Particle Symmetries Problem Set 4 (2012)

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Representation Lattices

1. For the simple roots, $\alpha_1 = (1, 0), \alpha_2 = (-1, 1)$ compute the Cartan matrix.

2. Identify the algebra which has this root system and find the fundamental weights.

3. Plot the weights of the representations [1, 0] and [0, 1] on a 2 dimensional lattice. What is the type of lattice which is generated by the fundamental weights?

4. What is the shape of the lattice for the representation \([n, 0]\)? What is the length measured by \(n\)? Compute the multiplicity of inner points on the lattice.

5. What is the shape of the lattice for the representation \([0, n]\)? What is the length measured by \(n\)? Compute the multiplicity of inner points on the lattice.

6. What is the shape of the lattice for the representation \([n_1, n_2]\)? What are the lengths measured by \(n_1, n_2\)? Compute the multiplicity of inner points on the lattice.