Imperial College London

MSci EXAMINATION (Today April 30, 2018)

This paper is also taken for the relevant Examination for the Associateship

QUANTUM FIELD THEORY

For 4th-Year Physics Students

(today April 30, 2018)

Answer THREE questions.
Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Complete the front cover of each of the THREE answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in THREE answer books even if they have not all been used.

You are reminded that Examiners attach great importance to legibility, accuracy and clarity of expression.

Go to the next page for questions

1. Question 1

[Total 0 marks]

2. Question 2

[Total 0 marks]

3. Question 3

[Total 0 marks]

4. Question 4

[Total 0 marks]

Useful Definitions

Units

Unless otherwise specified, natural units are used so $\hbar = c = 1$.

Metric

The metric is diagonal with $g^{00} = +1$ and $g^{ii} = -1$ for $i = 1, 2, 3$.

Annihilation and Creation Operators

The annihilation and creation operators obey the following commutation relations

$$\left[\hat{a}_{\boldsymbol{p}}, \hat{a}^\dagger_{\boldsymbol{q}}\right] = (2\pi)^3 \delta^3(\boldsymbol{p} - \boldsymbol{q}) \, , \qquad [\hat{a}_{\boldsymbol{p}}, \hat{a}_{\boldsymbol{q}}] = \left[\hat{a}^\dagger_{\boldsymbol{p}}, \hat{a}^\dagger_{\boldsymbol{q}}\right] = 0 \, . \tag{1}$$

The vacuum state $|\,0\rangle$ is destroyed by any annihilation operator, that is $\hat{a}_{\boldsymbol{p}}|\,0\rangle = 0$ for all $\boldsymbol{p}$.

Fields

In the following expressions $px \equiv p^\mu x_\mu = p_0 t - \boldsymbol{p}.\boldsymbol{x}$ where $p^\mu = (p_0, \boldsymbol{p})$, $x^\mu = (t, \boldsymbol{x})$ and $\boldsymbol{p}.\boldsymbol{x}$ is the usual three-vector scalar product.

In the interaction picture, a real field $\hat{\phi}(x)$ of mass m has the form

$$\hat{\phi}(x) \;=\; \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{1}{\sqrt{2\omega(\boldsymbol{p})}} \left(\hat{a}_{\boldsymbol{p}} e^{-ipx} + \hat{a}^\dagger_{\boldsymbol{p}} e^{+ipx}\right) , \tag{2}$$

$$\text{where} \quad p_0 = \omega(\boldsymbol{p}) = \left|\sqrt{\boldsymbol{p}^2 + m^2}\right| \, . \tag{3}$$

A complex field $\hat{\psi}(x)$ of mass M in the interaction picture has the form

$$\hat{\psi}(x) \;=\; \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{1}{\sqrt{2\Omega(\boldsymbol{p})}} \left(\hat{b}_{\boldsymbol{p}} e^{-ipx} + \hat{c}^\dagger_{\boldsymbol{p}} e^{ipx}\right) , \tag{4}$$

$$\text{where} \quad p_0 = \Omega(\boldsymbol{p}) = \left|\sqrt{\boldsymbol{p}^2 + M^2}\right| \, . \tag{5}$$

Both the $\hat{b}_{\boldsymbol{p}}, \hat{b}^\dagger_{\boldsymbol{p}}$ pairs and the $\hat{c}_{\boldsymbol{p}}, \hat{c}^\dagger_{\boldsymbol{p}}$ pairs of annihilation and creation operators obey similar commutation relations to those of (1) for the $\hat{a}_{\boldsymbol{p}}, \hat{a}^\dagger_{\boldsymbol{p}}$ pairs. Different types of annihilation and creation operator always commute e.g. $\left[\hat{a}_{\boldsymbol{p}}, \hat{b}^\dagger_{\boldsymbol{q}}\right] = 0$.