1 Chiral Superfield Models

We consider a 4d $\mathcal{N}=1$ theory with a collection of chiral superfields Φ^i,

$$\Phi^i = \phi^i + i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi^i + \frac{1}{4}\theta^2\bar{\theta}^2\partial^\mu\partial_\mu\phi^i + \sqrt{2}\theta\psi^i - \frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi^i\sigma^\mu\bar{\theta} - \theta^2 F^i \tag{1}$$

and a microscopic Lagrangian

$$\mathcal{L} = \mathcal{L}_\mathcal{K} + (\mathcal{L}_\mathcal{W} + h.c.) \tag{2}$$

where $\mathcal{L}_\mathcal{K}$ and $\mathcal{L}_\mathcal{W}$ are the suitable superspace integrals over the Kähler potential and the superpotential respectively. We pick the Kähler potential

$$\mathcal{K} = \bar{\Phi}_i\Phi^i \ , \tag{3}$$

(i) Show that in components

$$\mathcal{L}_\mathcal{W} = \partial_i\mathcal{W}F^i - \partial_i\partial_j\mathcal{W}\psi^i\psi^j \ , \tag{4}$$

where $\partial_i\mathcal{W} = \frac{\partial\mathcal{W}}{\partial\Phi^i}|_{\Phi^j=\phi^j}$.

(ii) Consider a theory with two chiral superfields Φ^1 and Φ^2 and the superpotential

$$\mathcal{W} = \frac{1}{2}m_{ij}\Phi^i\Phi^j + \frac{1}{3}\lambda_{ijk}\Phi^i\Phi^j\Phi^k \ . \tag{5}$$

Following what we saw in the lecture[1] show that the superpotential doesn't get renormalised, i.e. $\mathcal{W}_{\text{eff}} = \mathcal{W}$.

(iii) Show that in general an arbitrary superpotential for any amount of chiral fields doesn't get renormalised. [Hint: Introduce an additional spurious superfield Y with R-charge 2, s.t. $\mathcal{W}(\ldots, Y)$ for $Y=1$ gives back the original superpotential.]

(iv) Now consider a simpler model of only a single chiral superfield Φ and the superpotential

$$\mathcal{W} = \frac{1}{2}m\Phi^2 + \frac{1}{3}\lambda\Phi^3 \tag{6}$$

The Kähler potential receives a wave function renormalisation

$$\mathcal{K} = Z^2\bar{\Phi}\Phi \ , \tag{7}$$

since the superfield has a wavefunction renormalisation $Z\Phi$. Compute Z at 1-loop. What is the renormalisation of m and λ? [Hint: Write down the Lagrangian in component fields, then compute the wavefunction renormalisation of ψ, where ψ is the Weyl fermion of the chiral superfield. What is the relevant Feynman diagram?]

[1] you can also go to the original Seiberg paper 'Naturalness Versus Supersymmetric Non-renormalization Theorems' Phys.Lett.B318:469-475,1993

(v) Consider a theory with two chiral superfields, $\Phi^1 = L$ and $\Phi^2 = H$, with the superpotential

$$\mathcal{W} = \frac{1}{2}mH^2 + \frac{1}{2}\lambda L^2 H + \frac{1}{6}gH^3 \ . \tag{8}$$

For low energies

(a) Compute the effective mass of H.

(b) Integrate out H and obtain the effective potential

$$\mathcal{W}_{\text{eff}} = \frac{1}{3}\frac{m^3}{g^2}\left[\left(1 - 3/2\frac{\lambda g}{m^2}L^2\right) \mp \left(1 - \frac{\lambda g}{m^2}L^2\right)\sqrt{1 - \frac{\lambda g}{m^2}L^2}\right] \ . \tag{9}$$

For which energy scales is this a good effective potential? What is the meaning of the $\mp$ sign? What are the physically special points?

(vi) **O'Raifearthaigh's Model** consider a theory with 3 chiral superfields A, B, C and the superpotential

$$\mathcal{W} = \mu A + mBC + gAC^2 \ . \tag{10}$$

Compute the F-term equations and the scalar potential. Is supersymmetry spontaneously broken? What happens for (some of) $\mu, m, g \to 0$?

