Scalar Yukawa Model — SYTh

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Yukawa's original theory (see Brown's "Hideki Yukawa and the Meson Theory" Physics Today **39** (1986) 55) described the interaction between nucleons, protons and neutrons, by suggesting a new particle or field, the meson. We will adapt this theory by using a pair of charged scalar particles instead of the fermionic nucleons in the original theory. My motivation is to use this simple model to illustrate the important features found in Feynman rules when you have charge (quantum number) conservation. This is also one of the two main examples used by Tong [Tong (3.7), p.49].

Key Properties:-

- One spin zero ϕ-particle which is its own anti-particle.
  - Plays the role of the "meson" of Yukawa's theory.
  - Mass m, dispedrsion relation $\omega_{\boldsymbol{k}} = \sqrt{\boldsymbol{k}^2 + m^2}$
  - Number not conserved, i.e. charge 0.
  - Created by $\hat{a}^\dagger_{\boldsymbol{k}}$.
  - Represented by real scalar field $\hat{\phi}(x)$ where in the interaction picture

$$\hat{\phi}(x) \;=\; \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \frac{1}{\sqrt{2\omega(\boldsymbol{k})}} \left(\hat{a}_{\boldsymbol{k}} e^{-ikx} + \hat{a}^\dagger_{\boldsymbol{k}} e^{+ikx}\right), \quad k_0 = \omega(\boldsymbol{k}) \tag{1}$$

- One particle/anti-particle pair of spin zero particle, the ψ-particle and its anti-particle the $\bar{\psi}$.
  - These play the role that the nucleons had in Yukawa's theory.
  - Mass M, dispersion relation $\Omega_{\boldsymbol{k}} = \sqrt{\boldsymbol{k}^2 + M^2}$
  - Total ψ particle number conserved, i.e. $(\#\psi) - (\#\bar{\psi})$ is conserved.
    We may choose ψ to have charge $+1$ and $\bar{\psi}$ to have charge -1.
  - A ψ particle is created by $\hat{b}^\dagger_{\boldsymbol{k}}$, while a $\bar{\psi}$ quanta is created by $\hat{c}^\dagger_{\boldsymbol{k}}$.
  - Represented by complex scalar field $\hat{\psi}(x)$ and its hermitian conjugate $\hat{\psi}^\dagger(x)$ where $\hat{\psi}(x) \neq \hat{\psi}^\dagger(x)$ and in the interaction picture

$$\hat{\psi}(x) \;=\; \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \frac{1}{\sqrt{2\Omega(\boldsymbol{k})}} \left(\hat{b}_{\boldsymbol{k}} e^{-ikx} + \hat{c}^\dagger_{\boldsymbol{k}} e^{ikx}\right), \quad k_0 = \Omega(\boldsymbol{k}) \tag{2}$$

The dynamics of the scalar Yukawa theory is described by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m^2\phi^2 + (\partial_\mu\psi^\dagger)(\partial^\mu\psi) - M^2\psi^\dagger\psi - g\psi^\dagger(x)\psi(x)\phi(x)\,. \tag{3}$$

The annihilation and creation operators obey the usual commutation relations

$$\left[\hat{a}_{\boldsymbol{p}}, \hat{a}^\dagger_{\boldsymbol{q}}\right] = (2\pi)^3\delta^3(\boldsymbol{p}-\boldsymbol{q})\,, \qquad \left[\hat{a}_{\boldsymbol{p}}, \hat{a}_{\boldsymbol{q}}\right] = \left[\hat{a}^\dagger_{\boldsymbol{p}}, \hat{a}^\dagger_{\boldsymbol{q}}\right] = 0\,. \tag{4}$$

Both the $\hat{b}$, $\hat{b}^\dagger$ pair and the $\hat{c}$ and $\hat{c}^\dagger$ pair of annihilation and creation operators obey similar commutation relations to those of the $\hat{a}$ and $\hat{a}^\dagger$ pair. Different types of annihilation and creation operator always commute e.g. $\left[\hat{a}_{\boldsymbol{p}}, \hat{b}^\dagger_{\boldsymbol{q}}\right] = 0$.