENTS**

All modules within the Department are registered for G100, G103 Mathematics. To qualify for a degree a student must satisfy the overall College requirements.

To qualify for the BSc codings G102, G125, G1F3, G1G3, G1GH, GG31, a suitable number of modules must eventually be passed from subsets of the general list as follows: (this list may change from year to year and some of the modules listed may not be available in a particular year)

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<th>Course Code</th>
<th>Modules</th>
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<td>G102</td>
<td>4 from</td>
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<tr>
<td>G125</td>
<td>6 from</td>
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**Mathematics with Mathematical Computation**
- M3N3, M3N4, M3N7, M3N10, M3SC, M3R, M3C.

**Mathematics (Pure Mathematics)**
- M2PM5, M3P5, M3P6, M3P7, M3P8, M3P10, M3P11, M3P12, M3P14, M3P15, M3P16, M3P17, M3P18, M3P19, M3P20, M3P21, M3P22, M3P23, M360, M3R.
G1F3  6 from Mathematics with Applied Mathematics/Mathematical Physics
M2AM, M3A2, M3A4, M3A6, M3A7, M3A10, M3PA16, M3A21, M3F22, M3PA23, M3PA24, M3PA46, M3PA48, M3A25, M3A28, M3A29, M3PA34, M3M3, M3M6, M3M9, M3SC, M3R.

G1G3  6 from Mathematics with Statistics
M2S2, M3S1, M3S2, M3S3, M3S4, M3S7, M3S8, M3S9, M3S10, M3S11, M3S12, M3S14, M3S15, M3S16, M3S17, M3R.

G1GH  6 from Mathematics with Statistics for Finance
M2S2, M3A22, M3S1, M3S2, M3S3, M3S4, M3S7, M3S8, M3S9, M3S10, M3S11, M3S12, M3S14, M3S15, M3S16, M3S17, M3SC, M3R.

GG31  6 from Mathematics, Optimisation and Statistics
M2S2, M3S1, M3S2, M3S3, M3S4, M3S7, M3S8, M3S9, M3S10, M3S11, M3S12, M3S14, M3S15, M3S16, M3S17, M3P17, M3P22, M3N3, M3SC, M3R.

It is generally possible to swap between the various BSc codings, at a fairly late stage, even in the 3rd year.

As part of the continuing review of the undergraduate programme of study, amendments to this list can be expected. The above are the normal requirements – the Department has the discretion to modify them.

THIRD AND FOURTH YEAR

In the 3rd and 4th years there is a vast selection of optional modules, though the Third and Fourth Year syllabuses substantially overlap. Third and fourth year examinations are normally 2 and 2.5 hours respectively.

For details of the current Third and Fourth Year programmes, see the relevant online documentation online at: http://www.imperial.ac.uk/natural-sciences/departments/mathematics/study/students/undergraduate/programme-information/

M4R Project: In the 4th year of an MSci, a substantial M4R project is required, equivalent to 2 lectured modules.

G104: For the Mathematics with a Year in Europe course G104 the Third Year is spent studying at a host institution elsewhere in Europe. On the rare occasion that a G104 student performs very poorly in their year away, they may, at the discretion of the Senior Tutor, be transferred onto a BSc on their return.

G1EM: For the Maths with Education MSci, the project is only half the M4R length, and takes place only in the second term. During the first term students take Education modules rather than Mathematics ones.