Want to study theories with 32 supercharges. Consider toroidal compactification of \( \mathcal{N} = 1 \) \( d \times 10 \)-theory in Minkowski spacetime \( \mathbb{R}^{10} \times T^n \) \( n \)-dimensional torus. As the \( n \)-torus is compact it will have Fourier modes in every direction but dimensional reduction will tell us to restrict only to 0 modes; i.e., forget about dependence on torus directions and focus only on Minkowski space. Consider different cases for \( n \):

- \( n = 0 \) in \( (10+1) \)-d : have gravitino, gravitino 3-form [2000], [1001], [2010].

- \( n = 1 \) in \( (9+2) \)-d : have type IIA and in general 1 gravitino, p-forms, gravitinos, gravitinos, scalar fields.

p-forms give electric and magnetic sources, scalar fields have \( \phi > 0 \) that parametrizes manifold.

A relevant result is that the scalar manifold is of the form \( \frac{G_n}{H_n} \). \( G_n \) is the maximally non-compact group with algebra \( E_n \). \( H_n \) maximal compact subgroup of \( G_n \).

It follows that the number of scalar particles is \( \dim \frac{E_n}{H_n} \).

... Let's review some \( E_n \) algebras:

\[
E_0 = 0 \quad E_1 = A_1 \quad E_2 = A_2 \quad E_3 = A_3 \quad E_4 = A_4 \quad E_5 = D_5 \quad E_6 \quad E_7 \quad E_8
\]

Now, for \( A_n \) the maximally non-compact sigmae whatever is \( \text{SL}(n+1) \).

for \( D_n \) \( \text{SO}(n+1) \times \text{SO}(n) \)

for \( E_n \) \( \text{E}_{n+1} \)

We don't care about \( E_0 \) and \( E_6 \) because not finite dimensional representation.

So \( H \) will be \( \text{SO}(n+1) \) for \( A_n \) and \( \text{SO}(n) \times \text{SO}(n) \) for \( D_n \).
A study \( \frac{G_n}{H_n} \) for \( n = 1, \ldots, 8 \). Before doing this recall \( H_n \) is a symmetry group. All massless fields transform in irreducible representations of \( H_n \). So once we compute the massless fields we can fit them into the relevant representations.

1. Type \( \text{II A} \), \( E_1 = \text{SO}(1,1) = \mathbb{R}^+ \) with \( \text{dim} (\mathbb{R}^+) = 1 \), 1 scalar field, non-compact.

1'. Type \( \text{II B} \), \( E_2 = A_1 \), \( \frac{G_2}{H_1} = \frac{\text{SL}(2,\mathbb{R})}{\text{SO}(2)} = T \), \( \text{dim} T = 2 \), 2 scalar fields, 1 compact, 1 non-compact.

2. \( E_2 = A_1 \times U(1) \), \( \frac{G_2}{H_2} = \frac{\text{SL}(2,\mathbb{R})}{\text{SO}(2)} \times \mathbb{R}^+ \), so in \( (8\times 1) \) would have 3 scalars, 3 = (3-1) + 1.

3. \( E_3 = A_2 \times A_1 \), \( \frac{G_3}{H_3} = \frac{\text{SL}(3,\mathbb{R})}{\text{SO}(3)} \times \frac{\text{SL}(2,\mathbb{R})}{\text{SO}(2)} \), \( \text{dim} \frac{G_3}{H_3} = (8-3) + (3-1) = 7 \) scalars in \((7\times 1)\).

4. \( E_4 = A_4 \), \( \frac{G_4}{H_4} = \frac{\text{SL}(5,\mathbb{R})}{\text{SO}(5)} \), of dimension 14.

5. \( E_5 = D_5 \), \( \frac{G_5}{H_5} = \frac{\text{SO}(5,5)}{\text{SO}(5) \times \text{SO}(5)} \), of dimension 25.

6. \( E_6 \), \( \frac{G_6}{H_6} = \frac{E_{6,6}}{\text{SU}(k)} \), of dimension 42 = \( \frac{8 \times 9}{2} \). Recall adjoint in \( \text{SO}(10) \) is 162.

