How Universal can Turbulence be?

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London 2010
content

• comments on universality
• scaling properties
• characterization without scaling assumption
• how universal is turbulence?
How Universal can Turbulence be?

- concept of universality
  - only one model is required

Unsolved problems in physics

Is it possible to make a theoretical model to describe the behavior of a turbulent flow — in particular, its internal structures?
concept of universality

- **Kolmogorov 41**
  - inertial range
  - \( \eta \ll r \ll L \)

- **Kolmogorov Obukhov 62**
  - self-similar energy statistics
  - scaling law
    - \( \langle (\ln \epsilon_r)^2 \rangle = \Lambda_0^2 - \mu \ln(r) \)
    - \( \langle u_r^n \rangle \propto \langle \epsilon_r^{n/3} \rangle \propto r^{n/3} \propto r^{\xi_n} \)
• Kolmogorov 41
  inertial range

\[ \eta \ll r \ll L \]

• Kolmogorov Obukhov 62
  self-similar energy statistics

\[ \langle (\ln \epsilon_r)^2 \rangle = \Lambda_0^2 - \mu \ln(r) \]

scaling law ??

\[ \langle u_r^n \rangle \propto \langle \epsilon_r^{n/3} \rangle \propto r^{n/3} \]

\[ \propto r^{\xi_n} \]
content

• comments on
• universality is based on what?
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scaling properties

hints for scaling behaviour

• 4/5th law

\[ < u_r^3 > = -\frac{4}{5} < \epsilon_r > r + 6\nu \frac{\partial}{\partial r} < u_r^2 > \]
scaling properties

hints for scaling behaviour

• 4/5th law
• structure functions

\[ \langle u_r^n \rangle \propto r^{\xi_n} \]
scaling properties

hints for scaling behaviour

- 4/5th law
- structure functions
- ESS

\[ < u_r^n > \propto < u_r^3 >^{\xi_n} \]

ESS S6 vs. S3 von AuspS09

Steigung = 1.7313 (±1.695e-3) im Bereich von 15 bis 400 incr.


(Benzi et al. (1993))
scaling properties

hints for scaling behaviour

- 4/5th law
- structure functions
- ESS
- long / transversal

\[ < u_r^n > \propto r^{\xi_n} \]

Benzi et al. JFM (2010)
are there hints against scaling behaviour?

• Castaing

\[ \langle (\ln \epsilon_r)^2 \rangle \propto r^\beta \]

- Kolmogorov Obukhov 62

self-similar energy statistics

\[ \langle (\ln \epsilon_r)^2 \rangle = \Lambda_0^2 - \mu \ln(r) \]
are there hints against scaling behaviour?

- Castaing
- Re dependencies
- ...
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idea of a turbulent cascade:

large vortices are generating small ones

$\Rightarrow$ stochastic cascade process evolving in $r$

$\partial_r u_r$

$\partial_r p_r(u_r)$

$\Rightarrow$ stochastic cascade process evolving in $r$
The reconstruction of stochastic equation goes back to Kolmogorov work from 1931.

\[ D^{(n)}(u_r, r) = \lim_{\Delta \to 0} \frac{1}{n! \cdot \Delta} \int (\tilde{u}_r - u_r)^n p(\tilde{u}_r, r + \Delta|u_r, r) d\tilde{u}_r \]

\[ -\frac{\partial}{\partial r} p(u_r, r|u_0r_0) = \left[ -\frac{\partial}{\partial u_r} D^{(1)}(u_r, r) + \frac{\partial^2}{\partial u_r^2} D^{(2)}(u_r, r) \right] p(u_r, r|u_0, r_0) \]
measured Fokker-Planck equation further results

\[- \frac{\partial}{\partial r} p(u_r, r | u_0 r_0) = \left[ - \frac{\partial}{\partial u_r} D^{(1)}(u_r, r) + \frac{\partial^2}{\partial u_r^2} D^{(2)}(u_r, r) \right] p(u_r, r | u_0, r_0)\]

differential equations for the structure function

\[- \frac{\partial}{\partial r} \langle u_r^n \rangle = n \cdot \langle u_r^{n-1} D^{(1)}(u_r, r) \rangle + n \cdot (n - 1) \langle u_r^{n-2} D^{(2)}(u_r, r) \rangle\]
measured Fokker-Planck equation further results

\[-\frac{\partial}{\partial r} p(u_r, r|u_0 r_0) = \left[ -\frac{\partial}{\partial u_r} D^{(1)}(u_r, r) + \frac{\partial^2}{\partial u_r^2} D^{(2)}(u_r, r) \right] p(u_r, r|u_0, r_0)\]

differential equations for the structure function

\[-\frac{\partial}{\partial r} \langle u_r^n \rangle = n \cdot \langle u_r^{n-1} D^{(1)}(u_r, r) \rangle + n \cdot (n - 1) \langle u_r^{n-2} D^{(2)}(u_r, r) \rangle\]

