

Feynman Rules in Momentum Space for G

Physical particles are e/states of momentum so best to work in terms of p^μ

Rules

① Each internal line e is associated with

a) Momenta k_e^μ

b) $\int d^4 k_e$

c) $\Delta_F(k_e)$ for relevant field

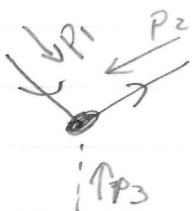
Follows since $\Delta(x_i - x_j) = \int d^4 k_e e^{-i k_e (x_i - x_j)} \Delta(k_e)$

② Each internal vertex has

a) $-ig$ (coupling constant)

b) Conserve 4-momenta at vertex

$\delta^4(\sum_e p_e)$ where p_e^μ is INTO vertex

e.g. 5/Th  $\equiv -ig \delta^4(p_1 + p_2 + p_3)$

Follows since $\int d^4 x_i \prod_e e^{-i p_e (x_i - x_{other})} = \int d^4 x e^{-i \sum_e p_e x_i} = \delta^4(\sum_e p_e)$

③ External Legs / Vertices

Since $G(\{p\}) = \left(\prod_i \int d^4 y_i e^{-i p_i y_i} \right) G(\{y_i\})$

we get a factor of

$$\begin{array}{c} y_i \\ \times \end{array} \text{---} \begin{array}{c} \times \\ \times \end{array} \Rightarrow \int d^4 y_i e^{-i p_i y_i} \underbrace{\Delta(y_i - x)}_{\int d^4 k e^{-i k(y_i - x)} \Delta(k)}$$

$$\Delta(p) \underbrace{e^{+i p x}}_{\substack{\text{Goes into } \times \text{ vertex} \\ \delta^4(p + k_1 + k_2 \dots)}}$$

$$\Rightarrow \begin{array}{c} p^\mu \\ \times \text{---} \end{array} = \Delta(p) \quad \text{for } G(\{p\})$$

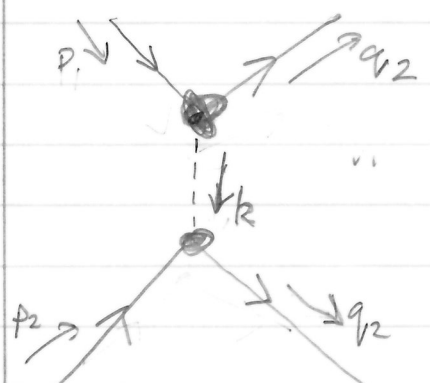
EP N.B. For $\mathcal{M}$ we have $\int d^3 y e^{-i p y} \omega(p) G(y, \dots)$
factor which sets $p^0 = i \omega p$
& REMOVES $\Delta(p)$ on external legs

$$\begin{array}{c} p^\mu \\ \times \text{---} \end{array} = 1 \quad p^0 = \omega(p)$$

④ Symmetry Factor

Multiply by $\frac{1}{S}$ as before.

Example $\psi\psi \rightarrow \psi\psi$ Scattering

$A =$

 $=$
 $\Delta(p_1) \Delta(q_1) \times$
 $-ig \delta^4(p_1 - k - q_1) \times$
 $\int d^4k \Delta(k) \times$
 $-ig \delta^4(p_2 + k - q_2) \times$
 $\Delta(p_2) \Delta(q_2)$

$$G_A(p_1, p_2, q_1, q_2) = \delta^4(p_1 + p_2 - q_1 - q_2) \leftarrow \text{Overall E/p cons}$$

$$\times \Delta(p_1) \Delta(p_2) \Delta(q_1) \Delta(q_2) \leftarrow \text{ext legs}$$

$$\times (-ig)^2 \Delta(k) \leftarrow \text{1PI / Truncated part.}$$

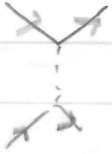
For $\mathcal{M}$

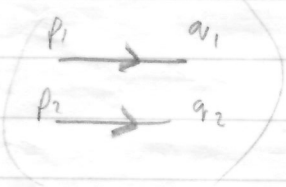
$$\mathcal{M}_A(\psi\psi \rightarrow \psi\psi) = \delta^4(p_1 + p_2 - q_1 - q_2) (-ig)^2 \Delta(k) \Big|_{p_i \xrightarrow{m=0} = \Omega(p_i) \text{ etc}}$$

Loop Momenta

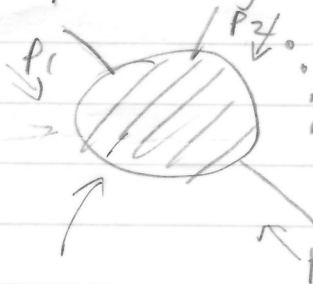
Connected Components/Diagrams

A connected diagram is one where you can find a path along edges & vertices to every other part.

e.g.  is connected ($C=1$)

e.g.  has two connected subdiagrams (components) here $(-)\Delta(-)$ ($C=2$)

Each connected component conserves momentum separately

 $\propto \delta\left(\sum_{i=1}^n p_i\right)$

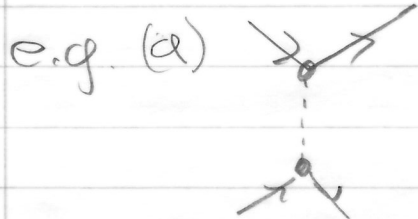
Shading = One connected component

Loop Momenta

Wernberg Ia(6.3.4) Number of momentum integrations δ_{p_i} left after δ functions at vertices accounted for is L

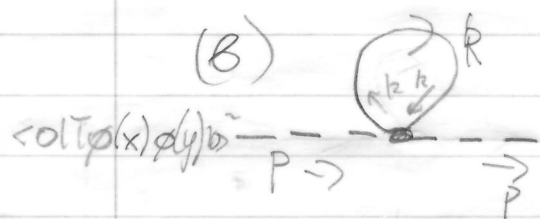
$$L = I - (V - C) = I - V + C$$

internal lines # vertices # components



$$I=1 \quad V=2 \quad C=1$$

$$\Rightarrow L=0$$



$$I=1 \quad V=1 \quad C=1$$

$$\Rightarrow L=1$$

- Loops = momenta NOT constrained by δ function
- Loops generate U.V. infinities of QFT

e.g. (b) $\sim \int d^4p \frac{i}{p^2 - m^2 + i\epsilon} \sim \int dp p \sim \Lambda^2$
for $p \sim \Lambda \gg m$