8. \( E_8 \), \( \frac{G_8}{H_6} = \frac{E_{8,8}}{\text{SO}(16)} \), of dimension 248 - 128 = 120 scalars.

\[ [1,1]_{\text{SO}(3)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \]

Side think of \( \text{SU}(3) \supset \text{SO}(3) \) : 

\[ 133 - 63 = 70 \text{ scalars} \]

\[ 248 - 128 = 120 \text{ scalars} \]
Last time: Theories with 32 supercharges

Some common theories are: \( N=8 \) supergravity, \( AdS_7 \times S^4 \), \( AdS_4 \times S^7 \), \( AdS_5 \times S^5 \)

So \( AdS_3 \times S^4 \) massless content is that for \( n=4 \) in the previous discussion

Let's take it further: \( E_6 \sim A_4 = SU(5) \)

\[ H = \frac{SL(5, \mathbb{R})}{SO(5)} \], little group is \( SO(5) \)

\( \Rightarrow \) massless states are in irreducible representations of \( SO(5) \times SO(5) \):

- bosons \( \begin{bmatrix} [20;00] & [02;10] & [10;02] & [00;20] \end{bmatrix} \)
  - graviton \( 11 \), gravitino \( 11 \), graviphoton \( 10 \), gravitorm \( 0 \)

Braids ending on braids

We saw some cases:

\[
\begin{array}{c|c|c}
F_1 & \text{massless} & \text{and magnetic dual} \\
\hline
m_1 & \text{de} & \text{dual} \\
\hline
m_2 & \text{in M theory} & \text{in M theory} \\
\hline
m_5 & \text{dual} & \text{dual} \\
\end{array}
\]

\( p \leq 3 \) (both in type \( I \), \( 0 \leq p \leq 6 \)

Let's write equations for the 1st case

\[ d \star_3 H = S^{1/3} \Omega - S^{(3-t)} \star_3 F \], can integrate from an action. Note now we are in \textbf{spacetime} perspective. So construct a low energy effective action, at most quadratic in \( B \) (derivatives).

\[
\int \left[ H \wedge_3 H + \int_b + \int B \wedge_3 F \right]
\]

quadratic \( B \) \quad \text{linear} \( B \) \quad \text{linear} \( B \) \quad \text{quadratic derivatives}
let's consider particle. Terms in action are:

\[ Q \frac{\partial}{\partial t} A(x) = \int \left( A(x) \frac{\partial A(x)}{\partial t} \right) dx^4 \]

( via pullback current)

\[ \oint \mathcal{J} = Q \int \mathcal{B} \wedge \mathcal{J} \]

2 parameters

charge that string carries with respect to 2-form

\((\mathcal{J})\)

in complete analogy term that tells me that the brane is electrically charged with respect to the form.

\(\mathcal{B}\) is coupling for kinetic term with a scalar field but mixing 2 scalar fields.

\[ \int \mathcal{G}_{\mu \nu}(x) \, \partial \mathcal{J}^\mu \mathcal{J}^\nu \]

in complete analogy term in spacetime perspective is how fundamental string couples to 2-form, for worldsheet perspective just coupling for kinetic term.

call vector x vector = \( A(x) \) (vector) + \( \mathcal{B}(x) \) (vector) + trace (ie gravity, 2-form, dilaton)

can write the trace \( \int dx^2 \delta(x) \partial^\mu \mathcal{J}^\mu \partial^\nu \mathcal{J}_\nu \)

the 3 fields (\( A, \mathcal{B}, \phi \)) always appear as couplings of the kinetic term.

for quantization have renormalization group flow, \( \beta \)-functions equal to 0 \( \leftrightarrow \) field equations in spacetime

for \( A(x) \) get Einstein's equations, for \( B \) the others, similarly for dilaton.
Now try to understand the other term. Recall gauge invariance of \( B \), \( \mathcal{B}_B = \mathcal{A}(1) \), \( H = dB \implies \mathcal{S}H = 0 \).

Do terms 1, 2, 3 on worldsheet give 1, 2, 3 in spacetime?

\[ \mathcal{S} \int_B = \int_{\mathcal{P}_1} \mathcal{A}_1 = 0 \]

But Stokes' theorem holds in absence of a boundary so the story is unfinished. \( \mathcal{P}_1 \) has a boundary (pointlike) and its end is source for 1-form. In presence of boundaries \( \int_B \) says that the end of the string is changed under the 1-form.

\[ \mathcal{S} \left( \int_B - \int_A \right) = \int_{\mathcal{P}_1} \mathcal{A}(1) - \int_{\mathcal{P}_1} \mathcal{A}(1) = 0 \] as \( \mathcal{B}_B = \mathcal{A}(1) \), \( \mathcal{B}^* = \mathcal{A}(1) \). So have to add this term to the action.