- closed equation for structure functions and scaling behaviour if

\[D^{(1)}(u_r, r) = d_1^u(r) u_r\]
\[D^{(2)}(u_r, r) = d_2(r) + d_2^u(r) u_r + d_2^{uu}(r) u_r^2\]

- and

\[d_1^u(r) \propto 1/r\]
\[d_2^{uu}(r) \propto 1/r\]

other terms zero => K62
multi-scale statistics -4-

differential equations for the structure function

\[ -\frac{\partial}{\partial r} \langle u_r^n \rangle = n \cdot \langle u_r^{n-1} D^{(1)}(u_r, r) \rangle + n \cdot (n - 1) \langle u_r^{n-2} D^{(2)}(u_r, r) \rangle \]

- violation of proper scaling

\[
D^{(1)}_1(u_r, r) = d^{u}_1(r) u_r \\
D^{(2)}(u_r, r) = d^{u}_2(r) + d^{uu}_2(r) u_r + d^{uu}_2(r) u_r^2
\]

\[
d^{u}_1(r) \propto 1/r \\
d^{uu}_2(r) \propto 1/r
\]
additive term in the diffusion term: \( \rightarrow \) additive noise

\[
\begin{align*}
    D^{(1)}_1(u_r, r) &= d_1^u(r) u_r \\
    D^{(2)}(u_r, r) &= d_2(r) + d_2^u(r) u_r + d_2^{uu}(r) u_r^2
\end{align*}
\]
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  • energy cascade
energy cascade $e_r$

2-dim Fokker-Planck analysis

$q(r) = \begin{pmatrix} u(r) \\ x(r) \end{pmatrix}$, $x(r) = \ln (\epsilon_r/\bar{\epsilon})$.

result in form of a Langevin equation for the cascade

\[ -\frac{\partial}{\partial r} u(r) = -\frac{1}{r} \gamma(r) u(r) + m \exp\left( \frac{\alpha_1}{2} x(r) \right) \Gamma_u(r), \]
\[ -\frac{\partial}{\partial r} x(r) = \frac{1}{r} G(r) x(r) + \frac{1}{r} F(r) + \sqrt{\frac{1}{r} D^{(2)}_{xx}(u, x, r)} \Gamma_x(r). \]

$\sigma = -\sigma_\infty$ (circles), $u = 0$ (squares) and $u = +\sigma_\infty$ (diamonds).

no multiplicative noise anymore!!
No interittency!!
result in form of a Langevin equation for the cascade

\[-\frac{\partial}{\partial r} u(r) = -\frac{1}{r} \gamma(r) u(r) + m \exp \left( \frac{a_1}{2} x(r) \right) \Gamma_u(r),\]
\[-\frac{\partial}{\partial r} x(r) = +\frac{1}{r} G(r) x(r) + \frac{1}{r} F(r) + \sqrt{\frac{1}{r} D^{(2)}_{uu}(u, x, r) \Gamma_x(r)}.\]
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  • longitudinal-transversal structure functions
turbulence: long/transversal

spatial correlation in different directions

Quantities

- **longitudinal** increment

\[ u_r(x) = \left[ \hat{u}(\vec{x} + \vec{r}) - \bar{u}(\vec{x}) \right] \cdot \hat{r} \]

- **transversal** increment

\[ v_r(x) = \left[ \hat{u}(\vec{x} + \vec{r}) - \bar{u}(\vec{x}) \right] \times \hat{r} \]
extended selfsimilaritiy ESS

supposed scaling laws

$$\langle |u_r|^n \rangle \propto \langle |u_r|^3 \rangle^{\xi^l_n}$$
$$\langle |v_r|^n \rangle \propto \langle |u_r|^3 \rangle^{\xi^t_n}$$
extended selfsimilarty ESS

supposed scaling laws

\[ \langle |u_r|^n \rangle \propto \langle |u_r|^3 \rangle ^{\xi_l} \]

\[ \langle |v_r|^n \rangle \propto \langle |u_r|^3 \rangle ^{\xi_t} \]

open problem: (Antonia 97, Benzi 97, van der Water 99, Grossman et.al. 97....)

