Imperial College London

MSc EXAMINATION May 2015

This paper is also taken for the relevant Examination for the Associateship

STRING THEORY

For Students in Quantum Fields and Fundamental Forces

Friday, 8 May 2015: 14:00 to 17:00

Please answer all 3 questions.
Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Complete the front cover of each of the 3 answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in 3 answer books even if they have not all been used.

You are reminded that Examiners attach great importance to legibility, accuracy and clarity of expression.

1. Branes ending on Branes.

Write down equations for a D1 brane ending on a NS brane as follows:

(i) First find the little group on the world volume of the NS brane, and the corresponding R symmetry group. [2 marks]

(ii) Find the massless spectrum on the NS brane and identify the representation of the massless fields under the little group and the R symmetry group. Use a highest weight notation for the representations. [4 marks]

(iii) Match each representation with a corresponding field in the field theory on the world volume of the brane. [4 marks]

(iv) Write the source equation for a D1 brane ending on a NS brane. [7 marks]

(v) Write the source equation for the end of the D1 brane in the world volume of the NS brane. [4 marks]

(vi) Identify the terms in the source equation of (iv) that can be derived from an action, and write each contribution to the action. [3 marks]

(vii) Find the gauge variation of the form that couples to the D1 brane, including gauge variations of all fields involved in the action from the previous question. [4 marks]

(viii) Write down an action that includes the terms in (vi) and is gauge invariant under these gauge variations. [4 marks]

[Total 32 marks]

2. String theory in 7+1 dimensions with 32 supercharges.

(i) What is the little group L for massless states in 7+1 dimensions? [1 mark]

(ii) Specify all of its irreducible representations and their dimension formula. [6 marks]

(iii) List the representations which appear in the supergravity multiplet, specify the names of the corresponding fields, and their corresponding dimensions. [30 marks]

(iv) What is the U duality group for this amount of supersymmetry, its non-compact form, and the corresponding maximally compact subgroup, H. [3 marks]

(v) Specify all the irreducible representations of H, and compute their dimension. [3 marks]

(vi) Find the multiplicity of fields in (iii) and fit them into representations of H. [30 marks]

(vii) Check that the number of bosonic and fermionic degrees of freedom in this theory match. [2 marks]

(viii) What is the dimension of the scalar manifold? [1 mark]

(ix) Write down the coset space, G/H which is the scalar manifold of the theory, with G the maximally non-compact version of the E_n algebra and H is its maximal compact subgroup. Verify it has the right dimension. [5 marks]

(x) Write down the supersymmetry algebra for this theory. First identify the transformation laws of the supercharges under the Lorentz group, and the R symmetry. [9 marks]

(xi) Find the different representations which appear in the supersymmetry algebra. [16 marks]

(xii) Finally write down the supersymmetry algebra using central charges and gamma matrices. [30 marks]

[Total 136 marks]

3. Dimensional reduction of supergravity multiplets with 16 supercharges in 5+1 dimensions.

(i) How many different super-algebras are there in 5+1 dimensions with 16 supercharges. Specify the corresponding R symmetry and the 5 brane on which these super-algebras are realized. [6 marks]

(ii) What is the little group for the massless states in 5+1 dimensions. Provide the dimension formula for irreducible representations of this group. Identify fundamental representations of the little group in terms of their dimension, highest weights, and the name used in physics. [9 marks]

(iii) Find the shortest massless multiplet for the non-chiral theory in 5+1 dimensions with 16 supercharges. Write the different massless fields in terms of highest weights for irreducible representations of the little group, as well as the R symmetry group. Specify the dimensions and name the different fields. [16 marks]

(iv) Using this multiplet find another massless supermultiplet in 5+1 dimensions with this type of supersymmetry. Use highest weights, dimensions, and name each field. [27 marks]

(v) As a check, verify that the number of bosonic degrees of freedom equals the number of fermionic degrees of freedom. [2 marks]

(vi) Using the corresponding brane, find the shortest massless multiplet for the chiral theory in 5+1 dimensions with 16 supercharges. Write the different massless fields in terms of highest weights for irreducible representations of the little group, as well as the R symmetry group. Specify the dimensions and name the different fields. Please provide the name for the supermultiplet you find. [10 marks]

(vii) Using this multiplet find another massless supermultiplet in 5+1 dimensions with this type of supersymmetry. Use highest weights, dimensions, and name each field. [12 marks]

(viii) As a check, verify that the number of bosonic degrees of freedom equals the number of fermionic degrees of freedom. [2 marks]

(ix) What is the little group for the massless states in 4+1 dimensions. Provide the dimension formula for irreducible representations of this group. Identify fundamental representations of the little group in terms of their dimension, highest weights, and the name used in physics. [6 marks]

(x) Write down the R symmetry for theories in 4+1 dimensions with 16 supercharges. Provide the dimension formula for irreducible representations of this group. Identify fundamental representations of the little group in terms of their dimension, highest weights, and the name used in physics. [10 marks]

(xi) Write down the massless supermultiplets in 4+1 dimensions with 16 supercharges. Write the different massless fields in terms of highest weights for irreducible representations of the little group, as well as the R symmetry group. Specify the dimensions and name the different fields. [32 marks]

(xii) As a check, verify that the number of bosonic degrees of freedom equals the number of fermionic degrees of freedom per each supermultiplet. [4 marks]

(xiii) Perform dimensional reduction on the 4 supermultiplets found in (iii), (iv), (vi), and (vii), and write them in terms of the supermultipets found in (xi). [4 marks]

(xiv) Compute the number of massless vector multiplets in the toroidal compactification of the Heterotic string on T^5. Based on this, and the results of (xiii) compute the number of matter multiplets in the chiral theory in 6 dimensions with 16 supercharges. [2 marks]

[Total 142 marks]