Direct and large-eddy simulation of rotating turbulence

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Multiscale Modeling and Simulation (Twente)
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Will

- connect DNS to experiment - rotating decaying turbulence
- show accuracy of regularization modeling - LES
- apply regularization modeling to high *Re* flow
- quantify cross-over from 3D to 2D dynamics as *Ro* $\downarrow$
- illustrate low *Ro* compressibility effects

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Outline

1. Rotation modulates turbulence
2. Rotation of incompressible fluid
3. Rotation at high Reynolds numbers
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Taylor-Proudman: columnar structuring

Strong rotation/small friction:

> "Motion of a homogeneous fluid will be the same in all planes perpendicular to the axis of rotation"

Taylor's experiment:

- When an object moves in a rotating flow, it drags along with it a column of fluid parallel to the rotation axis

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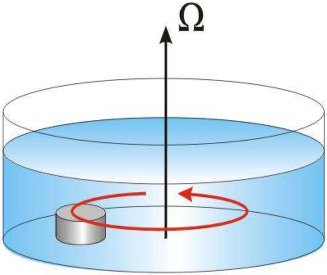

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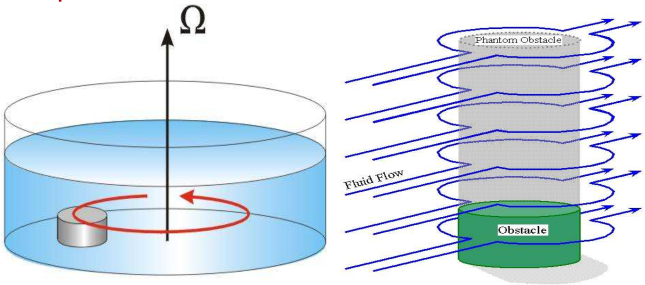

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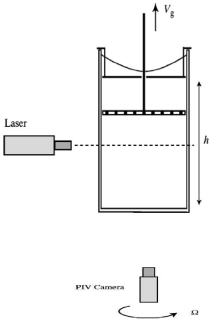
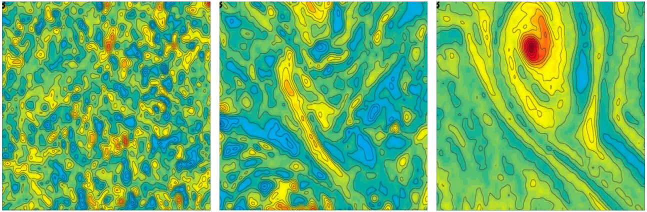
Guadalupe Islands

Taylor columns disturb cloud flow well above islands

Rotating turbulence in a container

Experiment: Morize, Moisy, Rabaud, PoF, 2005





$Ro = 6.4$ $\quad$ $Ro = 0.6$ $\quad$ $Ro = 0.3$

Vertical structuring of flow as $Ro = u_r/(2\Omega_r \ell_r)$ decreases

Modulation: Competition 3D turbulence – 2D structuring

Rotating turbulence in a container

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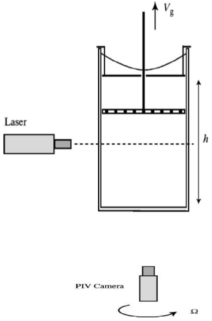