2 Supersymmetric Quantum Electrodynamics (SQED)

Let us consider the 4d $\mathcal{N} = 1$ analouge of QED. This is a theory with an abelian gauge field living in a vector superfield V, and a number of chiral superfields Φ^i of charge q_i.[2] The Lagrangian is

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{FI}} + \mathcal{L}_{\text{matter}} \tag{11}$$

where the individual terms are given as

$$\mathcal{L}_{\text{gauge}} = \mathcal{L}_{\mathcal{W}_\alpha} + h.c. = \frac{-i}{16\pi}\int d^2\theta \tau W^\alpha W_\alpha + h.c. \tag{12}$$

and

$$\mathcal{L}_{\text{FI}} = 2\int d^4\theta \kappa V \tag{13}$$

and

$$\mathcal{L}_{\text{matter}} = \sum_i \int d^4\theta \bar{\Phi}_i e^{q_i V}\Phi^i + (\mathcal{L}_{\mathcal{W}} + h.c.) \ , \tag{14}$$

where we have defined

$$W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V. \tag{15}$$

[2]We have to satisfy the anomaly conditions $\sum_i q_i = 0$ and $\sum_i q_i^3 = 0$. One way is to pick pairs of chirals with opposite charge only, but we could come up with more exotic field content.

The vector superfield is written in components as

$$V = B + \theta\chi - \bar{\theta}\bar{\chi} + \theta^2 C + \bar{\theta}^2\bar{C} - \theta\sigma^\mu\bar{\theta}A_\mu + i\theta^2\bar{\theta}(\bar{\lambda} + \frac{1}{2}\bar{\sigma}^\mu\partial_\mu\chi) \\ - i\bar{\theta}^2\theta(\lambda - \frac{1}{2}\sigma^\mu\partial_\mu\bar{\chi}) + \frac{1}{2}\theta^2\bar{\theta}^2(D + \partial^\mu\partial_\mu B) \tag{16}$$

(i) Remind yourself of the dynamical degrees of freedom in the vector multiplet.

(ii) What is the Wess-Zumino (WZ) gauge? What symmetry does it correspond to?

(iii) Show in the WZ gauge

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2g^2}D^2 - \frac{i}{g^2}\lambda\bar{\partial\!\!\!/}\bar{\lambda} + \frac{\vartheta}{32\pi^2}F_{\mu\nu}\widetilde{F}^{\mu\nu}\ . \tag{17}$$

(iv) Show that we can write

$$\mathcal{L}_{\text{FI}} = \frac{1}{2}\int d\theta^\alpha \kappa W_\alpha + h.c.\ . \tag{18}$$

(v) Without any gauge fixing, integrate out F^i and D to obtain the scalar potential

$$\mathcal{V} = \sum_i e^{-q_i B}|\partial_i \mathcal{W}|^2 + \frac{g^2}{4}(2\kappa + \sum_i q_i e^{q_i B}|\phi^i|^2)^2 + C(\sum_i q_i\phi^i\partial_i\mathcal{W}) + h.c. \tag{19}$$

fix the WZ gauge and compare with the result given in class.

(vi) What happens in the case of $\kappa \neq 0$?

(vii) Consider SQED with $\kappa \neq 0$, with two chirals $\Phi_\pm$ of opposite charge $\pm q$ and the superpotential

$$\mathcal{W} = m\Phi_+\Phi_- \tag{20}$$

In WZ gauge write down the superpotential for the dynamical components. Analyse the symmetry breaking behaviour and particle spectrum (masses) for:

(a) $m^2 > \frac{1}{2}q\kappa$,

(b) $m^2 < \frac{1}{2}q\kappa$.

(viii) Finally consider SQED with $\kappa \neq =$, with two chirals $\Phi_\pm$ of opposite charge $\pm q$, one neutral chiral Φ_0 and the superpotential

$$\mathcal{W} = \mu\Phi_0 + \frac{1}{2}\lambda\Phi_0^2 + m\Phi_+\Phi_- + g\Phi_0\Phi_+\Phi_- \tag{21}$$

Analyse the moduli space of this theory and check that it allows for gauge symmetry breaking while supersymmetry is unbroken.

