# Imperial College London

# MSc EXAMINATION May 2014

*This paper is also taken for the relevant Examination for the Associateship*

# STRING THEORY SPRING 2014

## For Students in Quantum Fields and Fundamental Forces

Friday, 2 May 2014: 14:00 to 17:00

*Please answer all 4 questions.*
*Marks shown on this paper are indicative of those the Examiners anticipate assigning.*

**General Instructions**

Complete the front cover of each of the 4 answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in 4 answer books even if they have not all been used.

**You are reminded that Examiners attach great importance to legibility, accuracy and clarity of expression.**



**Turn over for questions**

**1. Branes ending on Branes.**

Write down equations for a D3 brane ending on a NS brane as follows:

(i) First find the little group on the world volume of the NS brane, and the corresponding R symmetry group. [2 marks]

(ii) Find the massless spectrum on the NS brane and identify the representation of the massless fields under the little group and the R symmetry group. Use a highest weight notation for the representations. [4 marks]

(iii) Match each representation with a corresponding field in the field theory on the world volume of the brane. [3 marks]

(iv) Write the source equation for a D3 brane ending on a NS brane. [3 marks]

(v) Write the source equation for the end of the D3 brane in the world volume of the NS brane. [2 marks]

(vi) Identify the terms in the source equation of (iv) that can be derived from an action, and write each contribution to the action. [3 marks]

(vii) Find the gauge variation of the form that couples to the D3 brane, including gauge variations of all fields involved in the action from the previous question. [4 marks]

(viii) Ignoring self duality issues, write down an action that includes the terms in (vi) and is gauge invariant under these gauge variations. [4 marks]

[Total 25 marks]

**2. String theory in 3+1 dimensions with 32 supercharges.**

(i) What is the little group $L$ for massless states in 3+1 dimensions? [1 mark]

(ii) Specify all of its irreducible representations and their dimension formula. [3 marks]

(iii) List the most common representations which are discussed in physics, and the names of the corresponding fields. [5 marks]

(iv) What is the U duality group for this amount of supersymmetry, its non-compact form, and the corresponding maximally compact subgroup, $H$. [3 marks]

(v) Specify the basic representations of $H$, compute their dimension, and write their names. [21 marks]

(vi) Find the field content of the supergravity multiplet in 3+1 dimensions with 32 supercharges. Specify the different fields and their transformation laws under $L \times H$. [15 marks]

(vii) Check that the number of bosonic and fermionic degrees of freedom in this theory match. [2 marks]

(viii) What is the dimension of the scalar manifold? [1 mark]

(ix) Write down the coset space, $G/H$ which is the scalar manifold of the theory, with $G$ the maximally non-compact version of the $E_n$ algebra and $H$ is its maximal compact subgroup. Verify it has the right dimension. [2 marks]

[Total 53 marks]

**3. Supergravity multiplets with 16 supercharges in 4+1 dimensions.**

(i) What is the little group $L$ for massless states in 4+1 dimensions? [1 mark]

(ii) Specify all of its irreducible representations and their dimension formula. [2 marks]

(iii) List the most common representations which are discussed in physics, and the names of the corresponding fields. [5 marks]

(iv) What is the R symmetry for theories with 16 supercharges in 4+1 dimensions? [1 mark]

(v) Specify the basic representations of the R symmetry group, compute their dimension, and write their names. [6 marks]

(vi) Find the shortest massless multiplet in 4+1 dimensions with 16 supercharges. Write the different massless fields in terms of highest weights for irreducible representations of $L$, as well as of the R symmetry group. Specify the dimensions and name the different fields. [9 marks]

(vii) Using this multiplet find another massless supermultiplet in 4+1 dimensions with this amount of supersymmetry. Use highest weights, dimensions, and name each field. [15 marks]

(viii) As a check, verify that the number of bosonic degrees of freedom equals the number of fermionic degrees of freedom. [2 marks]

[Total 41 marks]

**4. S duality and Orientifold planes**

Consider a D3 brane that is parallel to an $O3$ plane, and is located a distance $L$ from the $O3$ plane.

(i) What are the 3 types of $O3$ planes? Write down their charges and their tensions. [7 marks]

(ii) What are the boundary conditions for the existence of a zero mode BPS state for a fundamental string which is stretched between the brane and its image, or the $O3$ plane? Specify the boundary conditions for each type of $O3$ plane, and the mass of the BPS state. [5 marks]

(iii) Based on the answers in (i), or any other argument, what is the behavior of the different objects under S duality of Type IIB? Include D3, F1, and D1. [4 marks]

(iv) Based on the answer in (iii) find the boundary conditions for a D1 stretched between the D3 brane and its image, or the O3 plane. Specify the mass of the BPS state. [5 marks]

(v) Write down the root system for the $B, C$, and $D$ Lie algebras. [6 marks]

(vi) Match a Lie algebra to an O3 plane with a collection of $n$ parallel D3 branes. [3 marks]

(vii) Match the magnetic spectrum of a gauge theory with $B, C$ or $D$ gauge group with the corresponding root system. [3 marks]

[Total 33 marks]