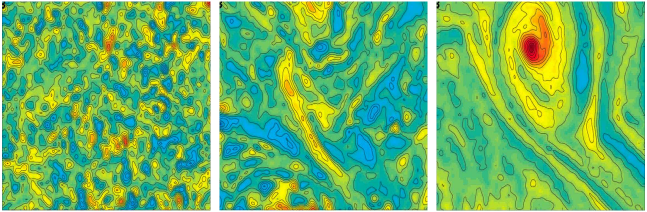

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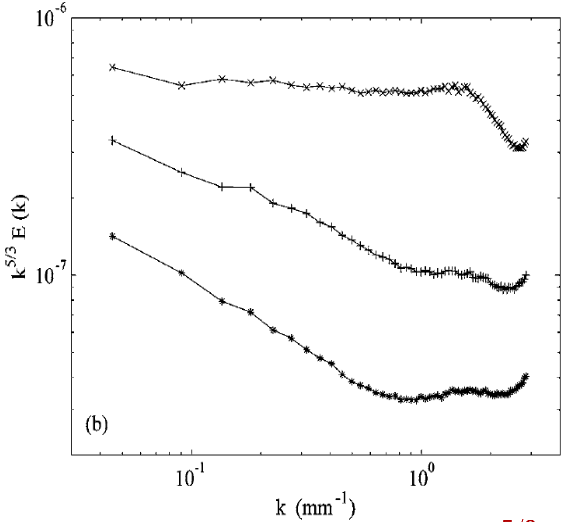
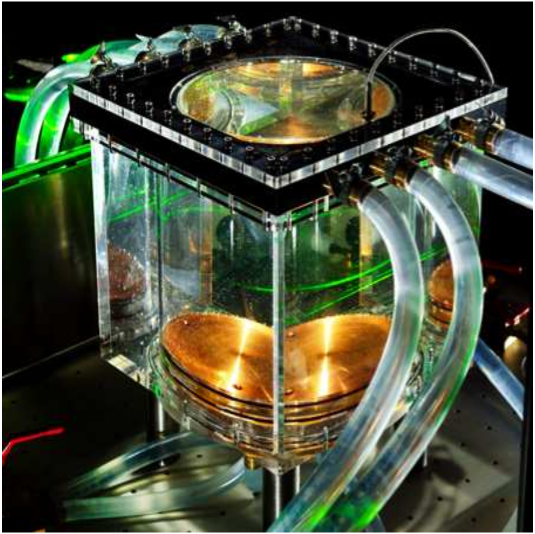


$\times$ $Ro = 5.5$
$+$ $Ro = 1.2$
$*$ $Ro = 0.35$

Alteration of energy-spectrum: from $k^{-5/3}$ to steeper $k^{-9/4}$

Rotating Rayleigh-Bénard problem



Collaboration with Rudie Kunnen, Herman Clercx - Eindhoven

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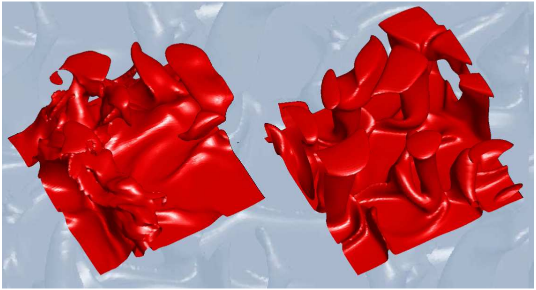

- Temperature: buoyancy dominated ($Ta = 0$ - left) and rotation dominated ($Ta > Ra$)

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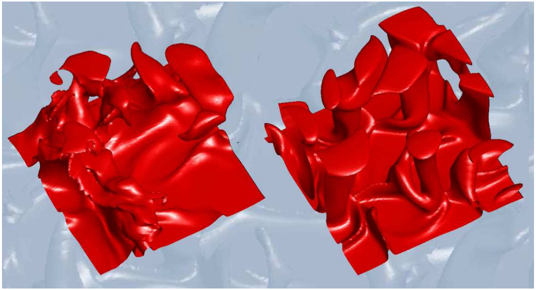

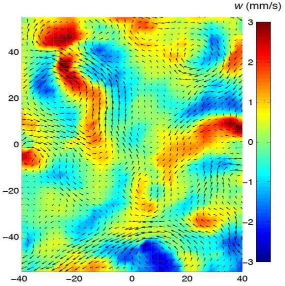

- Temperature: buoyancy dominated ($Ta = 0$ - left) and rotation dominated ($Ta > Ra$)
- Stereo PIV measurement of (u, v, w) in horizontal plane

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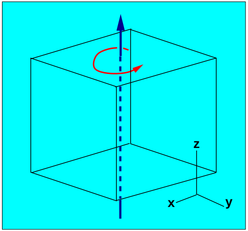
Governing equations

Incompressible formulation: rotating frame of reference

$$\nabla \cdot \mathbf{u} = 0$$
$$\partial_t \mathbf{u} + \nabla \cdot (\mathbf{u}\,\mathbf{u}) + \nabla p - \frac{1}{Re}\nabla^2 \mathbf{u} = \frac{\mathbf{u} \times \boldsymbol{\Omega}}{Ro}$$

Characteristic numbers:

$$Re = \frac{u_r \ell_r}{\nu_r} \quad ; \quad Ro = \frac{u_r}{2\Omega_r \ell_r}$$



Numerical method:

- Pseudo-spectral discretization: full de-aliasing, parallelized
- Helical-wave decomposition: Coriolis diagonalization

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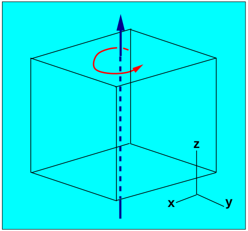

