Energy cascade in multi-mode stretched spiral vortex and viscoelastic effect

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Temporal development of coherent structures in isotropic turbulence box

Red isosurfaces: tubular object; White isosurfaces: planar object

Motivation

- Existence of organized vortical structures, termed ribbons, blobs, and worms has been known (e.g., Jimenez & Wray 1998).
- The primary elements of vortical structures are the tube-like object and the sheet (or layer)-like object. These objects are not separable since local dissipation is particularly strong, not within vortex tubes, but rather in their neighbourhood (e.g. Kerr 1985).
- A model of generalized Burgers vortices for the small-scale structure of turbulence was introduced in Lundgren (1982). In this model (LSV), vortex sheets are stretched in the spiral to continually tighten, and this mechanism causes an energy cascade. The LSV model gives the $k^{-5/3}$ energy spectrum.

Objective

- Extract LSVs and analyse their complete creation process in homogeneous isotropic turbulence.
- Explore the roles of the LSVs on generation of turbulence energy cascade and dissipation.
- Explore a possibility of achieving turbulence control through the suppression of formation of LSV (in polymer-diluted flow).

Identification method for turbulent structures (1)

Vortex tube:

\[ \Omega_{ii} \Omega_{jj} \gg S_{ij} S_{ij} \]

- Pressure, $p$.
- 2nd-order invariant of the velocity gradient tensor, $Q$

Vortex sheet:

\[ \Omega_{ij} \Omega_{ij} \sim S_{ij} S_{ij} \]

Eigenvalue of the 2nd-order tensor of the velocity gradient tensor:

\[ [A]_{ij} \cdot [A]_{ij} \equiv S_{ij} \Omega_{ij} + S_{ij} \Omega_{ij} \] (Horiuti & Takagi 2005, PoF)

(Kida & Yanase)
Identification method for turbulent structures (3)

Performance of identification method
Comparison with other fourth order velocity gradient invariants

Burgers' vortex layer

Burgers' vortex tube

Reordering of eigenvalues
Based on degree of alignment of their eigenvectors with vorticity vector $\omega$

\[ z \text{ or } s \text{ Maximally aligned with } \omega : \sigma_z, e_z \ (z \text{ or } s) \]
Largest among remainder: $\sigma_+, e_+$
Smallest eigenvalue: $\sigma_-, e_-$

(Andreotti 1993)

Vectors and tensors on the basis of strain rate eigenvectors

- Vector, Vortex stretching term
  \[ \omega_z = \omega \cdot e_z, \ \omega_s = \omega \cdot e_s, \ \omega_v = \omega \cdot e_v. \]
- Tensors: ex) Pressure Hessian term
  \[ \Pi_j = \nabla^T (\Pi_j), \ \Pi_j = \frac{\partial^2 p}{\partial x_i \partial x_j}, \ E = (e_x, e_y, e_z) \]

Alternatively, eigenvalues and eigenvectors of $[A_j]$ are used:

\[ [A_j], [A_j], [A_j], a_z, a_s, a_v. \]

Crossover of strain-rate tensor eigenvalues

Conventional ordering of eigenvalues: $\sigma_1 > \sigma_2 > \sigma_3$

Burgers' vortex tube

Burgers' vortex layer

Alignement of the eigenvector for the second largest eigenvalue with the vorticity vector (Kerr et al. 1985).
Profiles of homogeneous-isotropic DNS data

<table>
<thead>
<tr>
<th>Grid-point #</th>
<th>$R_+$</th>
<th>$k_{max}$</th>
<th>$&lt;K&gt;$</th>
<th>$&lt;\varepsilon&gt;$</th>
<th>$L$</th>
<th>$\lambda$</th>
<th>$\eta \times 10^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>256³ Decay</td>
<td>77.2</td>
<td>1.02</td>
<td>0.90</td>
<td>0.65</td>
<td>0.47</td>
<td>0.14</td>
<td>8.00</td>
</tr>
<tr>
<td>512³ Decay</td>
<td>76.9</td>
<td>2.05</td>
<td>0.90</td>
<td>0.65</td>
<td>0.47</td>
<td>0.14</td>
<td>8.00</td>
</tr>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1024³ Decay (Low Re)</td>
<td>77.4</td>
<td>4.09</td>
<td>0.90</td>
<td>0.65</td>
<td>0.47</td>
<td>0.14</td>
<td>8.00</td>
</tr>
<tr>
<td>1024³ Decay (High Re)</td>
<td>122.5</td>
<td>1.35</td>
<td>0.96</td>
<td>0.30</td>
<td>0.47</td>
<td>0.09</td>
<td>2.63</td>
</tr>
<tr>
<td>512³ Forced (1.0&lt;k&lt;2.5)</td>
<td>158.1</td>
<td>2.27</td>
<td>1.41</td>
<td>0.40</td>
<td>1.26</td>
<td>0.22</td>
<td>8.91</td>
</tr>
<tr>
<td>1024³ Forced (1.0&lt;k&lt;2.5)</td>
<td>243.3</td>
<td>2.49</td>
<td>1.43</td>
<td>0.39</td>
<td>1.14</td>
<td>0.15</td>
<td>4.89</td>
</tr>
</tbody>
</table>

Assessment using the DNS data

- Homogeneous isotropic turbulence (decaying)
  
  Initial energy spectrum:
  \[
  E(k) = C k_p^\alpha \left(\frac{k}{k_p}\right)^\beta \exp\left\{-2\left(\frac{k}{k_p}\right)^2\right\}, \quad k_p = 2
  \]

- Homogeneous shear turbulence

Advantage of use of eigenvalue, $[A_{ij}]_+$

- Vortex sheet is approximately spanned by the eigenvectors for $[A_{ij}]_+$ and $[A_{ij}]_-$, noting that $\text{grad}([A_{ij}]_+)$ is nearly perpendicular to the surface of the sheet.

Formation of vortex tube via conventional rolling-up of a (single) vortex sheet

The vortex tube is formed through focusing of vorticity along a single vortex sheet (Neu 1984, Kerr & Dold 1994).
Formation of vortex tube via conventional rolling-up
Vorticity vectors are always aligned with the longitudinal direction of the tube.

Origin of the tube in the development of the rolled-up vortex sheet

Traced back to the concentration of vorticity along the sheet in the initial velocity field. The vortex tube is formed through focusing process (Neu 1984).

Multi-mode stretched spiral vortex

3-dimensional rendering

Mode 1
By Kelvin-Helmholz instability

Mode 2
Through interaction of several sheets

Mode 3
(Pullin & Lundgren 2003)

Mode 1
(Lundgren 1982)
A process of formation of stretched spiral vortex

**Configuration in early stage**
- Consists of a lot of stagnation flows caused by vortex sheets. (Davila and Vassilicos 2003)
- Straining and stretching of the vortex blob along the sheets. (Gilbert 1993)
- Mostly in Mode 3, converted into Mode 1 or Mode 2 with lapse of time.

**Initial configuration**
- Appearance of the stagnation flow along the vortex sheets.
- Generation of recirculating flow through interaction with another sheet.
- Straining and stretching of the sheets by the recirculating flow.
- Reorientation the vorticity directions along the stretched sheets due to the action of the pressure Hessian term.
- Creation of the vortex tube by axial straining and concentration of the low pressure region in the recirculating flow.

**Summary of the process**
- Mostly in Mode 3
- Different from rolling up of the layer due to Kelvin-Helmholz instability.
- Created through the interaction of several sheets.
- Convergence of recirculating flow and concentration of its low-pressure region.

**Distribution of sheets at an early stage**
(Waleffe 2003)
A process of formation of stretched spiral vortex

- Different from rolling up of the layer due to Kelvin-Helmholz instability.
- Created through the interaction of several sheets.
- Similar to the process considered for wall turbulence by Waleffe (2003).