\[ \xi_t > \xi_l \]

are transversal structures more intermittent?
turbulence: long/transversal -3-

\[-r \frac{\partial}{\partial r} p(u, r | u_0, r_0) = \]

\[\left( - \sum_{i=1}^{n} \frac{\partial}{\partial u_i} D^{(1)}_i + \sum_{i,j=1}^{n} \frac{\partial^2}{\partial u_i \partial u_j} D^{(2)}_{ij} \right) p(u, r | u_0, r_0)\]
turbulence: long/transversal -3-

\[-r \frac{\partial}{\partial r} p(u, r|u_0, r_0) = \]

\[
\left( - \sum_{i=1}^{n} \frac{\partial}{\partial u_i} D_{i}^{(1)} + \sum_{i,j=1}^{n} \frac{\partial^2}{\partial u_i \partial u_j} D_{ij}^{(2)} \right) p(u, r|u_0, r_0)
\]

\[-\frac{\partial}{\partial r} \langle u^m v^n \rangle = \]

\[
+ m \langle u^{m-1} v^n D_1 \rangle + n \langle u^m v^{n-1} D_2 \rangle \\
+ \frac{m(m-1)}{2} \langle u^{m-2} v^n D_{11} \rangle + \frac{n(n-1)}{2} \langle u^m v^{n-2} D_{22} \rangle \\
+ mn \langle u^{m-1} v^{n-1} D_{12} \rangle.
\]
turbulence: long/transversal -3-

\[ -r \frac{\partial}{\partial r} p(u, r|u_0, r_0) = \]

\[ \left( - \sum_{i=1}^{n} \frac{\partial}{\partial u_i} D^{(1)}_i + \sum_{i,j=1}^{n} \frac{\partial^2}{\partial u_i \partial u_j} D^{(2)}_{ij} \right) p(u, r|u_0, r_0) \]

\[ \frac{\partial \langle u^2 \rangle}{\partial r} = (2d_1^{u} + d_{11}^{uu}) \langle u^2 \rangle + d_{11} + d_{11}^{vv} \langle v^2 \rangle \]

\[ \frac{\partial \langle u^3 \rangle}{\partial r} = (3d_1^{u} + 3d_{11}^{uu}) \langle u^3 \rangle + 3d_{11}^{u} \langle u^2 \rangle + 3d_{11}^{vv} \langle uv^2 \rangle \]

Compare with the Karman/Horwarth equations:

\[ \frac{\partial \langle u^2 \rangle}{\partial r} = -2 \frac{\langle u^2 \rangle}{r} + 2 \frac{\langle v^2 \rangle}{r} \quad \text{First Kármán} \]

\[ \frac{\partial \langle u^3 \rangle}{\partial r} = \frac{\langle u^3 \rangle}{r} - 6 \frac{\langle uv^2 \rangle}{r} \quad \text{Second Kármán} \]
turbulence: long/transversal -4- rescaling symmetry: $r \Rightarrow \frac{3r}{2}$
rescaling symmetry: \( r \rightarrow 3r/2 \)

\[
\langle |v(r)|^n \rangle \propto \langle |u(r)|^3 \rangle > \xi_n
\]

new ESST:

\[
\langle |v(r)|^n \rangle \propto \langle |u(3r/2)|^3 \rangle > \xi_n
\]
Assume • pure scaling: \( \langle u^n(r) \rangle = c_l^n r^{\xi_l^n} \) and \( \langle v^n(r) \rangle = c_t^n r^{\xi_t^n} \)

• and same intermittency \( d_{11}^{uu}(r) \equiv d_{22}^{vv}(r) \)

\[
\Rightarrow \langle v^n(r) \rangle = \langle u^n(\frac{3}{2}r) \rangle = c_t^n r^{\xi_t^n} = c_l^n (\frac{3}{2}r)^{\xi_l^n} \quad (\Rightarrow \xi_l^n = \xi_t^n)
\]

\[n = 2 \quad c_t^2 / c_l^2 \approx 1.33 \quad 2\% \text{ deviation from literature}
\]

\[n = 4 \quad c_t^4 / c_l^4 \approx 1.72 \quad 3\% \text{ deviation from literature}
\]

turbulence: long/transversal -4-

rescaling symmetry: \( r \Rightarrow 3r/2 \)

\[
< |v(r)|^n > \propto < |u(r)|^3 > \xi_n
\]