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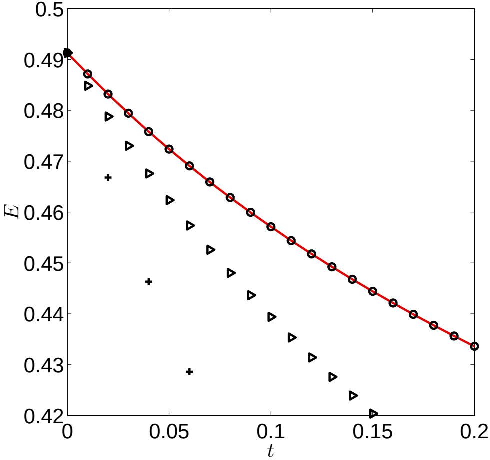
Benefit of helical wave decomposition (HWD)

Decay of kinetic energy:



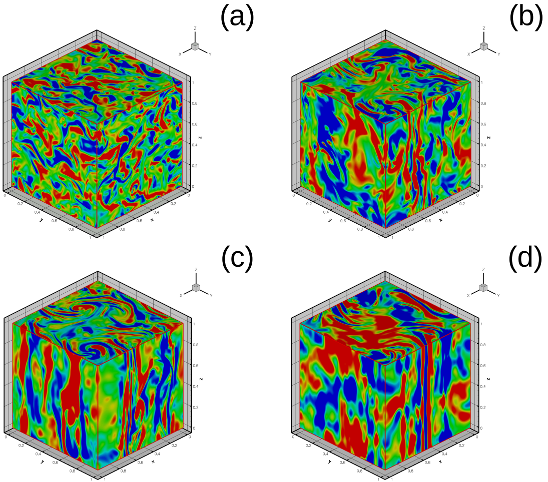
Rotation: $Ro = 10^{-2}$

RK4 with HWD (solid) at $\Delta t = 3.10^{-3}$

RK4 without HWD at
$\Delta t = 7.10^{-3}$ $(+)$
$\Delta t = 5.10^{-3}$ $(\triangleright)$
$\Delta t = 5.10^{-4}$ $(\circ)$

More robust and larger time steps possible

Vertical structuring - vorticity

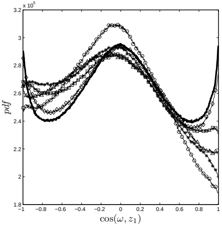


Decaying turbulence at $R_\lambda = 92$. Snapshot $t = 2.5$: $Ro = \infty$ (a), $Ro = 0.2$ (b), $Ro = 0.1$ (c), $Ro = 0.02$ (d)

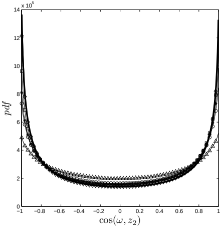
Geometrical statistics - alignment

Eigenvalues/vectors S_{ij}: $(\lambda_i, \mathbf{z}_i)$

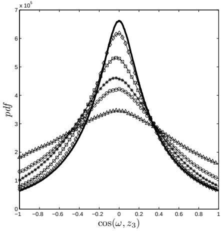
$$(\textit{positive})\ \ \lambda_1 > \lambda_2 > \lambda_3\ \ (\textit{negative})$$



(1)



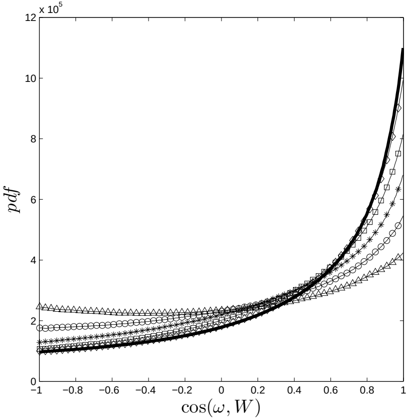
(2)



(3)

- $\cos(\omega, \mathbf{z}_i)$
  - $i = 1$ $Ro \to \infty$ – aligned or perpendicular, rotation reduces parallel alignment
  - $i = 2$ alignment ω and $\mathbf{z}_2$ – reduced by rotation
  - $i = 3$ $Ro \to \infty$ – mainly perpendicular, reduced preference with rotation