Contours of vortex sheets and pressure

Gray scale: vortex sheet, Vectors: velocity

- Initial configuration consists of by a stagnation flows caused by vortex sheets, (Davila and Vassilicos 2003)
- The following process is composed of by the three phases.
  1. Genesis phase
  2. Growth phase
  3. Annihilation phase

Genesis phase of LSV

Generation of recirculating flow by convergence of the stagnation flow.

Interaction with the vortex on the third sheets.

In Mode 3

Straining and stretching of vortex sheets by recirculating flow and the swirling flow caused by the vortex along S3.

Formation of lower (L) and upper (U) sheets
Creation of the vortex tube in the core region of LSV

- Absorption of the low pressure region in the recirculating flow into the lower sheet L.
- Stretching due to axially straining fields induced by the vortices in near neighbors.
- Concentration of the vorticity in the low pressure region.

Creation of vortex tube by axial straining and concentration of low pressure region.

- Entrainment of vortex sheets by the tube, causing the sheets to form a spiral.
- Lowering of pressure and intensification of swirling motion.
- This spiral tightens and form spiral turns.

Fractal properties of spiral (Vassilicos & Brasseur 1996)

Growth phase of LSV

- Decrease of the area of the cross section of the tube
- Concentration of the vorticity
- Further stretching of lower and upper sheets
- Entrainment of vortex sheets by the tube, causing the sheets to form a spiral
- This spiral tightens and form spiral turns

Generation of intense dissipation along the spiral arms

Schematic sketch of streamwise vortex formation process in sheared turbulence (Waleffe 2003)
Inter-mode transition in stretched spiral vortex

Initial configuration: Mode 3
⇒ Occurrence of reorientation of vorticity vector

Vortex stretching term: \( \sigma_z \omega_z^2 \)

Converted to Mode 2

Mechanism for occurrence of reorientation of vorticity direction

Governing equation for \( \sigma_z \) on the \( e_z, e_+, e_- \) basis

\[
\frac{D}{Dt} \sigma_z = -\sigma_z^2 + \frac{1}{4} (\omega_z^2 + \omega_z^2) \rho \frac{\Pi}{\sigma_z^2} \]

Horiiuti & Fujisawa (2008)

On upper sheet U:
\[\tilde{\Pi}_z < 0 \Rightarrow \frac{D}{Dt} \sigma_z > 0 \Rightarrow \sigma_z \xrightarrow{\rightarrow} \sigma_z > 0\]

On lower sheet L:
\[\tilde{\Pi}_z > 0 \Rightarrow \frac{D}{Dt} \sigma_z < 0 \Rightarrow \sigma_z \xrightarrow{\rightarrow} \sigma_z < 0\]

Inter-mode transition of stretched spiral vortex

Example of Mode 3 – Mode 1 transition

Formation of recirculating flow by an interaction of three sheets

\( \tilde{\Pi}_z > 0 \) on both sheets
⇒ Occurrence of reorientation on both sheets

Appearance for Mode 3-2 and 3-1 transitions

Mode 3 – 2 transition
Pressure distribution is convex near the branching point (B) of pressure.

\( \tilde{\Pi}_z < 0 \) on upper sheet
\( \tilde{\Pi}_z > 0 \) on lower sheet
⇒ Occurrence of reorientation only on lower sheet

Mode 3 – 1 transition
Pressure distribution is concave on both sheets.

\( \tilde{\Pi}_z > 0 \) on both sheets
⇒ Occurrence of reorientation on both sheets
Inter-mode transition in stretched spiral vortex

Initial configuration: Mode 3
⇒ Occurrence of reorientation of vorticity vector direction

Vortex stretching term: $\sigma_z \omega_z^2$

Velocity direction on S3

The same as those on S1 and S2:
Mode 3 ⇒ Mode 1
Opposite to those on S1 and S2:
No reorientation takes place

$$\frac{D}{Dt} \sigma_z = -\sigma_z^2 + \frac{1}{4}(\omega_x^2 + \omega_y^2) - \Pi_{zz}$$

Velocity direction on S3

The same as those on S1 and S2:

Governing equation for $\sigma_z$

$$\frac{D}{Dt} \sigma_z = -\sigma_z^2 + \frac{1}{4}(\omega_x^2 + \omega_y^2) - \Pi_{zz}$$

Occurrence of reorientation of vorticity vector direction

Persistence of three modes

Mechanism for stretching of the vortex sheet and formation of spiral turns

Differential rotation induced by the tube and that self-induced by the sheet ⇒ stretching, thinning and spiralling of vortex sheets to extreme length.

(Lundgren 1982)

A measure for the strength of the differential rotation

$$D = r \frac{\partial}{\partial r} \left( \frac{\mu}{r^2} \right)$$

Persistence of three modes

Schematic of configuration on lower and upper sheets in Mode 2

Intense azimuthal velocity is induced by the vortex sheet on the lower sheet $L$.

Differential rotation induced on the two sheets:
Lower sheet >> Upper sheet
Persistence of Mode 1 configuration
Estimate of average thickness of the vortex sheet, $\delta$

Decrement $\delta(t)$ obtained by fitting the energy spectra with the functional form as $E(k, t) = c(t) k^n \omega^{-2\delta\omega(k)}$

(Passot et al. 1995)

Asymptotic values

- Run 1 ($k_{\text{max}} = 4.0$): $2.05 \bar{T}$
- Run 2 ($k_{\text{max}} = 2.0$): $2.34 \bar{T}$
- Run 3 ($k_{\text{max}} = 1.0$): $3.21 \bar{T}$

No apparent tendency for convergence of the average thickness observed.

Process of formation of stretched spiral vortex (3)

Magnitude of vorticity implies reduction of strain (Tsinober et al. (1997)).

Existence of a region with large (concentrated) vorticity: indispensable for transformation of flat sheet region into vorticity-dominant region

Interaction of strain and vorticity

Enhancement of vorticity implies reduction of strain (Tsinober et al. (1997))

Existence of a region with large (concentrated) vorticity: indispensable for transformation of flat sheet region into vorticity-dominant region
Dissipation and vortex-stretching terms

The term representing the energy cascade

Governing equations of the strain rate and enstrophy

\[
\frac{D}{Dt} \left( \frac{1}{2} S_{ij} S_{ji} \right) = -S_{ik} S_{kj} S_{ji} - \Omega_{ik} \Omega_{kj} S_{ji} - S_{ij} \frac{\partial^2 p}{\partial x_i \partial x_j} + \nu S_{ij} \frac{\partial^2 S_{ij}}{\partial x_i \partial x_k}
\]

\[
\frac{1}{2} \frac{D}{Dt} \left( \frac{1}{2} \omega_{ij} \omega_{ji} \right) = 2\Omega_{ik} \Omega_{kj} S_{ji} - 2\nu \frac{\partial^2 \Omega_{ij}}{\partial x_i \partial x_k}
\]

- Strain production term + enstrophy production term

Cf. Estimate obtained using the nonlinear SGS model in LES

Estimate of the magnitude of energy cascade into small scale

Intense vortex-stretching along the vortex sheets

Superposition of \( \Omega_{ik} \omega_{kj} S_{ji} \) term and vortex sheets

PIV measurements

Correlation between vortex sheet and vortex-stretching term

\(-S_{ik} S_{kj} S_{ji} + \Omega_{ik} \Omega_{kj} S_{ji}\) term on the vortex sheet

PIV measurements
Distributions for indicator for the (small scale) turbulence generation.

