Nuclear Thermal-Hydraulics

PWR Question

(a) Typical PWR temperature distributions

(b)

\[ \dot{m}C_p \frac{dT}{dz} = \dot{q}'(z) \]

So the coolant temperature at any location is given by:

\[ T(z) - T_{in} = \frac{1}{\dot{m}C_p} \int_{-L}^{z} \dot{q}'(z) \, dz \]

For a cylindrical reactor

\[ \dot{q}'(z) = \hat{q}' \cos \left( \frac{\pi z}{L} \right) \]

so using this

\[ T(z) - T_{in} = \frac{\hat{q}'}{\dot{m}C_p} \int_{-L}^{z} \cos \left( \frac{\pi z}{L} \right) \, dz \]

or
\[ T(z) - T_{in} = \frac{\hat{q}'}{\dot{m}C_p \pi} \left[ \sin \left( \frac{\pi z}{L} \right) - \sin \left( \frac{-\pi L}{2} \right) \right] = \frac{\hat{q}'}{\dot{m}C_p \pi} \left[ \sin \left( \frac{\pi z}{L} \right) - \sin \left( \frac{-\pi}{2} \right) \right] \]

or

\[ T(z) - T_{in} = \frac{\hat{q}'}{\dot{m}C_p \pi} \left( \sin \left( \frac{\pi z}{L} \right) + 1 \right) \]

At any location \( z \) the difference between clad surface temperature \( T_c \) and the bulk coolant temperature \( T \) is proportional to the local surface heat flux: That is

\[ (T_c(z) - T(z)) 2\pi r_c h = \hat{q}'(z) \]

where \( h \) is the heat transfer coefficient and \( r_c \) the clad outer radius.

So

\[ T_c(z) = T(z) + \frac{1}{2\pi r_c h} \hat{q}'(z) \]

Using \( T(z) \) from above, and \( \hat{q}'(z) \), we can write

\[ T_c(z) = T_{in} + \frac{\hat{q}'}{\dot{m}C_p \pi} \left( \sin \left( \frac{\pi z}{L} \right) + 1 \right) + \frac{1}{2\pi r_c h} \hat{q}' \cos \left( \frac{\pi z}{L} \right) \]

or

\[ T_c(z) = T_{in} + \frac{\hat{q}'}{\dot{m}C_p \pi} \left( \sin \left( \frac{\pi z}{L} \right) + 1 \right) + \frac{1}{2r_c h} \hat{q}' \cos \left( \frac{\pi z}{L} \right) \]

Differentiating the above we get:

\[ \frac{\partial T_c(z)}{\partial z} = \frac{\hat{q}'}{\pi} \left[ \frac{L}{\dot{m}C_p \pi} \cos \left( \frac{\pi z}{L} \right) - \frac{\pi}{2r_c h} \frac{L}{\dot{m}C_p \pi} \sin \left( \frac{\pi z}{L} \right) \right] \]

Setting to zero we have at the maximum-temperature location:

\[ \frac{z}{L} = \frac{1}{\pi} \arctan \left( \frac{2r_c h L}{\dot{m}C_p \pi} \right) \]

(c) numbers:

**Physical properties**

- Coolant density: kg/m³ 7.13000E+02
- Specific heat: J/(kg K) 5.79400E+03
- Viscosity: kg m⁻¹ s⁻¹ 9.07000E-05
- Conductivity: W/mK 5.41000E-01

**Problem data**

- Pin length: m 3.66000E+00
- Pin od: m 9.50000E-03
- Pin square pitch: m 1.26000E-02
- Mass flow rate per pin: kg/s 3.66000E-01
- Peak linear rating: W/m 4.13000E+04
Coolant inlet temperature K 5.66500E+02

Computed geometric quantities
flow area per pin m^2 8.78797E-05
wetted perimeter m 2.98451E-02
hydraulic diameter m 1.17797E-02

Dimensionless numbers
Reynolds number - 5.40830E+05
Prandtyl No. - 9.71379E-01
Nu from D-B - 8.77267E+02

Heat transfer coefficient W m^-2 K^-1 4.02961E+04

Convenient intermediate nos.
q-dot-dash-max/m-dot Cp*(L/pi) 2.26894E+01

Location of T clad max 6.80211E-01
Value of T clad max 6.30349E+02

(d) Unchanged. Heat generation not function of flow rate.