In the following questions, the integrals are Minkowskian, the dimensionality of spacetime is $d = 4 - \epsilon$, and terms of order $O(\epsilon)$ and higher are ignored in the final answers. You are also given that

$$I_n(m) := \mu^\epsilon \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - m^2 + i\epsilon)^n} = \frac{i(-1)^n m^{d-2n}}{16\pi^2} \left( \frac{4\pi \mu^2}{m^2} \right)^{\epsilon/2} \frac{\Gamma(n-2+\frac{1}{2}\epsilon)}{\Gamma(n)} .$$

so that up to term of order $O(\epsilon)$

$$I_1(m) = \frac{i m^2}{16\pi^2} \left( \frac{2}{\epsilon} + \log \frac{4\pi \mu^2}{m^2} + 1 - \gamma \right) ,$$

$$I_2(m) = \frac{i}{16\pi^2} \left( \frac{2}{\epsilon} + \log \frac{4\pi \mu^2}{m^2} - \gamma \right) .$$

1. Use Lorentz invariance to show that

$$\mu^\epsilon \int \frac{d^d k}{(2\pi)^d} \frac{k^\mu}{k^2 - m^2} = 0 .$$

2. Given that, expanding in small $z$ to order $O(z)$, we have

$$\Gamma(z) = \frac{1}{z} - \gamma , \quad \Gamma(-2 + z) = \frac{1}{2} \left( \frac{1}{z} - \gamma + \frac{3}{2} \right) ,$$

argue that

$$\lim_{n \to 0} \frac{\Gamma(n-2+\frac{1}{2}\epsilon)}{\Gamma(n)} = 0 ,$$

and hence in dimensional regularisation $I_0(m) = 0$.

3. Using the expression for $I_1(m)$, that $I_0(m) = 0$ and the identity $g_{\mu\nu} g^{\mu\nu} = d$, show that

$$\mu^\epsilon \int \frac{d^d k}{(2\pi)^d} \frac{k^\mu k^\nu}{k^2 - m^2} = \frac{m^2 g^{\mu\nu}}{d} I_1(m) = \frac{im^4 g^{\mu\nu}}{64\pi^2} \left( \frac{2}{\epsilon} + \log \frac{4\pi \mu^2}{m^2} + \frac{3}{2} - \gamma \right) .$$

4. The Feynman parameter expression reads

$$\frac{1}{AB} = \int_0^1 \frac{dx}{[Ax + B(1-x)]^2} .$$

Taking $A = (k + p)^2 - m^2 + i\epsilon$ and $B = k^2 - m^2 + i\epsilon$ and changing variables to $k'^\mu = k^\mu + xp^\mu$ show that

$$\mu^\epsilon \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - m^2 + i\epsilon)((k + p)^2 - m^2 + i\epsilon)} = \int_0^1 dx I_2(\sqrt{m^2 - x(1-x)p^2}) = \frac{i}{16\pi^2} \left( \frac{2}{\epsilon} + \int_0^1 dx \log \frac{4\pi \mu^2}{m^2 - x(1-x)p^2} - \gamma \right) .$$

5. Hence show that in the massless limit ($m^2 = 0$)

$$\mu^\epsilon \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 + i\epsilon)((k + p)^2 + i\epsilon)} = \frac{i}{16\pi^2} \left( \frac{2}{\epsilon} + \log \frac{4\pi \mu^2}{-p^2} + 2 - \gamma \right) .$$
6. Write $p_\mu k^\mu$ in terms of $p^2$, $k^2$ and $(p + k)^2$. Hence show that

$$\mu^\epsilon \int \frac{d^d k}{(2\pi)^d (k^2 - m^2 + i\epsilon)((k + p)^2 - m^2 + i\epsilon)} = -\frac{1}{2} p^2 \int_0^1 dx I_2 (\sqrt{m^2 - x(1 - x)p^2}).$$

7. What is

$$\mu^\epsilon \int \frac{d^d k}{(2\pi)^d (k^2 - m^2 + i\epsilon)((k + p)^2 - m^2 + i\epsilon)} k^\mu?$$