Production term for strain and vorticity: $-S_{ik} S_{kj} S_{ji} + \Omega_k \Omega_{ij} S_{ji}$

Mechanism for stretching of the vortex sheet and formation of spiral turns

Differential rotation induced by the tube and that self-induced by the sheet
$\Rightarrow$ stretching and spiralling of vortex sheets
(Lundgren 1982)

A measure for the strength of the differential rotation
$$D = r \frac{\partial}{\partial r} \left( \frac{u_r}{r} \right)$$

Stretching and thinning of the spiral sheet to extreme length.
$\Rightarrow$ intense turbulent energy cascade and dissipation

Note: Differential rotation induced on the sheets in Mode 2:
- Lower sheet $\gg$ Upper sheet
- Persistence of Mode 1 configuration

Energy cascade in multi-mode stretched spiral vortex

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Y. Takagi, T. Fujisawa, K. Saitou, K. Kawamura, K. Matsumoto

Dissipation field in decaying homogeneous isotropic turbulence

PDF of dissipation rate from three Runs.

Objective
- Identify the vortical structure responsible for causing turbulent energy dissipation.
- Reveal the formation process for the identified structure.
- Examine the grid resolution requirement for the structures.

Model for turbulence energy cascade

Dissipation field in decaying homogeneous isotropic turbulence

$\text{Re}_i \sim 87.0$

Grid resolution:
- Run 1: $k_{\text{max}} = 4.0$ (1024$^3$)
- Run 2: $k_{\text{max}} = 2.0$ (512$^3$)
- Run 3: $k_{\text{max}} = 1.0$ (256$^3$)

$\overline{\nu}$: Averaged Kolmogorov scale

1st generation: LSV with intense vorticity
Confinement of large circulation in the recirculating flow into small cross section

2nd generation: LSV carrying smaller vorticity
Stretching of the spiral arms by 1st generation LSV $\Rightarrow$ Instability of the spiral sheets

3rd generation:
Straining and stretching of the vorticity blobs $\Rightarrow$ Tertiary instability $\Rightarrow$ Rolling up of sheets

Formation of hierarchical cluster of self-similar LSV networks
Intermittent cascade of energy to small scales

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Formation of hierarchical cluster of self-similar LSV networks
Intermittent cascade of energy to small scales
Scenario for turbulence energy cascade

- Formation of hierarchical cluster of self-similar LSV networks
- Cascade of energy to small scales

Frisch et al. (1978)

Richardson’s scenario for cascade

Decomposition of the strain production and vortex-stretching terms in the three regions (t =2.75)

<table>
<thead>
<tr>
<th>Region</th>
<th>Strain production term fraction</th>
<th>Vortex stretching term fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curved sheet</td>
<td>0.32</td>
<td>0.09</td>
</tr>
<tr>
<td>Flat sheet</td>
<td>0.41</td>
<td>0.36</td>
</tr>
<tr>
<td>Tube-core</td>
<td>0.27</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Decomposition of turbulence statistics in three regions

<table>
<thead>
<tr>
<th>Region</th>
<th>Grid point fraction</th>
<th>Energy fraction</th>
<th>Dissipation fraction</th>
<th>Energy individual</th>
<th>Dissipation individual</th>
<th>Taylor micro scale Re</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curved</td>
<td>0.26</td>
<td>0.27</td>
<td>0.33</td>
<td>1.04</td>
<td>0.80</td>
<td>80.5</td>
</tr>
<tr>
<td>Flat</td>
<td>0.39</td>
<td>0.38</td>
<td>0.39</td>
<td>1.00</td>
<td>0.65</td>
<td>85.8</td>
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<tr>
<td>Core</td>
<td>0.35</td>
<td>0.34</td>
<td>0.28</td>
<td>0.97</td>
<td>0.50</td>
<td>95.1</td>
</tr>
</tbody>
</table>
Correlation between the vortex sheet and dissipation rate

Resolution of spiral turns

Requirement for the grid point numbers: $R_{\lambda}^{2}$ is more feasible than $R_{\lambda}^{3/2}$.

(Sreenivasan 2004)

Formation process of LSV in homogeneous isotropic turbulence

Formation process of spiral vortices in homogeneous isotropic turbulence

Horiuti & Fujisawa (2007)
Beltrami decomposition of velocity fields

\[ u = u^+ + u^- \]

Enstrophy (positive helicity)  
Enstrophy (negative helicity)

At high Re, the stretched sheets are thinner, and spiral has more turns.  
\( \Rightarrow \) Instability of sheets  
\( \Rightarrow \) Creation of extra LSVs along the stretched sheets.

Formation process different from that due to Kelvin-Helmholz instability

Appearance of spiral turns

Generation of intense dissipation along the stretched spiral sheets

Inter-mode transition of stretched spiral vortex

Vortex stretching term:

\[ \sigma_z \omega_z^2 \]

Initial configuration: Mode 3

\( \Rightarrow \) Occurrence of reorientation of vorticity direction along lower sheet

Mode 3 – Mode 2 transition

\[ \omega_z = \omega \cdot e_z, \; \omega_+ = \omega \cdot e_+, \; \omega_- = \omega \cdot e_- \]
A process for formation of vortex tube along flat sheet

Strain-rate eigenvalue $\sigma_z$ (negative)

Occurrence of compression in the stretching ($z$-) direction along the flat sheet.

Distributions of decomposed vortex-stretching terms

<table>
<thead>
<tr>
<th>Stretaching ($z$) direction (negative)</th>
<th>Azimuthal (+) direction (positive)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_z \omega_z^2$ (-7500)</td>
<td>$\sigma_z \omega_z^2$ (12000)</td>
</tr>
</tbody>
</table>

Roles of the pressure for the sheet-tube transformation process

Relaxation of occurrence of compression through the pressure Hessian terms.

Pressure Hessian $\Pi_{zz}$

Cross-section of pressure and flat sheet

Generation of local minimum pressure $\Rightarrow$ Formation of the core region of tube
$\Rightarrow$ Generation of vortex tube with transverse vorticity.

Role of the pressure Hessian term for vorticity generation

Governing equations for vorticities, $\omega_z$, $\omega_\theta$, $\omega_\phi$

\[
\begin{align*}
\frac{D}{Dt} \left( \frac{1}{2} \omega_z^2 \right) &= \sigma_z \omega_z^2 - \frac{1}{4} \frac{\omega_z \omega_\theta}{\sigma_z - \sigma_\theta} \tilde{\Pi}_{zz} < 0 \\
\frac{D}{Dt} \left( \frac{1}{2} \omega_\theta^2 \right) &= \sigma_z \omega_z^2 + \frac{1}{4} \frac{\omega_z \omega_\theta}{\sigma_z - \sigma_\theta} \tilde{\Pi}_{zz} > 0 \\
\omega_\theta \omega_z \tilde{\Pi}_{zz} < 0 : \frac{\omega_\theta \omega_z}{\sigma_z - \sigma_\theta} \tilde{\Pi}_{zz} > 0
\end{align*}
\]

Pressure Hessian term reacts to relax an occurrence of compression in $z$-dir. by converting the $\omega_z$ vorticity into the transverse component, $\omega_\theta$. 

Role of the pressure for the sheet-tube transformation process

Relaxation of occurrence of compression through the pressure Hessian terms.

Pressure Hessian $\Pi_{zz}$

Cross-section of pressure and flat sheet

Generation of local minimum pressure $\Rightarrow$ Formation of the core region of tube
$\Rightarrow$ Generation of vortex tube with transverse vorticity.
Statistical measure for frequency of occurrence of mode transitions

\[ \tilde{\Pi} \] (Conditionally sampled in the sheet region)

\[ \text{p.d.f. of } \sigma_z \omega_z^2 \]

At an early stage: skewed to positive values
- Occurrence of reorientation of vorticity direction
- The vorticity in the converted direction grows.

