

Tong 3.7
P75
NOT considering 20.7

Full Vacuum

$|0\rangle$ is the vacuum of the free theory $\Rightarrow \hat{a}_p |0\rangle = 0$

$|\Omega\rangle$ is the ^{PHYSICAL} vacuum of the full (interacting) theory
 $\Rightarrow H|\Omega\rangle = E_0$ lowest energy, with $E_0 \neq 0$
 NOT THE SAME! $|\Omega\rangle$ is full of virtual particles
 $|0\rangle$ is empty NOT (time) inv.

We want initial/final states built on $|\Omega\rangle$ NOT $|0\rangle$
 $\Rightarrow$ We want $M = \langle \Omega | S | \Omega \rangle$ NOT $\tilde{M} = \langle 0 | S | 0 \rangle$

Lemma

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For any state $|\psi, t\rangle_S$

$\hat{a}|0\rangle$

then $\lim_{t_i \rightarrow -\infty} \langle \psi, t | \phi_i \rangle_S = \lim_{t_i \rightarrow -\infty} \langle \psi, t | U(t, t_i) | 0 \rangle_I$

& we find $\lim_{t_i \rightarrow -\infty} \langle \psi, t | \phi_i \rangle_S \rightarrow \langle \psi | \Omega \rangle \langle \Omega | 0 \rangle$

& $\lim_{t_f \rightarrow +\infty} \langle 0, t_f | \psi, t \rangle_S \rightarrow \langle 0 | \Omega \rangle \langle \Omega | \psi, t \rangle$

20.23

Meaning

For long time separations only the overlap between $|0\rangle$ & $|\Omega\rangle$ is significant, ^{contribution from} higher energy states insignificant (cancel out)

So for all

we can substitute $|\Omega\rangle \equiv \frac{1}{Z^{1/2}} |0\rangle$

where $Z = \langle 0 | S | 0 \rangle$

$$\Rightarrow \langle \Omega | T(\text{Anything}) | \Omega \rangle = \frac{\langle 0 | T(\text{Anything}) | 0 \rangle}{\langle 0 | S | 0 \rangle}$$

(All I pic.)

time notation : ()

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Proof

Consider $\langle \psi, t | 0, t_i \rangle =$

$$\Rightarrow = \langle \psi | U(t, t_i) | 0 \rangle \text{ in I.p.c}$$

$$= \langle \psi | \underbrace{U_{\text{full}}}_{H_{\text{full}} = H_0 + H_{\text{int}}} e^{-i \int_{t_i}^t H_{\text{full}}(\tau) d\tau} | 0 \rangle \text{ with } H = H_0 + H_{\text{int}}$$

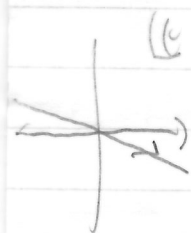
as $H_0 | 0 \rangle = 0$ (wlog, ignoring c-numbers)

Insert complete set of energy e/states using $| \alpha \rangle$ as vacuum with $\{ | n \rangle \}$ as excited states $H | n \rangle = E_n | n \rangle$

$$= \langle \psi | U_{\text{full}}(t, t_i) [| \alpha \rangle \langle \alpha | + \sum_n | n \rangle \langle n |]$$

$| 0 \rangle$

$$= \langle \psi | \alpha \rangle \langle \alpha | 0 \rangle$$



$$+ \sum_n e^{-i E_n (t - t_i)} \langle \psi | n \rangle \langle n | 0 \rangle$$

As $E_n > 0$

OR

Riemann-Lebesgue theorem for

THEN send

$t_i \rightarrow -\infty$ (i.e.)

& this term

damps away

$$\lim_{b \rightarrow \infty} \int_a^b dx f(x) e^{imx} = 0$$

"well" behaved func

when energy continuous.