Geometrical statistics - vortex stretching



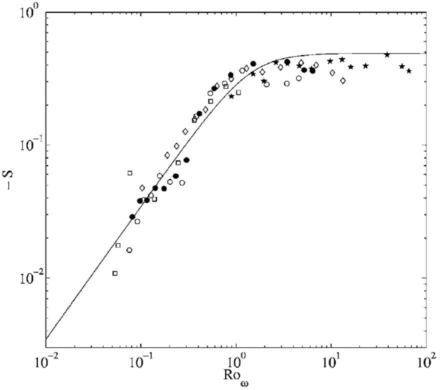
- $\cos(\omega, \mathbf{W})$

Ro $\rightarrow \infty$ vortex stretching dominant over vortex compression

Ro $\downarrow$ decreasing dominance vortex stretching

Depleted skewness at strong rotation

Experimental observation - reduced skewness at high $1/Ro$:

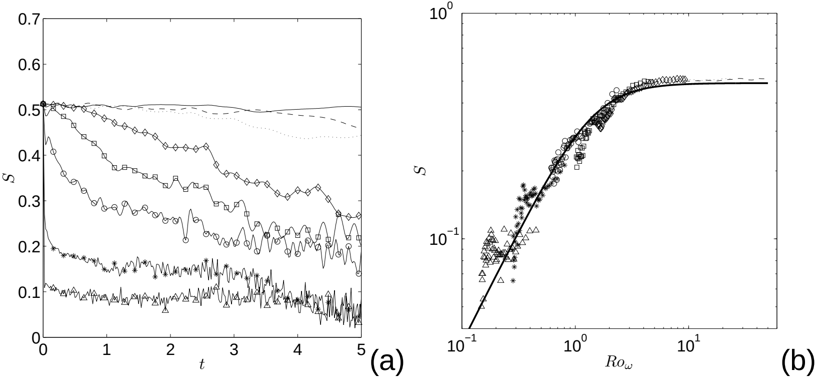


Cambon et al.

$$S = \frac{A}{(1+2Ro^{-2})^{1/2}} \quad ; \quad S \sim Ro \sim \Omega^{-1} \ \ as\ Ro \ll 1$$

Depleted skewness at strong rotation

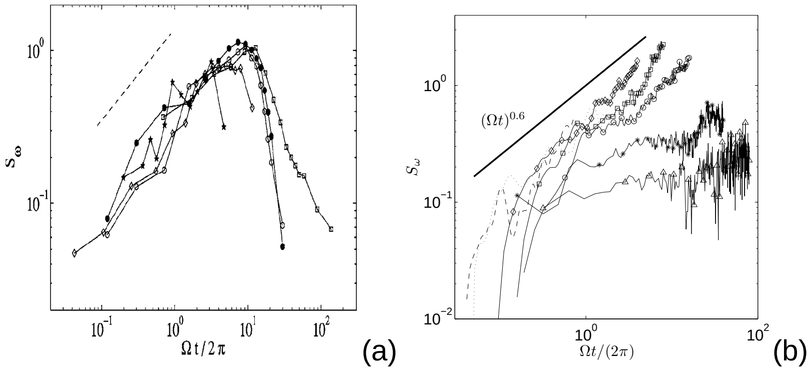
Simulated skewness: DNS at 512^3



Velocity skewness versus t (a) and versus Ro (b)

General agreement Cambon and experiment

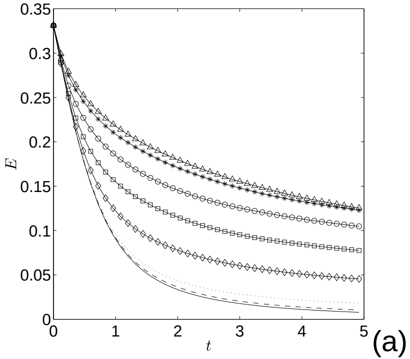
Vorticity skewness



Experimental (a) and simulated (b) vorticity skewness at various rotation rates Ω

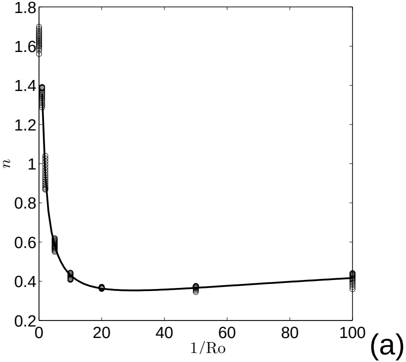
Algebraic decay of kinetic energy

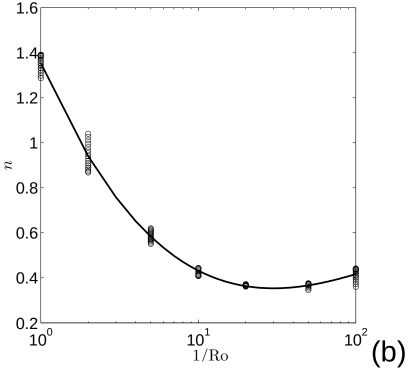
DNS at 256^3:



Decay of kinetic energy - slower decay at stronger rotation

Algebraic decay exponent





Exponent computed over various time-intervals $t_2 - t_1$.