3 Symmetry breaking

You're given a collection of (schematic) scalar potentials of a 4d $\mathcal{N} = 1$ gauge theory, Figure 1. Can you immediately tell whether supersymmetry or gauge theory is broken? State what is broken for

(i) ,

(ii) ,

(iii) ,

(iv) .

Can you find a model of SQED for all four cases?

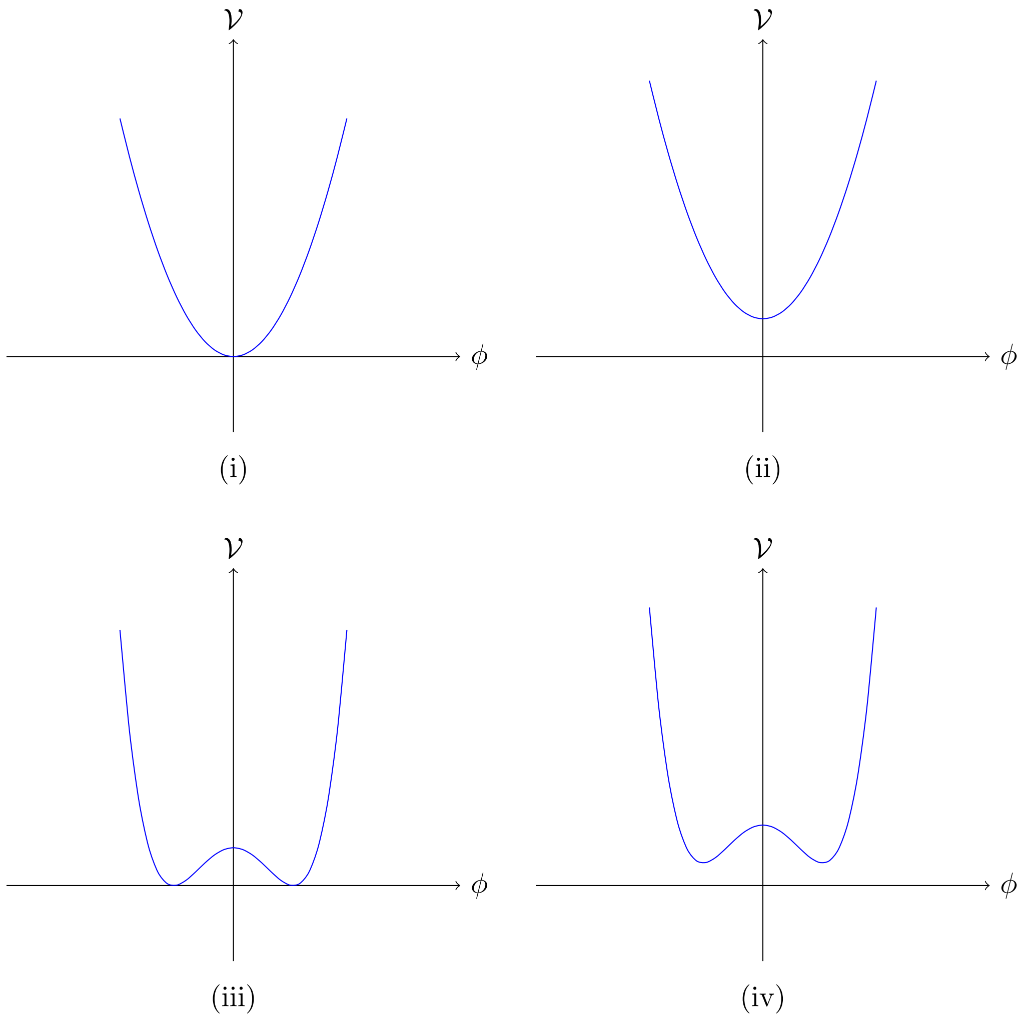

Figure 1: Different scalar potentials.