At a later time: skewed to negative values
- Appearance of a markedly large proportion of negative \( \sigma_z \omega_z^2 \)
- Occurrence of reorientation of vorticity direction

Inter-component energy exchange in Mode 2

Governing equations for energy: \( u_z^2, u_+^2 \)

\[
\begin{align*}
\frac{D}{Dt} \left( \frac{1}{2} u_z^2 \right) &= -\frac{1}{4} \frac{u_z u_z}{\sigma_z - \sigma_+} \left( \omega_z \omega_z - \tilde{\Pi}_{zz} \right) \\
\frac{D}{Dt} \left( \frac{1}{2} u_+^2 \right) &= +\frac{1}{4} \frac{u_z u_z}{\sigma_z - \sigma_+} \left( \omega_z \omega_z - \tilde{\Pi}_{zz} \right) \\
-\frac{1}{4} \frac{u_z u_z}{\sigma_z - \sigma_+} \left( \omega_z \omega_z - \tilde{\Pi}_{zz} \right) &< 0, \quad (\sigma_z - \sigma_+ < 0)
\end{align*}
\]

on upper sheet

Energy transfer from the z-component, \( u_z^2 \), to the transverse component, \( u_+^2 \).

Mode 1 LSV is more persistent than Mode 2 LSV.

Correlation between the vortex sheet and dissipation rate

Isosurfaces of dissipation

Isosurfaces of vortex sheet

Requirement for the grid point numbers: \( R_\lambda^2 \) is more feasible than \( R_\lambda^{3/2}. \) (Sreenivasan 2004)

Resolution of spiral turns

Run 2 (\( k_{max} \overline{\nu} = 2.0 \))

Run 1 (\( k_{max} \overline{\nu} = 4.0 \))
LSV formation at higher Reynolds number

Run 4: Initial velocity field from Run 1 at $t=1.75$, $\nu=0.00138 \rightarrow 0.00024$

Vortex-stretching term
Run 4 ($Re\sim 122.5$)

At high Re, the stretched sheets are thinner, and spiral has more turns.
→ Instability of sheets ⇒ Creation of extra LSVs along the stretched sheets.

Inter-mode transition of stretched spiral vortex in forced turbulence

Mode 3

Topological transformation: helicity $h = u \cdot \omega$

Mode 3

Yellow: $h>0$
Blue : $h<0$
Topological transformation: helicity production term $P_h$

Represented by

\[
\frac{Dh}{Dt} = \frac{\partial}{\partial x} \left( \rho \left( \frac{1}{2} \omega \cdot \mathbf{u}_h \right) \right) = -P_h
\]

(Yellow: $h > 0$
Blue: $h < 0$
(Holm & Kerr 2002)
Derivation of -7/3 energy spectrum (1)

Pullin and Lundgren (2001):

\[ E(k) = \frac{1}{1^{2/3}} \left( \frac{2/3}{2/3} \right) \left( \frac{v}{v a^2} \right) ^2 \frac{E}{k^3} + \frac{1}{1^{1/3}} \left( \frac{2/3}{2/3} \right) \left( \frac{v}{v a^2} \right) ^{1/2} \frac{E}{k^2} \]

Define the stretching parameter \( a \) using the Kolmogorov scaling: \( a = \left( \frac{\varepsilon}{\nu} \right)^{1/3} \)

\[ E(k) = \frac{1}{1^{2/3}} \left( \frac{2/3}{2/3} \right) \left( \frac{v}{v a^2} \right) ^2 \frac{E}{k^3} + \frac{1}{1^{1/3}} \left( \frac{2/3}{2/3} \right) \left( \frac{v}{v a^2} \right) ^{1/2} \frac{E}{k^2} \]

Diverge as \( v \rightarrow 0 \) unless \( \varepsilon \rightarrow 0 \).

Removal of divergence by using a large scale shear rate, \( S \).

\[ E(k) \sim \varepsilon^{1/3} S k^{-7/3} \quad \left( a = \left( \frac{\varepsilon}{\nu} \right)^{1/3} \right) \]

\[ E_{12}(k) \sim \varepsilon^{1/3} S k^{-7/3} \]

(Ishihara et al. 2002)

Derivation of -7/3 energy spectrum (2)

Cascade picture for the evolution of the vorticity blob (Gilbert 1993)

- Stretching of axial vorticity \( \omega_z \rightarrow E(k) \propto k^{-5/3} \)
- Stretching of azimuthal vorticity \( \omega_\theta \rightarrow \propto k^{-7/3} \) (Ohkitani 2004)
- Stretching of azimuthal vorticity \( \omega_r \rightarrow \propto k^{-9/3} \)

Enstrophy spectrum:

\[ \Omega(k) \approx \left( l_0^{11} \omega_0^{10} / a^7 \right)^{1/3} k^{-1/3} \]

\[ \dot{K} = -\varepsilon \rightarrow \dot{\varepsilon} \approx -\frac{\varepsilon^2}{K} \]

\[ \left( \frac{l_0^{11} \omega_0^{10}}{a^7} \right)^{1/3} \approx \frac{\varepsilon^2}{K} \varepsilon^{-2/3} \approx \dot{\varepsilon} \varepsilon^{-2/3} \]

\[ E(k) \propto \dot{\varepsilon} \varepsilon^{-2/3} k^{-7/3} \]

\( \dot{\varepsilon} \): Time derivative of \( \varepsilon \)

\( K \): turbulent energy

Hierarchical spectrum and multi-mode spiral vortex

\[ E(k) \approx K_0 \varepsilon^{2/3} k^{-5/3} + C_1 \dot{\varepsilon} \varepsilon^{-2/3} k^{-7/3} \]

Spiral vortex in Mode 1

Mode 3

Mode 2: intermediate between -5/3 and -7/3
Phase 1 \[\longrightarrow\] Phase 2

Mode 3 \[\longrightarrow\] Mode 1

- Mode 3 (or 2) tends to be converted to Mode 1 (Horiuti \textit{et al.} 2008)
- Rolling-up of the stretched sheet $\Rightarrow$ Creation of Mode 1 spiral vortices
  $\Rightarrow$ Mode 1 spiral vortex predominates in Phase 2.

\begin{align*}
\text{Mode 3} & \quad \text{Mode 1} \\
\text{Phase 1} & \quad \text{Phase 2}
\end{align*}

Time variations of energy and dissipation rate

Mode 3 \quad \text{isotropic turbulence}

\begin{align*}
K & \quad \varepsilon \\
\text{Cross-correlation of } K \text{ and } \varepsilon
\end{align*}

Period of oscillation $\sim$ Eddy turnover time due to forcing: $T$

\begin{align*}
\text{Time lag} & \sim \text{Cascade time scale} \\
& \sim \text{Eddy turnover time according to integral scale } L
\end{align*}

(Wan, Xiao, Meneveau \textit{et al.} 2010)

Conditional sampling of energy spectrum

Extracted spectra

- Forced isotropic turbulence
- $E(k)$,
- $|E(k)|$, $|E(k)|/2$

$C_k \sim 1.61$

$C_1 \sim C_k$

Heisenburg: $C_1 \sim 3/7$ $C_k$

Energy flux and transfer function: Average in Phases 1 and 2

Phase 1 \[\longrightarrow\] Phase 2

\begin{align*}
\text{Energy flux } \Pi(k) \\
\text{Transfer function } T(k)
\end{align*}

Forward

Backward

\begin{align*}
\text{Feedback} & \quad \text{Theory}
\end{align*}
Flux and transfer function: Average in Phase T

Conditional sampling of energy spectrum

Extracted spectra

Forced isotropic turbulence

\[ C_\kappa \approx 1.61 \]

\[ C_1 \approx C_\kappa \]

Heisenburg: \[ C_1 \approx 3/7 C_\kappa \]