Look at \( \mathcal{B}_B = \mathcal{A}(1) \), \( \mathcal{B}_A = \mathcal{A}(1) \), \( F = dB \Rightarrow \mathcal{S}F : \mathcal{A}(1) \mathcal{A}(1) \) \( B \) is not gauge-invariant

\( \mathcal{A} \) is not gauge-invariant

\( F \) is not gauge-invariant

However \( \mathcal{S}(F - B) = 0 \) so \( F - B \) is gauge invariant.

Now can write effective action for gauge field on brane \( \int (F - B) \wedge \ast_p (F - B) \)

If there was no 2-form as source it was \( F \wedge F \).

Now because of gauge invariance when expand find \( \mathcal{B}_A \wedge \mathcal{F} \) but also \( \mathcal{B}_A \wedge \mathcal{B}_B \).
recall from last time 3 interaction terms in the action

\[ \int d^2 \sqrt{g} \, \varepsilon^{\alpha \beta} \, \partial_\alpha X^\mu \partial_\beta X^\nu \mathcal{L}_{\mu \nu}(x) \]

interaction term for the metric on worldsheet

gives rise to Einstein's equations

\[ \int d^2 \varepsilon^{\alpha \beta} \partial_\alpha X^\mu \partial_\beta X^\nu \mathcal{L}_{\mu \nu}(x) = \int \mathcal{B} \]

interaction term for the 2-form, pull-back of 2-form
into world sheet

\[ \mathcal{B} \text{ first depends only on spacetime coordinate, then depends} \]
on parametrization of worldsheet

Relevant equations from \( \beta \)-functions is

\[ \int d^2 \sqrt{g} \, \varepsilon^{\alpha \beta} \partial_\alpha X^\mu \partial_\beta X^\nu \phi(x) \]

Side: how to perform dimensional reduction from 5d to 4d i.e. \( \text{SO}(3) \supset \text{SO}(2) \)

think of \( \text{SO}(3) \) as the Cartan generator inside \( \text{SO}(3) \)

So, we have 5 cases; in every case let weight of \( \text{SO}(3) \) become 2 \( \pm h \) in \( \text{SO}(2) \) (twice the helicity)

- Graviton \( [4] = x^4 + x^2 + x^{-2} + x^{-4} \) weights \( 4, 2, 0, -2, -4 \Rightarrow \) helicities \( 0, \pm 1, \pm 2 \)

  as always \( \text{graviton}_5 \rightarrow \text{graviton}_4, \text{1-form}_4, \text{scalar}_4 \)

  helicity: \( \pm 2, \pm 1, 0 \)

- Gravitino \( [3] = x^3 + x + x^{-1} + x^{-3} \) weights \( \pm 3 \Rightarrow \) helicities \( \pm 3/2 \) : gravitinos

  weights \( \pm 1 \Rightarrow \) helicities \( \pm 1/2 \) : gravitinaus

  so \( \text{gravitino}_5 \rightarrow \text{gravitino}_4, \text{gravitinaus}_4 \)

- 1-form \( [2] = x^2 + x + x^{-1} \)

  analogously as before \( \text{1-form}_5 \rightarrow \text{1-form}_4, \text{scalar}_4 \)
\[ [1] = x + x^{-1} \]
\[ \text{fermion} \quad \rightarrow \quad \text{fermion} \]
\[ \text{fermion} \quad \rightarrow \quad \text{fermion} \]
\[ \text{scalar} \quad \rightarrow \quad [0] = 1 \]
\[ \text{scalar} \quad \rightarrow \quad \text{scalar} \]

Note: fields always come in pairs for CPT invariance.

So now we know how to compute all supergravity multiplets in any dimension with 32 supercharges. We see there is always a single graviton, collection of gravitinos, p-form fields, fermions, scalars.