\[
< |v(r)|^n > \propto < |u(3r/2)|^3 > \xi_n
\]

consistent with Karman equation:

\[
-r \frac{\partial}{\partial r} \left< u_r^2 \right> = 2 \left< u_r^2 \right> - 2 \left< v_r^2 \right>
\]

or

\[
\left< v_r^2 \right> = \left< u_r^2 \right> + \frac{r}{2} \frac{\partial}{\partial r} \left< u_r^2 \right>
\]

taken as Taylor series

\[
\left< v_r^2 \right> \approx \left< u_{3/2r}^2 \right>
\]

rescaling symmetry: \( r = 3r/2 \)

\[
< |v(r)|^n > \propto < |u(r)|^3 > \xi_t^n
\]

\[
< |v(r)|^n > \propto < |u(3r/2)|^3 > \xi_n
\]

striking result

\[
< u_{3r/2}^n > \propto (3r/2)^\xi_n \propto r^{\xi_n}
\]

- this is only possible if the scaling laws does not hold
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• universal turbulence?
  • Reynolds dependencies
universality of turbulence: Re dependence?

\[ D^{(1)}(u_r, r) = d_1^u(r)u_r \]
\[ D^{(2)}(u_r, r) = d_2(r) + d_2^u(r)u_r + d_2^{uu}(r)u_r^2 \]


experimental data sets
Universality of turbulence: Re dependence?

\[ D^{(1)}(u_r, r) = d_1^u(r) u_r \]
\[ D^{(2)}(u_r, r) = d_2(r) + d_2^u(r) u_r + d_2^{uu}(r) u_r^2 \]

\[ d_1^u(r) = -\frac{1}{r} \gamma(r) \]
\[ \gamma(r) = \frac{2}{3} + c \sqrt{\frac{r}{\lambda}} \]

\[ D^{(1)} \text{ universal} \]
universality of turbulence: Re dependence?

\[ D^{(1)}(u_r, r) = d_1^u(r)u_r \]
\[ D^{(2)}(u_r, r) = d_2(r) + d_2^u(r)u_r + d_2^{uu}(r)u_r^2 \]

\[ d_2(r) \propto Re^{-3/8}/r \]

\[ d_2^{uu}(r) = f(Re) \]
universality of turbulence: Re dependence?

\[ D^{(1)}(u_r, r) = d_1^u(r)u_r \]
\[ D^{(2)}(u_r, r, Re) = d_2(r, Re) + d_2^u(r, Re)u_r + d_2^{uu}(r, Re)u_r^2 \]

\[ d_2(r) \propto Re^{-3/8}/r \]

\[ d_2^{uu}(r) = f(Re) \]

\[ \beta(r) \]
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• universal turbulence?
• Reynolds dependencies
• flow dependencies
different type of flows

wake behind cylinder and free jet

**universality of turbulence -2-**

- **different type of flows**

wake behind cylinder and free jet

\[-\frac{\partial}{\partial r} p(u_r, r | u_0 r_0) = \left[ -\frac{\partial}{\partial u_r} D^{(1)}(u_r, r) + \frac{\partial^2}{\partial u_r^2} D^{(2)}(u_r, r) \right] p(u_r, r | u_0, r_0)\]

\[
D^{(1)}(u_r, r) = d_1^u(r) u_r
\]

\[
D^{(2)}(u_r, r) = d_2(r) + d_2^u(r) u_r + d_2^{uu}(r) u_r^2
\]

Laupichler thesis. OI (2007)
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fractal grid turbulence - Christos Vassilicos
fractal grid turbulence - Christos Vassilicos

- properties do not change with Re

at L and L/2:
- conditional pdf
- Drift \( D^{(1)} \) and diffusion \( D^{(2)} \)
do not depend on Re

\[ R_{\lambda} = 150...750 \]

fractal grid turbulence - Christos Vassilicos

Re dependency

\[ D^{(1)}(u_r, r) = d_1^u(r)u_r \]
\[ D^{(2)}(u_r, r) = d_2(r) + d_2^u(r)u_r + d_2^{uu}(r)u_r^2 \]

=> Exp: cascade process depends on Re

summary

• universality is based on what?
• scaling properties one scale property
• characterization with multi-scale statistics
  • additive noise
  • energy conservation
  • longitudinal-transversal structure functions
• universal turbulence?
  • Reynolds dependencies
  • flow dependencies
  • new class of fractal generated turbulence
n-scale; n-point statistics
=> deeper insight into the nature of turbulence

are there different classes of turbulence?