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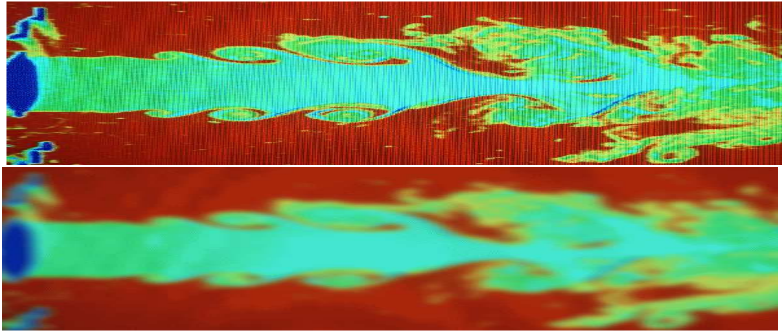
DNS and LES in a picture



Classical problem: wide dynamic range

- capture both large and small scales: resolution problem

$$N \sim Re^{9/4} \;\; ; \;\; W \sim Re^3$$

If $Re \to 10 \times Re$ then $N \to 175N$ and $W \to 1000W$

- Coarsening/mathematical modeling instead: LES

Filtering Navier-Stokes equations

Convolution-Filtering: filter-kernel G

$$\overline{u}_i = L(u_i) = \int G(x-\xi)u(\xi)\,d\xi$$

Application of filter:

$$\partial_t \overline{u}_i + \partial_j(\overline{u}_i\overline{u}_j) + \partial_i\overline{p} - \frac{1}{Re}\partial_{jj}\overline{u}_i - \frac{1}{Ro}(\overline{\mathbf{u}} \times \mathbf{\Omega})_i = -\partial_j\tau_{ij}$$

Turbulent stress tensor: $\tau_{ij} = \overline{u_i u_j} - \overline{u}_i\overline{u}_j$

Shorthand notation: Closure problem

$$NS(\mathbf{u}) = 0 \quad \Rightarrow \quad NS(\overline{\mathbf{u}}) = -\nabla\cdot\tau(\mathbf{u},\overline{\mathbf{u}}) \;\Leftarrow\; -\nabla\cdot M(\overline{\mathbf{u}})$$

Closed LES formulation

$$Find\ v: \quad NS(v) = -\nabla\cdot M(v)$$

Various closure models – (dynamic) eddy-viscosity, similarity, ...

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Regularization models

Alternative modeling approach

- 'first principles' – derive implied subgrid model
- achieve unique coupling to filter, e.g., Leray, LANS-α

Example: Leray regularization - alteration convective fluxes

$$\partial_t u_i + \overline{u}_j \partial_j u_i + \partial_i p - \frac{1}{Re}\Delta u_i = 0$$

LES template:

$$\partial_t \overline{u}_i + \partial_j(\overline{u}_j \overline{u}_i) + \partial_i \overline{p} - \frac{1}{Re}\Delta \overline{u}_i = -\partial_j \left(m^L_{ij} \right)$$

Implied Leray model

$$m^L_{ij} = L\left(\overline{u}_j L^{-1}(\overline{u}_i)\right) - \overline{u}_j \overline{u}_i = \overline{\overline{u}_j u_i} - \overline{u}_j \overline{u}_i$$

- Rigorous analysis available ($\sim$ 1930s)
- Provides accurate LES model ($\sim$ 03)

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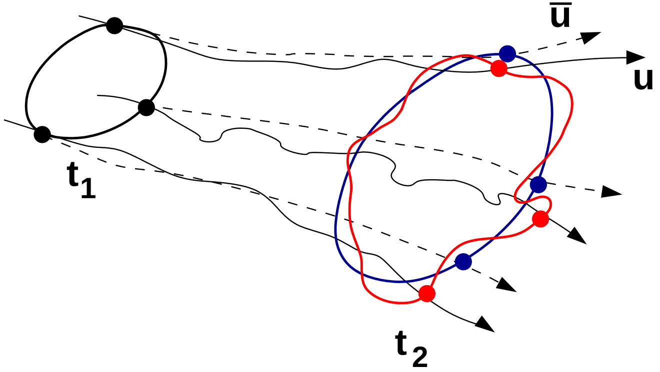
NS-α regularization