Non-equilibrium energy spectrum (Kovasnay model)

\[ E(k) \approx C_\kappa e^{2/3} k^{-5/3} + \frac{2}{3} C_\kappa^2 \varepsilon \varepsilon^{-1/3} k^{-7/3} + \frac{1}{3} C_\kappa^3 \varepsilon^{-1} (\varepsilon)^2 e^{-2} k^{-9/3} + \ldots \]

\[ = C_\kappa e^{2/3} k^{-5/3} + \frac{2}{3} C_\kappa^2 \varepsilon \frac{d(\log \varepsilon)}{dt} e^{1/3} k^{-7/3} + \frac{1}{3} C_\kappa^3 \frac{d^2(\log \varepsilon)}{dt^2} e^{-1} k^{-9/3} + \ldots \]

Objective

- Extract the \( k^{-7/3} \) and \( k^{-9/3} \) spectrum using conditional sampling of the spectrum in quasi-steady turbulence DNS data.
- Discuss on the effect of non-locality in the transfer function.
- Elucidate the roles of the \( k^{-7/3} \) and \( k^{-9/3} \) spectrum in generation of energy transfer and examine the non-equilibrium/unsteady effect.
Multi-mode stretched spiral vortex

3-dimensional rendering

Mode 1
Lundgren spiral vortex

Mode 2
Intermediate

Mode 3

Horiuti & Fujisawa (2008)

Multi-mode stretched spiral vortex

3-dimensional rendering

Mode 1
Lundgren spiral vortex

Mode 2

Intermediate

Mode 3
(Pullin & Lundgren 2003)

Possibility in reduction of turbulence generation by means of termination of the occurrence of LSV formation

Tom’s effect: Drag reduction in the polymer-diluted flows

Investigation of the effect of viscoelasticity on the formation process

Characteristic features of the viscoelastic fluids

1. Inhibition of the vortex generation.
3. Shear-rate dependent viscosity (shear thinning).

Weissenburg effect (Rod climbing)

In this video clip a dilute solution (0.025 wt%) of a high molecular weight (25-10^6 g/mol) polystyrene polymer (Polysciences Inc) is dissolved in a low molecular weight (~100 g/mol) Newtonian viscous (~30 Pa.s) solvent (Piccolastic, Hercules Inc).

In the video clip a rod is rotated with its end immersed in the fluid outlined above. In the Newtonian case inertia would dominate and the fluid would move to the edges of the container, away from the rod. Here however the elastic forces generated by the rotation of the rod (and the consequent stretching of the polymer chains in solution) result in a positive normal force – the fluid rises up the rod. The bulbous shape remaining at the end of the video is the onset of instability as the mass that has been forced up the rod a) relaxes and b) overcomes the force pushing from below.

How these viscoelastic features affect the LSV formation process.

Weissenburg effect

Weissenburg effect (Rod climbing)

In this video clip a dilute solution of polystyrene polymer is dissolved in Newtonian solvent (Piccolastic).

(HP: Prof. McKinley, MIT)
**Incorporation of non-affine effect into the constitutive equation**

**Assumption of complete affinity**

\[
\dot{Q} = Q \cdot \nabla u
\]

**Upper-convective Oldroyd-B constitutive equation**

\[
\frac{D}{Dt} \langle Q \cdot Q \rangle = \langle Q \cdot \dot{Q} \rangle + \nabla \cdot \left[ \frac{1}{2} \nabla (\nabla Q : \nabla Q) \right]
\]

**Introduction of non-affinity**

\[
\dot{Q} = Q \cdot [\nabla u - 2\alpha S]
\]

**Johnson-Segalman constitutive equation**

\[
\dot{Q} = Q \cdot \nabla u
\]

**Parameters of the viscoelastic homogeneous-isotropic DNS data**

- Grid point numbers: 128³
- Molecular viscosity: \( \nu \approx 0.004 \)
- Polymer relaxation time: \( \lambda \approx 0.45 \) (Taylor: 0.66, Kolmogorov: 0.098)
- Non-affine slip parameter: \( \alpha = 0.0, 0.5, 1.0 \)
- Solvent viscosity contribution: \( \beta = 0.8 \)
- Artificial viscosity: \( \kappa = 0.05 \)
- External forcing: Random phase with an energy spectrum

\[
E_r(k,t) = \begin{cases} 
C_r(1.0 \leq k \leq 2.5) \\
0, & \text{otherwise}
\end{cases}
\]

- Initial condition: Newtonian steady turbulence (\( R^2 \approx 90.0 \))
- No damping function for the polymer stress is employed
- Work provided by the forcing to sustain the steady state

<table>
<thead>
<tr>
<th>( (\nu, f) )</th>
<th>( \alpha = 0.0 )</th>
<th>( \alpha = 1.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newtonian</td>
<td>0.470</td>
<td>0.468</td>
</tr>
</tbody>
</table>

**Temporal development of vortex sheets and tubes**

**Constitutive equation for the polymer stress tensor** \( \tau_{ij} \)

\[
\frac{D \tau}{Dt} = \left( 1 - \alpha \right) \left( \tau_{ij, \dot{u}_j} + \tau_{ij, \dot{u}_i} - \alpha \right) \left( \tau_{ij, \dot{u}_j} + \tau_{ij, \dot{u}_i} - \alpha \right)
\]

\[
- \frac{1}{\lambda} \tau_{ij} \beta_{(1-\beta)}^2 \left( \frac{2 \delta_{ij} - \nabla \cdot \nabla}{\lambda} \right) \]

\( \tau_{ijk} : \) slip parameter

\( \alpha = 0.0 \): reduced to Oldroyd-B eq. (Review in Procaccia & Sreenivasan 2008)

\( \alpha = 1.0 \): reduced to Oldroyd-A eq.
Temporal development of vortex sheets and tubes (viscoelastic)

\[ \alpha = 0.0 \quad \alpha = 1.0 \]

Characteristic features of the viscoelastic fluids

Governing equation for radial momentum

\[ r \frac{d}{dr} (p - \tau_{zz}) = r \frac{d}{dr} (\tau_{rr} - \tau_{zz}) + (\tau_{rr} - \tau_{\theta\theta}) + \cdots \]


Determination of normal stress difference is required.

Polymer stress on the basis of \([A]_{ij}\) eigenvectors along the tube

\( a_+ \): in the radial direction \( a_- \): in the azimuthal direction \( a_s \): in the longitudinal direction

Roles of normal stress difference on the vortex tube generation

\[ (\tau_{rr} - \tau_{\theta\theta}) \approx (\tau_{++} - \tau_{--}) \]

Pressure gradient in the radial direction

\[ r \frac{d}{dr} (p - \tau_{zz}) = r \frac{d}{dr} (\tau_{rr} - \tau_{zz}) + (\tau_{rr} - \tau_{\theta\theta}) + \cdots \]

Distribution of \((\tau_{++} - \tau_{--})\) on the tube

\( (\tau_{++} - \tau_{--}) \) : predominantly negative

\[ \therefore \frac{dp}{dr} < 0 \]

Pressure bulges out in tube core region

\( \Rightarrow \) Reduction of lowering of pressure in the tube core.

\( \Rightarrow \) Reduction of growth of the tube.

Inhibition of the vortex generation
Effect of viscoelasticity on the pressure force

Decomposition of the pressure into those due to solvent $p_s$ and polymer $p_τ$

$$\Delta p_s = 2Q, \quad \Delta p_τ = -\frac{∂(\frac{∂τ}{∂x_i})}{∂x_j} = -T$$

Joint pdf of source terms $Q$ and $T$

PDF of angle between $\nabla p_s$ and $\nabla p_τ$

$\nabla p_τ$ tends to oppose to $\nabla p_s$ ⇒ Reduction of stretching of tube and sheet

Determination of normal stress difference along the sheet

First and second normal stress differences along the vortex sheet

First normal stress difference

$$\left(τ_{11} - τ_{22}\right) \approx \left(τ_{ss} - τ_{++}\right)$$

Second normal stress difference

$$\left(τ_{22} - τ_{33}\right) \approx \left(τ_{++} - τ_{--}\right)$$

Note: When the vorticity of sheet is large ⇒ First normal stress difference

$$\left(τ_{11} - τ_{22}\right) \approx \left(τ_{ss} - τ_{++}\right)$$

⇒ Second normal stress difference

$$\left(τ_{22} - τ_{33}\right) \approx \left(τ_{++} - τ_{--}\right)$$

Alignment of $\{A_i\}$ eigenvectors with vortex sheet

$a_+$ eigenvector is mostly perpendicular to the vortex sheet.