We realized that p-form fields lead to branes (generalized electromagnetism): \( D_p \), \(-1 \leq p \leq 9\)

F1
NS5
M5
M2

Massless fields:
\[ D_p \rightarrow \text{V-plot} 16 \text{ supercharges} \]
\[ F1 \rightarrow \text{scalars and fermions (no gauge fields)} \quad x^\mu, \psi^\mu \]
\[ \text{T5} \rightarrow \text{tensor multiplet; Little group } SO(4) \rightarrow 2 \text{ Dynkin labels } SO(4) \approx SO(2) \times SO(2) \]
\[ \text{R-Symmetry } Sp(2) \text{, Represent as } [Sp(4); Sp(2)] \]

Tensor multiplet is \([2,0,0,0] \oplus [10; 10] \oplus [00; 01]\)

Self-dual 2-form \(4 \) fermions \(5 \) scalars; recall 2nd rank traceless antisymmetric represent of \( Sp(2) \)

NS5 of type II B already did \[ D^9 \]

R-Symmetry
\[ SO(4) \approx Sp(1) \times Sp(1) \]

\[ [11; 00] \oplus [01; 10] \oplus [10; 01] \oplus [00; 11] \]

(Use this representation because in 6 dimension supergravity fermions are pseudoreal)

Vectors are fermions
Scalars of R-Symmetry

Always compute by dimensional reduction of V-plot on D9-brane. Note is same massless content as for D5-brane.
NS5 of type IIA. We face a puzzle: We don't have a D1-brane in type IIA (recall p even for type IIA, odd for type IIB) at best D2-brane. So there will be a string on the world-volume of NS5.

Moreover, how many transverse scalars? 4! That's different from the 5 we found on the M5-brane. There is a missing scalar. Let's see why. Consider:

end of 0-brane is pointlike in all directions (instanton), carries charge as usual so expect 0-form gauge field propagating on world-volume 4

But remember p=0 Gauge field is periodic (compact) scalar field

So we also should expect periodic scalars

Finally: 4 transverse scalars +1 compact scalar \( \rightarrow \) 5 scalars !!!

So we solved the problem of having in principle 5 scalars but only observing 4.

But let's talk about duality of type IIA and M-theory

Remember: M-theory \([2000]_y \oplus [1001]_y \oplus [0010]_y\) massless multiplet

Hank of M-theory compactified on \(S^1\) . Write action in 11d

\[ \int f_{g} R \, d^{10+1}x \]

Count dimensions and observe there is scale. \([R]=2\)

\([g]=0\)

\([x]=-1\) (space)

So introduce coefficient to make action dimensionless

\[ k \int d^{10+1}x \sqrt{g} \cdot R \]

\([k]=9\)

\[ k=\ell_{Pl}^{-3} \]

\( \ell_{Pl} \) is Planck's length. \( R \) radius of \( S^1 \) measured in units of \( \ell_{Pl} \)
Last time: compute the massless fields on the world volume of type IIA and NS5-brane.

D0
- instanton, carries charge, gauge field couples to it (0-form) aka compact scalar

NS5

D2
- boundary is string, couples to 2-form gauge field, in (5+1)d self-dual or anti self-dual because its field strength is 3-form, unbroken representation
  \( \Rightarrow \) string is dyonic (carries electric and magnetic charge)

NS5

D4
- end is 3-brane, couples to 4-form in (5+1)d, by electric-magnetic duality can be replaced by lower dimensional form

\[ 4\text{-form} \Rightarrow 5\text{-form field strength}, \text{dual } 1\text{-form field strength of } 0\text{-form compact scalar} \]

so 3-brane magnetically charged under the compact scalar \( \phi \rightarrow \phi^{(i)} \rightarrow \nabla \phi^{(i)} \Rightarrow 0 \)

localized in 2 space dimensions, is vortex

as always have 4 massless scalars, Goldstone bosons

Now we have to discuss why does this extra scalar (fifth) appear?

