Dimension formulas

1. Compute the dimension formula for an irreducible representation of $SO(7)$, with Dynkin labels $[n_1, n_2, n_3]$. Find the dimensions of the basic representations $[1, 0, 0], [0, 1, 0], \text{ and } [0, 0, 1]$ and identify their names.

2. Compute the dimension formula for an irreducible representation of $Sp(3)$, with Dynkin labels $[n_1, n_2, n_3]$. Find the dimensions of the basic representations $[1, 0, 0], [0, 1, 0], \text{ and } [0, 0, 1]$ and identify their names.

Dynkin labels

3. Find the Dynkin labels and the highest weight of the following representations:
   
   (a) 3rd rank antisymmetric representation of $SO(7)$.
   (b) $n$-th rank antisymmetric representation of $SO(2n + 1)$.
   (c) $(n - 1)$-th rank antisymmetric representation of $SO(2n)$.
   (d) $n$-th rank antisymmetric representation of $SO(2n)$.

Tensor products

4. Find the tensor product decomposition of
   
   (a) $\text{Sym}^k([1, 0]_{G_2})$
   (b) $\text{Sym}^k([0, 0, 1]_{B_3})$
   (c) $\text{Sym}^k([1, 0, \ldots, 0]_{B_n})$
   (d) $\text{Sym}^k([1, 0, \ldots, 0]_{D_n})$
   (e) $\wedge^k([1, 0]_{G_2})$