

CROSS SECTION σ

Collide two beams of particles (here $q+q$)

Sometimes they miss, sometimes they collide

Suppose we measure N scattering events

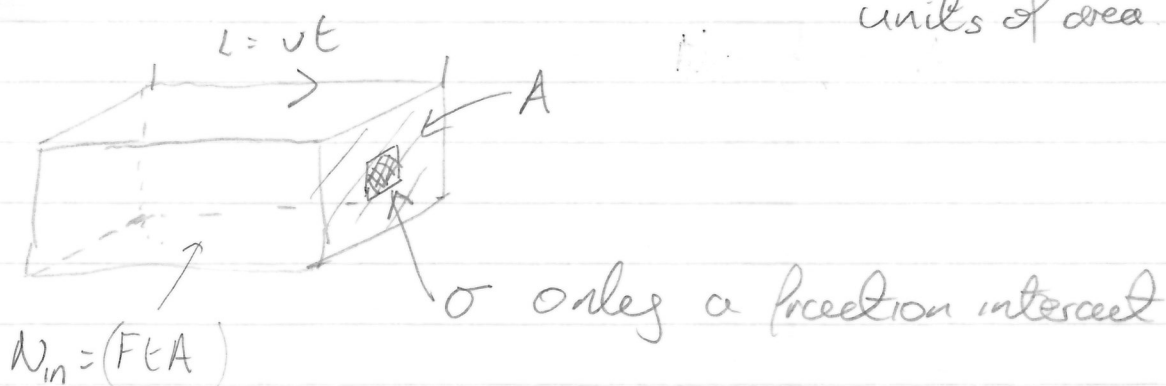
$\Rightarrow$ Rate of scattering = $\frac{dN}{dt}$

This should be proportional to Flux F of incoming particles, # particles per unit area per unit time

$$\frac{dN}{dt} = F \sigma$$

Definition of cross section

units of area



c.o.L24
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In general scattered particles occur at different rates in different directions
 so define DIFFERENTIAL CROSSSECTION $\frac{d\sigma}{d\Omega}$
 as crosssection $d\sigma$ in solid angle $d\Omega$ at (θ, ϕ)

$$\text{ie, } \sigma = \int_{\Omega} d\Omega \frac{d\sigma}{d\Omega} = \int_{\Omega} d\sigma$$

$$= \int_0^{\pi} d\theta \sin\theta \int_0^{2\pi} d\phi \cdot \frac{d\sigma}{d\Omega} \quad \text{if all directions detected.}$$

Long calculation involving kinematics etc.
 see Tong § 3.6 or more details in Peskin + Schroeder § 4.5.

One key point

$$\mathcal{M}_{fi} = \langle f | S | i \rangle = \mathcal{L}_{fi} \delta^4 \left(\sum_i p_i - \sum_f q_f \right)$$

$\uparrow$
 factor out
 overall
 Energy/Momentum

$$\text{but } \frac{d\sigma}{d\Omega} \propto |\mathcal{M}|^2 = |\mathcal{L}|^2 \delta^4(\sum p - \sum q) \underbrace{\delta^4(p^u=0)}_{VT=\infty!}$$

Interpret such $(\delta^4(p^u=0))$ through $VT=\infty!$

$$\begin{aligned} \delta^4(p^u=0) &= \int d^4x e^{-ipx} \Big|_{p^u=0} \\ &= \int d^3x \int dt \\ &= V \int dt \end{aligned}$$

VOLUME, TOTAL TIME

Example

- (i) Scattering Cross Section in CM (Centre of Mass) Frame for two particle scattering of equal mass particles e.g. $\phi\phi \rightarrow \phi\phi$ or $\phi\phi \rightarrow \phi\bar{\phi}$ we have

$$\frac{d\sigma}{d\Omega} = \frac{|A_{fi}|^2}{64\pi^2 E_{CM}} \quad \begin{matrix} \text{[PS (4.84)} \\ \text{Tong (3.92)]} \end{matrix}$$

↑
Centre of Mass Energy

Tong (3.56) p63 $|A_{fi}| = g^2 \left| \frac{1}{(p_1 - q_1)^2 - m^2 + i\epsilon} + \frac{1}{(p_1 - q_2)^2 - m^2 + i\epsilon} \right|$

* $G \rightarrow M$ no legs
if vacuum $\Rightarrow 1 \Rightarrow$



$$\frac{E_{CM}}{2} = \Omega_p = p_1^{n=0} = q_1^{n=0} \Rightarrow (a) \quad k^{n=0} = 0 \quad \text{NO Energy for } \phi$$

$$\Rightarrow (b) \quad |p_i| = |q_i|$$

$$\Rightarrow p_1 = -p_2, \quad q_1 = -q_2$$

$$\text{Let } p_1 \cdot q_1 = |p|^2 \cos \theta$$

$$\Rightarrow |A| = g^2 \left| \frac{1}{2p^2(1 - \cos \theta) + m^2} + \frac{1}{2p^2(1 + \cos \theta) + m^2} \right|$$

↓ Angular dependence } could reveal value of m
but $|p|^2$ dependence } BUT

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$\phi\bar{\phi} \rightarrow \phi\bar{\phi}$ needs better as
 $k = p_1 + p_2 \quad k^2 > m^2$

$\nearrow \dots \nwarrow$ is large
when $k^2 \approx m^2$
allowed by kinematics.

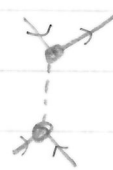
- $\Rightarrow$ (i) Angular dependence
 (ii) Variation with energy $p \Leftrightarrow |p|^2$
 (iii) ϕ field mass dependence

A careful fit of data (if accurate enough!) will show if this model works

$\Rightarrow$ can predict mass m for ϕ field
 & predict $\psi\psi^+$ interaction mediated by ϕ particle

Notes

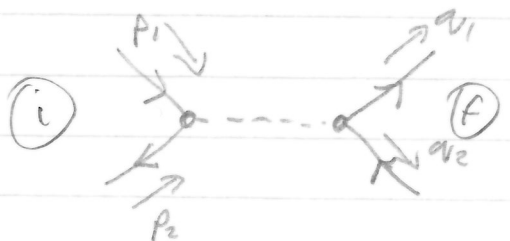
① Always need energy $\geq m$

If $|p|^2 \ll m^2 \Rightarrow$  $\approx \frac{(-ig)^2}{m^2} = -i \frac{\lambda}{2}$

Find low energy behaviour equivalent to theory of $\mathcal{L}_{int} = \frac{\lambda}{4} (\psi^\dagger \psi)(\psi^\dagger \psi)$

cut?

② Better to look at $\psi\bar{\psi} \rightarrow \psi\bar{\psi}$ as

(i)  $\sim \frac{i}{(p_1 + p_2)^2 - m^2 + i\epsilon} \sim \frac{i}{E_{cm}^2 - m^2}$

So $|A_{fi}| \rightarrow \infty$ as $E_{cm} \sim m^2$

In practice finite life time regulates

