Group Theory

This is a course on group theory, representation theory and their applications to quantum mechanics. The main part of the course deals with discrete finite groups, while a brief introduction into infinite continuous groups is given in the last part. The main objective is to make the participants fluent in the language of representation theory and confident in its applications to non-relativistic quantum mechanics.

The subjects covered are as follows:

1. Abstract group theory for finite groups: properties of the group, subgroups, cosets, Lagrange's theorem, invariant subgroup, factor-group, conjugate classes, Abelian vs non-Abelian groups, isomorphism, homomorphism, symmetric group, Cayley's theorem.

2. Symmetry groups, point groups and their classification.

3. Representation theory for finite groups: existence of unitary representation, reducible and irreducible representations, Schur's lemmas, orthogonality relations, characters and character tables, expansion in the basis functions of irreducible representations, representations of direct products.

4. Quantum mechanical applications: Wigner's theorem, molecular vibrations, molecular orbitals as linear combination of atomic orbitals, selection rules, crystal field.

5. Continuous groups: Lie groups, and Lie algebras, irreducible representations of SO(2) and SO(3), unitary groups.