Discuss Type IIA - M theory duality on a circle

In M theory there is 1 scale: Planck scale. Take 1 direction, compactify on a circle, have vacuum expectation value of a scalar: R

\[ [2000]_9 = [2000]_8 \oplus [1000]_8 \oplus [0000]_8 \]

think of \( g_{10,10}(x^{10}) \) = \( \sum a_n e^{i n x^{10}} \) Fourier decomposition, take 0-mode, associated to scalar

moreover write invariant length \( ds^2 = g_{\mu \nu} dx^\mu dx^\nu = g_{10,10} (dx^{10})^2 + \ldots \)

\[ \uparrow \]

coefficient interpreted as radius
Moduli space parameterized by value of $R: \mathbb{R}^+$

Type IIA: scalar, dilaton, on world-sheet perspective

same interaction $\int d\sigma d\chi$ but there is another term: $\int \frac{1}{2} \sum \partial \phi \partial \phi R$

for $\phi$ independent of $\sigma$ ($\phi_0$) have $\phi_0 \int \frac{1}{2} \sum \partial \phi \partial \phi R$. The integral $\int \frac{1}{2} \sum \partial \phi \partial \phi R$ is a very special integral, topological invariant = Euler character $\chi$

so $\int \frac{1}{2} \sum \partial \phi \partial \phi R = \phi_0 \chi$

For 2d surfaces there is a classification by Riemann (no boundaries) with integer genus $g$

$\chi = 2 - 2g$

look at case: $g = 0, \chi = 2$. Space with positive curvature topologically equivalent to $S^2$

$g = 1, \chi = 0$, flat compact 2-dimensional surface $T^2$

$g = 2, \chi = -2$

$g = \infty, \chi = -2g$

$g$ counts number of holes

now think of string perturbation theory and take these surfaces in their profile

eg $S^2$: closed string creates itself, propagates as closed string and then disappears

$T^1$: no string, then 1 string decomposes into 2 strings which propagate, annihilate from nothing again

all these are processes where strings interact. Holes: where strings generate and interact.

Connections to propagation of closed string

Can think of genus as measuring strength of interaction in perturbation theory
Look at the relevant term in the action:

\[ S = \ldots + \phi_0 \mathcal{L} + \ldots \]

So \( \phi_0 \) measures the strength of the interaction. \( e^{\phi} = g_5 \) string coupling, one for each spatial \( g_5 = e^{\langle \phi \rangle} \phi \text{ dilaton. All these surfaces have no boundaries.} \)

What if the surfaces have boundaries?

- Disc: \( S^2 \) with 1 boundary
- Cylinder: \( S^2 \) with 2 boundaries
- \( S^2 \) with 3 boundaries
- \( S^2 \) with 4 boundaries

Think of scattering of 2 strings.

In this case \( \chi = 2 - 2g - b \), so each boundary adds 1 more power to \( g_5 \).

Type IIA has 1 scalar field, moduli space is \( \mathbb{R}^+ \) parametrized by \( g_5 \).

\( g_5 \) doesn't have a length scale.

Effective term in action is

\[
\frac{1}{\ell_5^2} \int d^9 \sqrt{g} R \sim \ell_5 \text{ string scale}
\]

Look at other effective term:

\[
\frac{1}{\ell_5^2} \int \alpha^2 d\tau d\sigma d\chi d\xi = T \int d^2 \alpha d\chi d\xi \quad \text{as } \chi \text{ has scale}
\]

\( T = \frac{1}{\ell_5^2} \) has dimension of tension.

\( T \) is interpreted as tension of F1.
gravitational force \( \propto {r^{-1}} \) ? Measures space dimensions, \( \frac{1}{r^{d+1}} \)

To small scales all we can say is that it goes like \( \frac{1}{r^{(d+1)}} \), but we didn't go under the scale of 1mm, maybe at lower scales there are hidden dimensions.

Why only the gravitational force sees dimensions? Maybe because we live in a brane.

While electromagnetic sources are localized gravity is not localized on the brane.

1st time: M theory on \( S^1 \), and compare type IIA. By itself M theory has scale parameter, which measures all scales in \((10+1)\)d : \( L_p \). No scalar fields so that's the only scale.

But if we make one direction compact we introduce another parameter \( R \), radius of the circle, think of it as vacuum expectation value of scalar.

Type IIA has a natural scale, the tension of the string \( T_{pl} = \frac{1}{L_p^9} \) to make action dimensionless \( \frac{1}{L_p^8} \int \mathcal{L} dx \). T energy per unit length.

There is charge for coupling to p-form field, what about mass? So brane has energy per unit volume! For particle (point-like) have mass, and for string (1d) have energy per unit length, tension \( [T] = 2 \). For p-brane have tension \( [T_p] = p+1 \).