Kelvin's circulation theorem

$$\frac{d}{dt}\left(\oint_{\Gamma(\mathbf{u})} u_j\, dx_j\right) - \frac{1}{Re}\oint_{\Gamma(\mathbf{u})} \partial_{kk} u_j\, dx_j = 0 \quad \Rightarrow \quad NS - eqs$$



Filtered Kelvin theorem

NS-α regularization

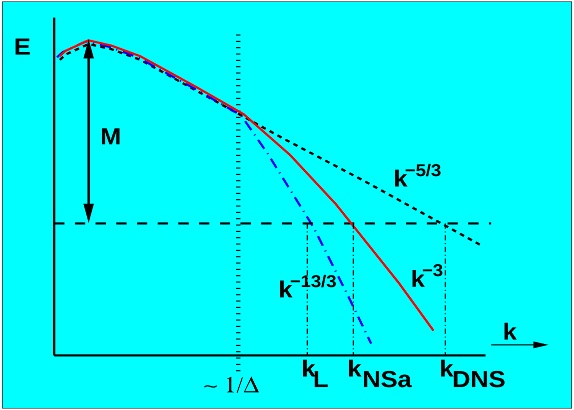
α-principle: Fluid loop: $\Gamma(\mathbf{u}) \to \Gamma(\overline{\mathbf{u}}) \Rightarrow$ Euler-Poincaré

$$\partial_t u_j + \overline{u}_k \partial_k u_j + u_k \partial_j \overline{u}_k + \partial_j p - \partial_j (\frac{1}{2} \overline{u}_k u_k) - \frac{1}{Re} \partial_{kk} u_j = 0$$

Rewrite into LES template: Implied subgrid model

$$\begin{aligned} \partial_t \overline{u}_i \quad &+ \quad \partial_j (\overline{u}_j \overline{u}_i) + \partial_i \overline{p} - \frac{1}{Re} \partial_{jj} \overline{u}_i \\ &= \quad -\partial_j \left(\overline{\overline{u}_j u_i} - \overline{u}_j \overline{u}_i \right) - \frac{1}{2} \left(\overline{u_j \partial_i \overline{u}_j - \overline{u}_j \partial_i u_j} \right) = -\partial_j m_{ij}^{\alpha} \end{aligned}$$

Cascade-dynamics – computability



- NS-α,Leray are dispersive
- Regularization alters spectrum – controllable cross-over as $k \sim 1/\Delta$: steeper than $-5/3$

Confined and nonconfined decay

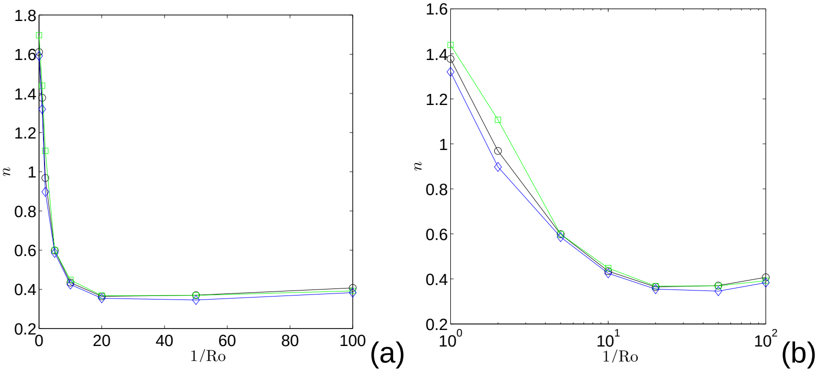
Decay exponent *n* and spectral exponent *p*

		Nonconfined	Confined
Nonrotating	$p = 5/3$	$n = 6/5$	$n = 2$
Rotating(I)	$p = 2$	$n = 3/5$	$n = 1$
Rotating(II)	$p = 3$	$n = 0$	$n = 0$

Rotating I – energy transfers time scale Ω^{-1}
Rotating II – totally inhibited energy transfer (Kraichnan)

LES: Algebraic decay at $R_\lambda = 92$

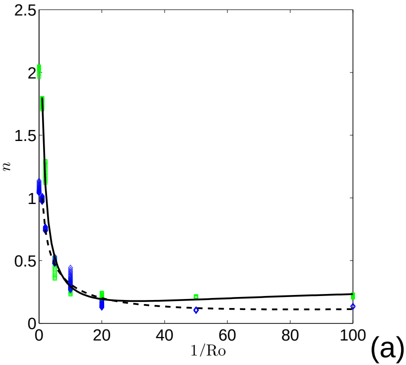
DNS at 256^3 – LES at 64^3:



- Decay exponent well captured: DNS, Leray, LANS-α
- Evidence of nonconfined decay; Rotating I or II ?