Fast Oscillating ($E_n > 0$) terms die away relative to lowest energy term.

$$\Rightarrow \lim_{t_i \rightarrow -\infty} \langle \psi, t | 0, t_i \rangle \rightarrow \langle \psi | \alpha \rangle \langle \alpha | 0 \rangle$$

(likewise for $t_f \rightarrow +\infty$)

Matrix Elements & Green Functions & Vacuum

We really want

$$M = \langle F | S | i \rangle$$

$$= \langle \Omega | (\text{annihilation}) \circ S (\text{creation}) | \Omega \rangle$$

$$G_c = \langle \Omega | T(\text{fields}) \circ S(\text{fields}) | \Omega \rangle$$

↑ Connected ↑ Full Interacting Vacuum

c.f. $G = \langle 0 | T(\text{fields}) S | 0 \rangle$

Lemma

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$$\langle \Omega | T(\text{fields}, S) | \Omega \rangle = \frac{\langle 0 | T(\text{fields}) S | 0 \rangle}{\langle 0 | S | 0 \rangle}$$

$$G_c(\{y\}) = \frac{1}{Z} G(\{y\})$$

Proof

$$\text{RHS} = \frac{\langle 0 | \Omega \rangle \langle \Omega | T(\text{fields}) | \Omega \rangle \langle \Omega | 0 \rangle}{\langle 0 | \Omega \rangle \langle \Omega | S | \Omega \rangle \langle \Omega | 0 \rangle}$$

$$\langle \Omega, t_f | \Omega, t_i \rangle = 1 \text{ Vacuum state stable } (E_0 = 0)$$

$$= \langle \Omega | T(\text{fields}) | \Omega \rangle$$

Matrix Elements

We really want to build on true physical vacuum.

$$\mathcal{M} = \langle F | S | i \rangle$$

$$= \langle \Omega | \left(\begin{matrix} \text{final} \\ \text{fields} \end{matrix} \right) S \left(\begin{matrix} \text{initial} \\ \text{fields} \end{matrix} \right) | \Omega \rangle$$

$$= \sum_{\text{ALL DIAGRAMS}} \langle \Omega | T(\text{fields}) | \Omega \rangle$$

$$\mathcal{M} = \frac{1}{Z} \sum_{\text{ALL DIAGRAMS}} \langle 0 | T(\text{fields}) | 0 \rangle$$

$$\uparrow$$

$$\langle 0 | S | 0 \rangle$$

Rules calculated above

What is $Z = \langle 0 | S | 0 \rangle$?

If $H_{int} = 0 \Rightarrow Z = \langle 0 | S | 0 \rangle = 1$

If $H_{int} \neq 0 \Rightarrow ?$ What is Z ?

Lemma Z is given by sum of all "vacuum diagrams" diagrams with no external legs.

e.o.l. 23
10/11/15

Example SYTh

$$Z = 1 + \text{diagram} + \text{etc.} + \left(\begin{matrix} \text{another} \\ O(g^2) \end{matrix} \right) + \dots$$

$$(55) \quad = 1 + \infty g^2 + g^4(\infty)$$

$$= \infty !$$

Corollary

Physical Vacuum expectation values of time ordered products are given by sums of diagrams WITHOUT any vacuum diagrams.

Proof (Sketch of)

Consider any diagram without vacuum diagrams
i.e. every part connected to at least one external leg

$$\langle 0 | T \phi_1 \phi_2 \dots \phi_n | 0 \rangle = \text{Diagram with } n \text{ external legs} + \dots + \langle 0 | T \phi_1 \phi_2 \dots \phi_n | 0 \rangle \text{ Expansion}$$

- N vertices $[O(V) \text{ term in}]$

$\Rightarrow$ We also get this with every possible vacuum diagram

Diagram
 $D_{AB} = D_A \cdot D_B$
disconnected

$\Rightarrow$
 $V_{AB} = V_A \cdot V_B$
Factorise

$$G = \text{Diagram} \left(1 + \text{all vacuum diagrams} \right) + \dots$$

* * *
 $O(V+U)$ term
in Expansion

PS3 Q6? SYTH

Combinatorics: If have $(Q \text{ } \text{Diagram})$ with V & U vertices

$$\frac{1}{(V+U)!} \cdot \frac{(V+U)!}{V! U!} = \frac{1}{V!} \cdot \frac{1}{U!}$$

Such diagrams give a result equal to the value of Q alone multiplied by vacuum contributions

$\Rightarrow$ The vacuum diagrams are completely cancelled by $\frac{1}{Z}$ contribution