Vortex sheet is approximately spanned by the $a_s$ and $a_-$ eigenvectors.
First and second normal stress differences along the vortex sheet

First: \( (\tau_{11} - \tau_{22}) \approx (\tau_{nn} - \tau_{ss}) \)

Second: \( (\tau_{22} - \tau_{33}) \approx (\tau_{ss} - \tau_{nn}) \)

Predominantly positive

Predominantly negative

Stretching and alignment of the polymer molecules along the streamlines

⇒ Extra tension exerted along the sheet ⇒ Snap back of the sheet to the original form

Elongational (or extensional) viscosity

The most likely reason for drag reduction:
Enhanced extensional viscosity leads to increased resistance to extensional motions of the turbulent flow (Lumley 1969)

Toonder et al. (1995) Strain parameter to identify occurrence of elongation

Sureshkumar et al. (1997) the maximum polymer extension to

L in FENE-P model

\( \alpha = 1.0 \)

Elongational \( \eta \)

Effect of viscoelasticity on the occurrence of a role reversal between the eigenvalues

\[
\begin{aligned}
\frac{DS_y}{Dt} &= -\frac{1}{2} \left( \frac{\partial u_{ik}}{\partial x_i} \frac{\partial u_k}{\partial x_j} + \frac{\partial u_k}{\partial x_i} \frac{\partial u_{ik}}{\partial x_j} \right) - T_y - \frac{1}{2} \Pi_y

T_y &= \frac{\partial^2 \tau_{bb}}{\partial x_i \partial x_j} + \frac{\partial^2 \tau_{bb}}{\partial x_i \partial x_j} \\
T &= E \Pi_y, \quad E = (e_+ e_-, e_z)
\end{aligned}
\]

Off-diagonal component of pressure Hessian

\[
\frac{De_+ \bullet e_z}{Dt} = \left( \frac{1}{\sigma_+ - \sigma_z} \right) \left\{ -\Pi_{zz} - \frac{1}{2} T_{zz} \right\}
\]

The rate of rotation in the plane defined by \( e_+ \) and \( e_z \) (Nomura & Post 1998)

The occurrence of a role reversal is inhibited by the polymer stress

Computed cases in pipe flow DNS

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Parameter} & \text{Newtonian} & \text{Johnson-Segalman model} \\
\hline
\alpha & 0.0, 0.1, 0.5, 0.9, 1.0 \\
\text{Re}_{10} & 180 (\text{Re}_{10} \approx 5300) & 180 \\
\text{We}_{10} & - & 25 \\
\text{Domain} & 10R \times R \times 2\pi R & 20R \times R \times 2\pi R \quad \text{Re}_{10} = \frac{\mu R}{v_0} \\
\text{Lz} & 1,800 & 3,600 \\
\text{Grid} & 128 \times 64 \times 64 & 256 \times 64 \times 64 \quad \text{We}_{10} = \frac{Atv^2}{v_0^3} \\
\beta & 1.0 & 0.9 \\
\Delta t u_t / R & 2.0 \times 10^{-4} & 2.0 \times 10^{-5} \\
\hline
\end{array}
\]

Peterlin damping function (FENE-P)

\[
f(r) = \frac{L^2}{L^2 - 3} + \frac{W_{10}}{\text{Re}_{10} \frac{L}{2\alpha}} \left( 1 - 2\alpha \right) \frac{\Pi_{zz}}{\sigma_z}
\]
**Governing Equations**

**Continuity Equation**
\[
d\frac{\partial u_i}{\partial x_i} = 0
\]

**Momentum Equation**
\[
d\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\beta}{\text{Re}_{\text{in}}} \frac{\partial^2 u_i}{\partial x_j^2} + \left(1 - \frac{\beta}{\text{Re}_{\text{in}}} \frac{\partial^2 u_i}{\partial x_j^2} + \Delta p \delta_{ij}\right)
\]

**Constitutive Equation**
\[
\frac{\partial \tau_{ij}}{\partial t} + u_j \frac{\partial \tau_{ij}}{\partial x_i} = (1 - \alpha) \left( \tau_{ij} \frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial x_j} \tau_{ij} - \alpha \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \tau_{ij} \right) \right)
\]

**Johnson-Segalman Model**
\[
f(r) = \frac{L^2}{L^2 - 3} \frac{\text{We}_{\tau}}{\text{Re}_{\tau}} \left[1 - 2 \alpha \tau_{ij}\right] \text{dumbbellの無限伸長を抑制するために導入したdumping項(Peterlin function)}
\]

\[
f(r) = 1 \to \text{Oldroyd-Bモデル(\alpha=0)}\]

\[
\text{We}_{\tau} = \frac{\lambda u^2}{v_0}
\]

**Mean velocity profiles in pipe flow**

**Mean shear stress profiles in pipe flow**

Budget of the shear stress
\[
\frac{r}{2} \frac{\partial \tau}{\partial z} = u' u' + \frac{\beta}{\text{Re}_{\text{in}}} \frac{\partial u_i}{\partial r} + \frac{1 - \beta}{\text{Re}_{\text{in}}} \tau_i
\]

**Dependence of drag reduction rate on slip parameter**

<table>
<thead>
<tr>
<th>Run</th>
<th>α</th>
<th>%DR</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-1</td>
<td>0.0</td>
<td>22</td>
</tr>
<tr>
<td>V-2</td>
<td>0.1</td>
<td>3</td>
</tr>
<tr>
<td>V-3</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>V-4</td>
<td>0.9</td>
<td>7</td>
</tr>
<tr>
<td>V-6</td>
<td>1.0</td>
<td>86</td>
</tr>
</tbody>
</table>

Non-affinity: Drag reduction
Non monotonous
非アファイン粘弾性流体における渦構造

ニュートン性流体

>多数の細かい渦構造

$\alpha=0.0$

>細かい縦渦構造の減少

$\alpha=0.5$

>ニュートン性流体の渦構造と類似

（赤：渦管，白：渦層）

$\alpha=1.0$（過渡期）

>渦構造の消失

過渡後期 >層流化

抵抗削減機構 ($\alpha=0.0$): Bulge out effect

Drag Reduction Mechanism at $\alpha=0.0$: Bulge out effect

圧力に対する高分子寄与

$\frac{\partial^2 p'_i}{\partial \chi_i^2} = 1 - \beta \frac{\partial^2 \tau'_i}{\partial \chi_i \partial \chi_j}$

$\tau'_i$: 高分子圧力

（等値面）

白：渦管
青：高分子圧力

(等高線)

黒：渦管
青→赤：高分子圧力

圧力低下を抑制するbulge out効果

Roles of normal stress difference on the vortex tube generation in pipe

$\left(\tau_{\rho r} - \tau_{\rho \theta}\right) \approx \left(\tau_{zz} - \tau_{ss}\right)$

Pressure gradient in the radial direction

$r \frac{d}{dr} \left( p - \tau_r \right) = r \frac{d}{dr} \left( \tau_{\rho r} - \tau_{\rho \theta} \right) + \left( \tau_{zz} - \tau_{ss} \right) + \cdots$

$\left(\tau_{zz} - \tau_{ss}\right)$: predominantly negative

$\therefore \frac{dp}{dr} < 0$

Pressure bulges out in tube core region

$\Rightarrow$ Reduction of lowering of pressure in the tube core.

$\Rightarrow$ Reduction of growth of the tube.