LES: Algebraic decay at $R_\lambda = 200$

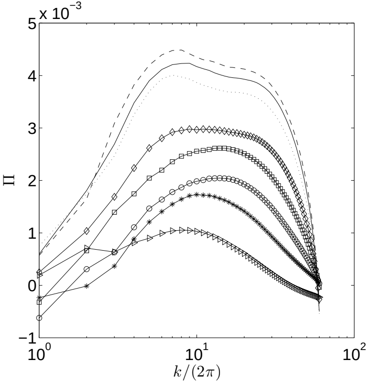
LES at 128^3:



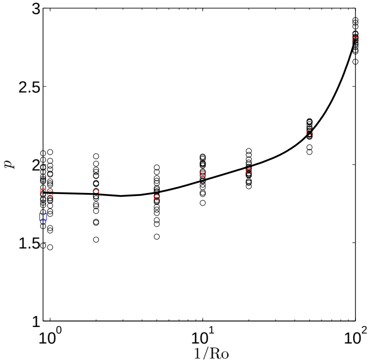
- Leray - dashed, LANS-α – solid
- Leray closer to ‘nonconfined’, LANS-α closer to ‘confined’
- Evidence Rotating II - Kraichnan

LES: Leray spectra at $R_\lambda = 200$

LES at 128^3: totally inhibited energy transfer regime



(a)



(b)

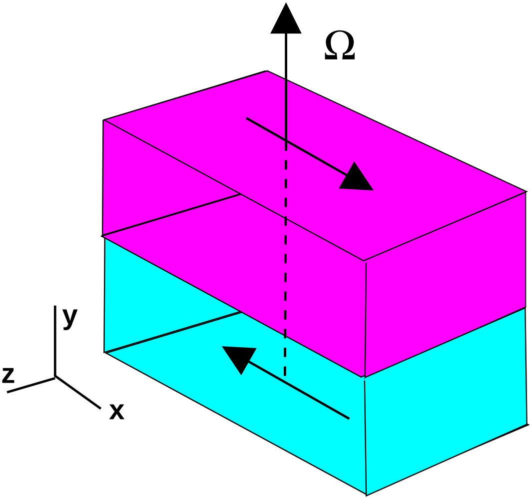
(a): Average transport-power spectra

(b): Energy-spectrum exponent showing transition -5/3 ($Ro \to \infty$) to -3 ($Ro \to 0$) – Kraichnan

Outline

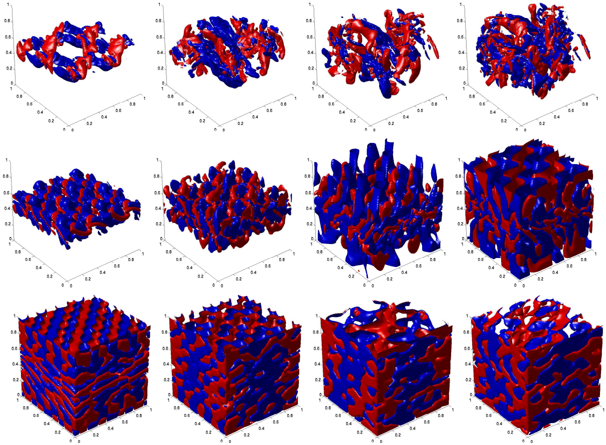
1. Rotation modulates turbulence
2. Rotation of incompressible fluid
3. Rotation at high Reynolds numbers
4. **Rotation induced compressibility effects**
5. Concluding remarks

