ODEBND_GSL (version 1.0)

ODEBND_GSL is a C++ class that computes enclosures of the reachable set of parametric nonlinear ordinary differential equations (ODEs) of the form

\[
\forall t \in [t_0, t_i] \quad \dot{x}(t) = f(x(t), p) \quad \text{with} \quad x(t_0) = h(p),
\]

where \( p \in P \subset \mathbb{R}^{n_p} \) denote the parameter vector; \( x(t) \in \mathbb{R}^{n_x} \), the state vector at a given time \( t \); \( f: \mathbb{R}^{n_x} \times \mathbb{R}^{n_p} \to \mathbb{R}^{n_x} \), the right-hand side function; and \( h: \mathbb{R}^{n_p} \to \mathbb{R}^{n_x} \), the initial value function. It is assumed that \( f \) and \( h \) are factorable and sufficiently many times continuously differentiable.

The algorithms in ODEBND_GSL rely on continuous-time set-propagation methods, whereby a set of auxiliary ODEs is constructed whose solutions provide the desired enclosures. Two different approaches are implemented, which enable the propagation of convex enclosures and nonconvex enclosures, based on interval arithmetic and Taylor model arithmetic, respectively. In either approaches, various contractors can be used, including:

- parallelepiped contractor
- ellipsoidal contractor [Houska et al., 2012]

Combination of these contractors with Taylor models are described, for instance, in [Chachuat & Villanueva, 2012; Villanueva et al., 2013a]. Moreover, the construction and convergence analysis of these methods is described in [Villanueva et al., submitted], whereby a generalized differential inequality is introduced whose solutions yield such support functions for a convex enclosure of the reachable set. In particular, it is shown that existing continuous-time propagation techniques based on standard differential inequalities or ellipsoidal set propagation techniques can be recovered as special cases of this generalized differential inequality, and a way of extending this approach for the construction of nonconvex enclosures is also described based on Taylor models with convex remainder bounds.

Getting and Using ODE BND_GSL
ODEBND_GSL is released as open source code under the Eclipse Public License (EPL). A link to the current version is: ODEBND_GSL_1.0.tgz

ODEBND_GDL uses a number of third-party libraries. The following ones are mandatory:

- Library **MC++** (version 1.0) for convex/concave relaxation, Taylor model bounding, and spectral bounding of factorable functions
- Library **FADBAD++** (version 2.1) for automatic differentiation (AD) using forward, backward and Taylor expansion methods
- ODE Integrator gsl_odeiv2 in GNU Scientific Library (GSL, version 1.15)
- Libraries **BLAS** and **LAPACK** (version 4.3) for linear algebra computations

Optional libraries include:

- Libraries **PROFIL** and/or **FILIB++** for verified interval arithmetic