We think of the different branes we've seen \((F4, D_p, N5, N5*, N2) \); \( L_s^2 = \alpha ' \)

\[ T_{F4} = \frac{1}{L_s^8} ; \quad T_{D_p} = \frac{1}{L_s^{p+1}} \]

Why have \( \frac{1}{L_s^8} \)? Remember \( g_s \) measures strength of derivative expansion. If \( g_s \ll 1 \) then perturbation theory, processes with high genus are suppressed. If \( g_s \gg 1 \) non perturbative effects.

\[ g_s \ll 1 \] have to take into account all topologies, perturbation theory not very reliable.
Now, recall $D_p$-brane naturally associated to open string (worldsheet with a boundary).

What is the simplest worldsheet with a boundary? Disc! Euler characteristic $= 1$, so power of $\frac{1}{g_s}$.

NS5 brane couples to NS2 mode (closed string), now topology is sphere so $T_{NS5} \approx \frac{1}{g_s^3}$.

Now compare $T_{D5}$ to $T_{NS5}$, $T_{D5} \ll T_{NS5}$ for $g_s \ll 1$. So should think in perturbation theory of NS5 brane as extremely heavy object compared to D5.

$T_{F4} \approx T_{D4}$ because $T_{F4}$ has no $g_s$ dependence $\frac{1}{g_s^3}$ $\approx \frac{1}{g_s^2}$.

So we are left to determine $T_{M5}$ but that's easy because M-theory has no scale.

$T_{M5} = \frac{1}{2\pi}$ $T_{M2} = \frac{1}{2\pi^3}$. We are omitting factors of $2$ and $\pi$. To fix them just use Dirac quantization. Note funny features like $T_{M2} = \sqrt{T_{M5}}$.

Now think of M-theory over $S^1$. Recall have 3-form $C(5)$, $C_{\mu\nu\rho}$. Now reduction to type II

$$C_{\mu\nu\rho} \left\langle \begin{array}{c} C_{\mu\nu} \text{ 3-form} \\ B_{\tau, \lambda} \text{ 2-form} \end{array} \right.$$ We know M2 and M5 brane couple to $C(5)$. But in type IIA $F_{4}$ and NS5 couple to 2-form and D2 and D4 couple to 3-form. So do (D2, D4, F4, NS5) descend from (M2, M5) in (10+1)d?

Think M2 brane can wrap $S^1$ (either transverse to the circle or along the circle). In limit of very small $S^1$ it loses a dimension and becomes $F_{4}$. If it does not wrap it stays 2d and becomes D2. Same for M5, if it wraps it loses a direction and becomes D4. If it does not wrap become NS5. So branes of type IIA can derive from branes of M-theory by either wrapping them or not.
Recall \( T_{11} = \frac{1}{\ell_s^2} = T_{10} \cdot \ell_s \).

Radius of circle, because when brane wraps a circular direction just need to multiply length by tension. There will be a \( 2\pi \).

\[
\frac{1}{\ell_s^2} = T_{11} = \frac{R}{\ell_p^3} \quad \rightarrow \quad \ell_s^2 = \ell_p^3
\]

\[
T_{12} = \frac{1}{\ell_s^3} = T_{12} = \frac{1}{\ell_p^4} \quad \rightarrow \quad \ell_s^2 = \frac{1}{\ell_p^3}
\]

\( g_s \ell_s = R \)

\[
T_{14} = \frac{1}{g_s \ell_s^5} = T_{15} = \frac{R}{\ell_p^6} \quad \rightarrow \quad \text{consistent previous relations}
\]

SELF CONSISTENT

\( g_s \ell_s^6 = T_{15} = \frac{1}{\ell_p^6} = T_{10} = \frac{1}{\ell_s^6} \)

- parameters 2 relations are enough (was obvious as formulated with \( \frac{1}{\ell_s^6} \))

Equivalence between M on S\(^1\) and type IIA

\( g_s \ell_s^2 = R \)

\( g_s \ell_s^5 = \ell_p^3 \)

Strong - Weak duality (depending on strength of brane the string coupling)

\( s \ll 1 \rightarrow R \ll 1 \) (type IIA) perturbative regime

\( s \gg 1 \) type IIA not valid, no perturbation regime, BUT Can do computations in M-theory because can effectively \( (R \text{ big}) \leftrightarrow (10+1) \text{d with explicit map parameters.} \)