Compressible rotating mixing layer



- Consider range of *Ro*

Vorticity along axis of rotation: $\omega_2 = \partial_3 u_1 - \partial_1 u_3$



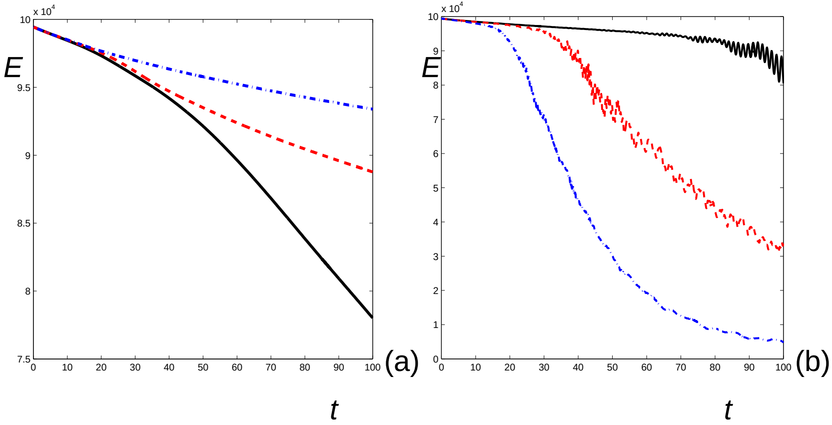
$Ro = \infty$

$Ro = 10$

$Ro = 2$

(30) (60) (80) (90)

Kinetic energy decay



(a): $Ro = \infty$ (solid), $Ro = 10$ (dashed), $Ro = 5$ (dash-dotted)
(b): $Ro = 1$ (solid), $Ro = 0.5$ (dashed) and $Ro = 0.2$ (dash-dotted)

Contributions to decay rate

Total decay:

$$\frac{dE}{dt} = D - W \quad ; \quad D = \langle p\, \nabla \cdot \mathbf{u} \rangle \quad ; \quad W = \frac{1}{2Re} \langle \mathbf{S} : \mathbf{S} \rangle$$

D: pressure/velocity divergence, W dissipation

- incompressible: $D = 0$, $W \geq 0 \rightarrow$ monotonous decay of E
- compressible: $W \geq 0$, $D \neq 0 \rightarrow$ non-monotonous decay

Effect of rotation:

- no direct influence
- contributions to dD/dt and dW/dt

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Effect of rotation:

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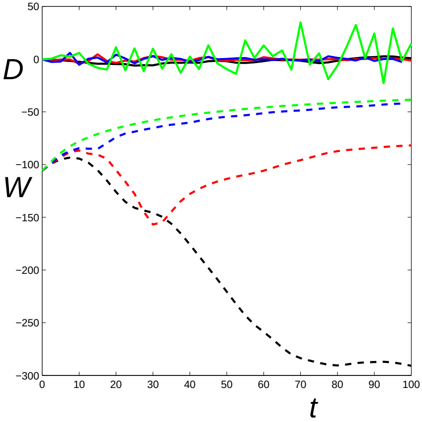
- contributions to dD/dt and dW/dt

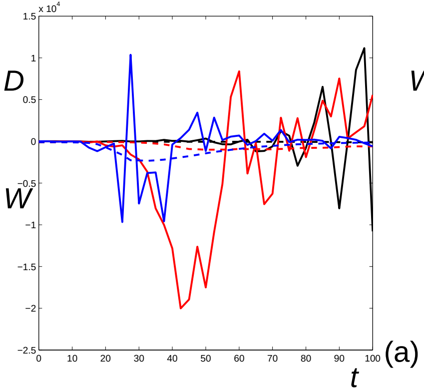
Slow rotation – incompressible limit

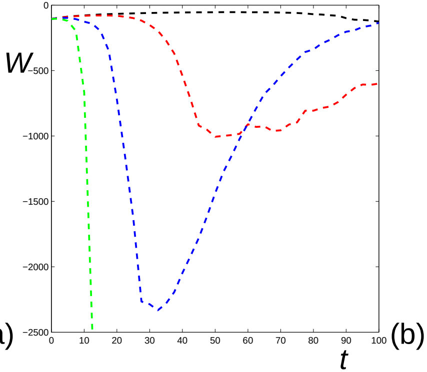


Dissipation and pressure/velocity divergence:

- $Ro = \infty$, $Ro = 10$, $Ro = 5$, $Ro = 2$
- reduced decay-rate as $Ro \downarrow$

Rapid rotation – compressibility





Dissipation W and pressure/velocity divergence D:

- $Ro = 1$, $Ro = 0.5$, $Ro = 0.2$, $Ro = 0.1$
- dominance D over W as $Ro \downarrow$
- non-monotonous decay

Outline

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5. **Concluding remarks**

Concluding remarks

- Established general correspondence DNS and experiments MMR
- Illustrated relevance helical wave decomposition
- Accurate predictions – Leray and LANS-α at $R_\lambda = 92$
- Leray at $R_\lambda = 200$ – exponent spectrum from -5/3 to -3 as $Ro \downarrow$: 3D - 2D
- Compressibility effects induced by Coriolis force: non-uniform decay, indirect rotation effect