Inhibition of the vortex generation

Streaky structure in pipe flow

Isosurfaces of $u<-\omega>$

$\Rightarrow$ First normal stress difference

$\left(\tau_{T_1} - \tau_{T_2}\right) \approx \left(\tau_{zz} - \tau_{ss}\right)$

$\Rightarrow$ Second normal stress difference

$\left(\tau_{T_2} - \tau_{T_3}\right) \approx \left(\tau_{ss} - \tau_{rr}\right)$
First and second normal stress differences along the vortex sheet

Similar to homogeneous isotropic turbulence

First: \( \tau_{11} - \tau_{22} \approx (r_- - r_+) \)

Second: \( \tau_{22} - \tau_{33} \approx (r_+ - r_-) \)

Predominantly positive

Stretching and alignment of the polymer molecules along the streamlines

\( \Rightarrow \) Extra tension exerted along the sheet

\( \Rightarrow \) Snap back of the sheet to the original form

Approximate solution of the JS model

Approximate solution with \( \tau_{ij} = 0 \) at \( t=0 \) (Bird et al. 1987)

\[
\tau_{ij}(t) = -\frac{v(1-\beta)}{\lambda} \int_0^{t/T} e^{-\frac{x-t}{T}} 2S_{ij}(s) ds
+ 2\frac{v(1-\beta)}{\lambda} (1-\alpha) \int_0^{t/T} ds \int_0^{s/T} e^{-\frac{x-s}{T}} \left( S_{ik}(s) \frac{\partial u_i}{\partial x_k}(r) + \frac{\partial u_i}{\partial x_k}(r) S_{ij}(s) \right) ds dr
- 2\frac{v(1-\beta)}{\lambda} \alpha \int_0^{t/T} ds \int_0^{s/T} e^{-\frac{x-s}{T}} \left( S_{ik}(s) \frac{\partial u_i}{\partial x_k}(r) + \frac{\partial u_i}{\partial x_k}(r) S_{ij}(s) \right) ds dr
\]

Assume the steady state (the solution up to 2nd-order)

\[
\tau_{ij} \approx -v(1-\beta)S_{ij} + \lambda v(1-\beta)[(1-2\alpha)4S_{ik}S_{ij} + (1-2\alpha)2S_{ik}S_{ij} + (1-2\alpha)-(1-2\alpha)(S_{ik}S_{ij} + S_{ij}S_{ik}) - (S_{ik}S_{ij} + S_{ij}S_{ik})]
+ 2(1-\beta)A_{ij} A_{ij}
\]

Non-monotonical dependence on \( \alpha \)

Production term of the elastic energy

\[
P_e = \pm (1-2\alpha)(-P_k) \begin{cases} + : \alpha \leq 0.5 (k_p = \pm \frac{1}{2} \tau_{ii}) \\ - : \alpha > 0.5 \end{cases}
\]
Approximate solution of the JS model (up to 2nd-order)

Production term of the solvent kinetic energy

\[ \tau_y S_{ij} \approx -\nu (1-\beta) 2 S_{ij} - 4 \lambda \nu (1-\beta) (1-2\alpha) S_{ik} S_{kj} S_{ij} \]

Effective shear viscosity

Shear thinning

\[ \alpha = 0.0 \]

\[ \tau_y S_{ij} \approx -\nu (1-\beta) 2 S_{ij} - 4 \lambda \nu (1-\beta) S_{ik} S_{kj} S_{ij} \]

Effective shear viscosity

\[ \alpha = 1.0 \]

\[ \tau_y S_{ij} \approx -\nu (1-\beta) 2 S_{ij} + 4 \lambda \nu (1-\beta) S_{ik} S_{kj} S_{ij} \]

Comparison of energy production terms (Full JS model)

Solvent kinetic energy

\[ P_s = \tau_{ij} S_{ji} < 0 \]

Elastic energy

\[ P_e = -(1-2\alpha) \tau_{ik} S_{ik} = 0 \]

\[ P_e = -(1-2\alpha) \tau_{ik} S_{ik} < 0 \]

\[ P_e = -(1-2\alpha) \tau_{ik} S_{ik} > 0 \]

Limitations of 2nd-order approximate JS model

2nd-order steady solution of the JS model

\[ \tau_y \approx 2 \lambda \nu (1-\beta) \left( S_{ij} - 2 S_{ik} S_{kj} \right) \]

3rd-order steady solution (Bird et al. 1987)

\[ \tau_y \approx 2 \lambda \nu (1-\beta) \left( S_{ij} - 2 S_{ik} S_{kj} \right) \]

\[ -2 \lambda \nu \left( S_{ik} + S_{kj} \Omega_{ij} + S_{ji} \Omega_{kj} \right) \]

\[ -2 \lambda \nu \left( 2\omega_i - (1-2\alpha) \right) \left( S_{ik} + S_{kj} \Omega_{ij} + S_{ji} \Omega_{kj} \right) \]

Elastic energy production terms (3rd-order)

\[ P_e = \frac{1-\beta}{\text{Re}_b} \tau_{ij} S_{ji} = \frac{1-\beta}{\text{Re}_b} \left( S_{ij} - 2 S_{ik} S_{kj} \right) \]

\[ \approx + \left(1-2\alpha\right) \left( S_{ik} + S_{kj} \Omega_{ij} + S_{ji} \Omega_{kj} \right) \]

\[ + \left(1-2\alpha\right) \left( S_{ik} + S_{kj} \Omega_{ij} + S_{ji} \Omega_{kj} \right) \]

\[ 3\text{-order}: \quad P_e = 0 \text{ for } 0 < \alpha < 0.5, \quad P_e = 0 \text{ for } 0.5 < \alpha < 1 \]
### Comparison of energy production terms (2nd-order model)

<table>
<thead>
<tr>
<th></th>
<th>$\alpha=0$ (Oldroyd-B)</th>
<th>$\alpha=0.5$</th>
<th>$\alpha=1$ (Oldroyd-A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solvent kinetic energy</td>
<td>$P_s = \tau_{ik}S_{ik} &gt; 0$</td>
<td>$P_s = \tau_{ik}S_{ik} = 0$</td>
<td>$P_s = \tau_{ik}S_{ik} &lt; 0$</td>
</tr>
<tr>
<td>Elastic energy</td>
<td>$P_e = -\tau_{ii}/2$</td>
<td>$P_e = -(1 - 2\alpha)\tau_{ik}S_{ik} = 0$</td>
<td>$P_e = \tau_{ik}S_{ik} &lt; 0$</td>
</tr>
</tbody>
</table>

Enhancement of turbulence    Close to Newtonian    Reduction of turbulence

### Summary

- A stretched spiral vortex is identified using DNS data for homogeneous isotropic turbulence. Its genesis, growth and annihilation are elucidated.
- Existence of two symmetric modes and a third asymmetric of configurations is extracted. They are achieved through the interaction of several sheets.
- Mechanism of mode transition and persistence of each mode is shown.
- By tightening of the spiral turns, spiral sheets are stretched to extreme lengths. Intense energy cascade and dissipation occurs along the spiral sheets.
- Effect of viscoelasticity on the formation of spiral vortex is studied using the constitutive equation for the polymer stress. It is shown that viscoelasticity works to resist extensional motions of the turbulent flow.

### Interaction of multiple tubular vortical structures

(Transverse: Holm & Kerr 2002; Anti parallel: Goto 2008)

Reconnection of two orthogonally offset cylindrical vortices

Initial condition: Boratav, Pelz and Zabusky (1991) $Re_\Gamma = \Gamma/\nu = 1392.0$, Equal circulation.

$t=0$ $t=0.5$
Transition of topology during the reconnection process

Time evolution of helicity density and $P_n$ term

<table>
<thead>
<tr>
<th>$t$</th>
<th>Helicity density</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td></td>
</tr>
</tbody>
</table>

Helicity density

- Yellow: Positive
- Blue: Negative

Intense dissipation event via an interaction and reconnection of the two vortices

Intense dissipation is generated along the stretched sheets in the vicinity of the reconnection point.

