Feynman Rules for SYTh in Coordinate Space

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The scalar Yukawa theory for a real scalar field ϕ of mass m and a complex scalar field ψ of mass M has a cubic interaction with real coupling constant g and the Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2 + (\partial_\mu \psi^\dagger)(\partial^\mu \psi) - M^2 \psi^\dagger \psi - g \psi^\dagger(x) \psi(x) \phi(x) \, . \tag{1}$$

The Feynman rules for scalar Yukawa theory of (1) Green functions in coordinate space are as follows.

0. Draw all graphs (i.e. Feynman diagrams) which are topologically distinct and which differ in their external vertex labelling[1].

1. Each line is associated with a factor of $\Delta_f(x-y)$ for a field f of mass m_f where $\Delta_f(x-y)$ is the relevant contraction (i.e. the relevant Feynman propagator, in momentum space this is $\Delta_f(p) = i(p^2 - m_f^2 + i\epsilon)^{-1}$), and x and y are the coordinates associated with vertices at the end of each line.

   (a) For field ϕ this is a scalar propagator $\Delta_\phi(x-y)$ with mass $m_f = m$ and will be denoted by a dashed line with no arrows as ϕ is its own anti-particle.

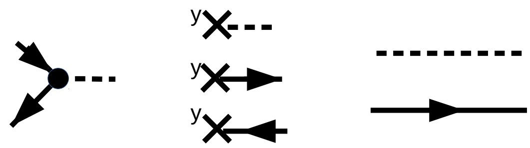
   (b) For field ψ this is a scalar propagator $\Delta_\psi(x-y)$ with mass $m_f = M$ and will be denoted by a solid line with an arrow from a $\psi^\dagger$ field at a vertex to a ψ field at another vertex[2] due to the distinction between ψ and $\psi^\dagger$.

2. There is one type of **internal vertex**, unlabelled for purposes of symmetry factor calculations but associated with some dummy coordinate x_i and a factor of $-ig \int d^4 x_i$. This vertex has three legs: one ϕ propagator leg, one ψ propagator leg (arrow in), and one $\psi^\dagger$ propagator leg (arrow out) to be consistent with my convention on lines.

3. An **external vertex** is labelled with the of the associated explicit field in the expectation value so is one of the coordinates of the Green function. I use coordinates y_i or z_f for these fields. We link these to a particle in an initial or final state particle in the corresponding matrix element $\mathcal{M}$.
   Note in the Green function and so in these rules there is no integration over these coordinates. So this is not an x_i dummy coordinate of a field coming from the S-matrix expansion. Also note that the same diagram for a Green function can be used as a contribution for several different processes so any one external leg can be linked to different initial or final states when one uses it to calculate matrix elements.

4. We divide by the **symmetry factor** $\mathcal{S}$ for the diagram. This is the number of permutations of internal lines which leave the diagram invariant.

The diagrammatic elements are where (a) is the the only internal vertex (no labels), (b) are the three external vertices (distinct labels), and (c) are the two propagators $\Delta_f(x-y)$.



(a) (b) (c)

[1]Note the external vertices/legs carry a label but the other vertices/legs do not. We can sometimes swap these labels to produce a diagram of the same shape but with a different labelling and this represents a distinct contribution.

[2]The opposite convention for arrow directions also works but you must be consistent in your choice.