Green isosurface: Dissipation rate > 0.5

Candidate for non-affine polymers

1. DNA: Exhibits marked drag reduction
2. Surfactant (with high concentration)
Conclusion

- A stretched spiral vortex is identified using DNS data for homogeneous isotropic turbulence. Its genesis, growth and annihilation are elucidated.
- Aside from the two symmetric modes of configurations studied in previous works, a third asymmetric mode is extracted, which is achieved through the interaction of several sheets.
- By tightening of the spiral turns, spiral sheets are stretched to extreme lengths. Intense dissipation occurs along the spiral sheets. The local dissipation rate exhibits a strong intermittency.
- At a higher Reynolds number, the hierarchical cluster of spiral vortices is formed due to the instability cascade induced by the stretching of vortex sheets.
- Similarity in the fractal properties of the vortex sheet region and the dissipative region is shown.

Classification of structures in turbulent flows

- Tube-like structure similar to Burgers' vortex tube
- Sheet-like structure similar to Burgers' vortex layer

Comparison of energy production terms (2nd-order model)

<table>
<thead>
<tr>
<th></th>
<th>$\alpha=0$ (Oldroyd-B)</th>
<th>$\alpha=0.5$</th>
<th>$\alpha=1$ (Oldroyd-A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solvent kinetic energy $\frac{u^2}{2}$</td>
<td>$P = \tau_{ik} S_{ik} S_{ij} S_{jk} &gt; 0$</td>
<td>$P = \tau_{ik} S_{ik} = 0$</td>
<td>$P = \tau_{ik} S_{ik} = 4(1-\beta)\frac{u^2}{2} S_{ij} S_{jk} &lt; 0$</td>
</tr>
<tr>
<td>Elastic energy $-\frac{\tau_{ij}}{2}$</td>
<td>$P = \tau_{ik} S_{ik} S_{ij} S_{jk} &gt; 0$</td>
<td>$P = (1-2\alpha) \tau_{ik} S_{ik} = 0$</td>
<td>$P = \tau_{ik} S_{ik} = 4(1-\beta)\frac{u^2}{2} S_{ij} S_{jk} &lt; 0$</td>
</tr>
<tr>
<td>Enhancement of turbulence</td>
<td>Close to Newtonian</td>
<td>Reduction of turbulence</td>
<td></td>
</tr>
</tbody>
</table>
Eigenvalues for $A_{ij}$

- Characteristic equation
  $$x^3 - \frac{1}{2}(A_{ij}A_{ji})x - \frac{1}{3}(A_{ij}A_{jk}A_{ki}) = 0$$
  $$\text{tr}[A_{ij}] = 0$$

  where
  $$A_{ij}A_{ji} = -6S_i\Omega_i\Omega_jS_j\Omega_i + S_{ij}\Omega_j\Omega_{ij}$$
  $$= \frac{\omega^2}{2}(\sigma_+ - \sigma_-)^2 + \frac{\omega^2}{2}(\sigma_+ - \sigma_-)^2 + \frac{\omega^2}{2}(\sigma_+ - \sigma_-)^2,$$
  $$A_{ij}A_{ji} = \frac{3}{4}(\sigma_+ - \sigma_-)(\sigma_+ - \sigma_-)(\sigma_+ - \sigma_-)\omega_+\omega_-\omega_-.$$

- DNS data shows that $A_{ij}A_{ji} \gg A_{ij}A_{jk}A_{ki}$, thus
  $$[A_{ij}]_k \approx \pm \sqrt{A_{ji}A_{ji}/2}, \quad [A_{ij}]_z \approx 0$$

Invariants of fourth-order moments of velocity gradients

- $I_1 = (S_{ik}S_{ki})(S_{jl}S_{lj})$
- $I_2 = -2S_{ik}S_{kj}\Omega_{ij}\Omega_{lk}$
- $I_3 = 4S_{ik}\Omega_{kj}\Omega_{ji}S_{lj}^{-2}S_{ik}\Omega_{kj}\Omega_{lj}$
- $I_4 = 8\Omega_{ik}\Omega_{kj}\Omega_{lj}\Omega_{ji}$ (Sigia, 1981)

All fourth-order moments are linear combination of $I_i$ ($i=1,2,3,4$).

- $A_{ij}A_{ji} = I_2 - \frac{3}{2}I_3$

Fractal properties of the vortex sheet and dissipation region (1)

Box counting for individual dissipative structures

$N_C$: Number of boxes containing some point of large dissipative structures

$N_C(L) \sim L^{-d}$

$d$: Fractal dimension

A set of adjacent points satisfying the thresholding criterion

Moisy and Jimenez (2004)

Fitting in the range, $6/\bar{\eta} < L < L_{\text{max}}$

Fractal properties of the vortex sheet and dissipation region (2)

Mean value of $d$: averaged over structures as a function of threshold and fractal dimension for $[A_{ij}]$

Correlation between the vortex sheet and dissipation rate

C.C~ 0.83
Statistical property of the educed region: Fractality

Statistical property of the educed region: Fractality of $[A_{ij}]_+$

Fractal dimension of $[A_{ij}]_+ \sim 1.7$, close to that of strain rate. $[A_{ij}]_+$ educes the region in which intense dissipation takes place.

$S(512^3, \tau=4.2 \, \text{2nd}, D=2.01)$

$W(512^3, \tau=3.0 \, \text{2nd}, D=1.96) \ W(512^3, \tau=8.0 \, \text{1nd}, D=1.55)$
マルチフラクタル解析

- Subbox average
  \[ p_i = \left( \frac{\varepsilon^{(1)}(r)}{r} \right)^d \quad (d = 3) \]

- 一般化次元
  \[ D_q = \lim_{r/L \to 0} \frac{\ln \sum p_i^q}{(q-1) \ln(r/L)} \]

\[ D_q \] は \((\sum p_i^q)^{1/(q-1)}\) と \(r/L\) のスケーリング関係より求まる。

マルチフラクタル特性

- \(q\)-Dq曲線
  - 散逸構造と渦層構造は相似な分布

- 等\(\alpha\)集合のフラクタル次元 \(f(\alpha(q))\)
  \[ p_i \sim \left( \frac{r}{L} \right)^{\alpha - 1} \]
  \[ \alpha(q) = \frac{d}{dq} \left( (q-1)(D_q - d + 1) \right), \]
  \[ f(\alpha(q)) = q\alpha(q) - (q-1)D_q + q(d-1). \]

散逸構造と渦層構造は相似なマルチフラクタル特性を有している。
ひずみ速度と渦度のジョイント・マルチフラクタル

・ 結合一般化次元 $D(q, p)$

\[
\left( \frac{r}{L} \right)^{d(q,p)} \sum_i \left( \frac{s_i^{(q)}}{s_i^{(p)}} \right)^{D(q,p)} = \left( \frac{r}{L} \right)^{-(q-1)(p-1)D(q,p)}, \quad \frac{r}{L} \to 0
\]

Subbox 内の平均ひずみ速度、平均渦度：$s_i^{(q)}$, $w_i^{(p)}$

・ 等 $\alpha, \beta$ 集合のフラクタル次元 $f(\alpha, \beta)$

\[
\tau(q, p) = -(q-1)(p-1)D(q, p)
\]

\[
\alpha(q, p) = \frac{\partial}{\partial q} \tau(q, p) + 1 - d, \quad \beta(q, p) = \frac{\partial}{\partial p} \tau(q, p) + 1 - d, \quad f(\alpha, \beta) = -\tau(q, p) + (\alpha - 1 + d)q + (\beta - 1 + d)p.
\]

DNS の結果

Menevau et al. (1990)の境界層実験の結果