Application of wall forcing methods in a turbulent channel flow using Incompact3d

S. Khosh Aghdam

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Application of wall forcing methods in a turbulent channel flow using
Most of large scale engineering flows are turbulent

- Atmosphere
- Transportation (automobile, airplanes, ships,···)
- Blood flow in heart

Aim of flow control → modify the characteristics of a flow field favourably

- Suppression or enhancement of turbulence
- Dissipation of kinetic energy by turbulent flow around objects
- Increase of resistance to their motion → Drag
- Component of the force experienced by a body, parallel to the direction of motion
Enhancement of turbulence

- Mixture in combustion: quality of the fuel-air mixture determines power generation efficiency
- Process industry: quality of mixtures affects chemical reaction rates and purity of final products

Reduction of turbulence

- Drag reduction techniques $\Rightarrow$ energy consumption issues
- Half of the total drag experienced by an aircraft accounts for skin-friction
- Aircraft industry demonstration test: Coverage of fuselage surface with riblet films $\Rightarrow$ resistance by 2%
- Fuel cost savings (Airbus A320) $\Rightarrow$ $5 \times 10^4$ L/year $\iff$ saving 200 million $$/year
Difficulty in control design ⇒ turbulence → multiscale phenomenon ⇔ coupling of system macroscopic size ($L$) & Kolmogorov scale ($\eta$) by the chaotic process of vortex stretching

- Two main groups ⇒ active and passive
- Categorisation relying on energy expenditure
  - Passive ⇒ no energy added in the flow → longitudinal grooves or riblets on a surface
  - Active ⇒ input of energy in the flow → blowing and suction jets in opposition control [1]
- Based on the control loops → active techniques categorisation:
  - open-loop (predetermined)
  - closed-loop (interactive)

Flow control

Drag reduction

Maths framework

Simulations

Summary

Control techniques

Passive

Active

Closed-loop

Feedback

Feedforward

Control theory

Optimal control

Open-loop
- Skin-friction coefficient

\[ C_f = \frac{2\tau_w^*}{\rho^* U_b^2} \] (1)

- Friction velocity

\[ u_\tau^* = \sqrt{\frac{\tau_w^*}{\rho^*}} = \sqrt{\nu^* \frac{\partial u^*}{\partial y^*}} \bigg|_{\text{wall}} \] (2)

- Reduction of velocity gradient ⇒ reduction in drag
- Spanwise wall oscillations (active/open-loop)
- Steady rotating discs [2] (active/open-loop)
- Oscillating rotating discs (active/open-loop)
- Hydrophobic surfaces

Applying control to

Navier-Stokes - continuity equations ⇒ (PDEs)

- PDEs state-space ⇒ infinite-dimensional → \( u_x = 0 \) ⇒ any \( f(y) \) solution ⇒ infinite dimensional solutions space ≠ ODEs
  - state-space → \( dy/dt = 0 \) ⇒ solutions in \( \mathbb{R}^p \)

- Right framework to deal with infinite-dimensional state space solutions ⇒ Functional analysis

- **Functional analysis framework**: functions studied as part of normed and complete + inner product ⇒ Hilbert

- Why Functional Analysis? ⇒ Banach-\( L^p \) spaces too broad for analysing PDEs solution

- Regularity properties not always verified in \( L^p \) spaces

- Further assumptions ⇒ higher order derivatives to ensure regularity (and boundedness) of solutions

- "Higher-order" spaces ⇒ Sobolev → energy spaces
Motivation: design control laws to stabilise a specified equilibrium for the NSE

- NSE → nonlinear ⇒ nonlinear stability analysis
- Depart from a Lyapunov function → energy of the system
- Choose the right norm
- Example: function $f(t, x)$ (perturbed variable) with $x \in (0, 1)$, within $L^2(0, 1) →$ prove that:

$$\|f(t)\|_{L^2(0,1)} \leq C_1 e^{-C_2 t} \|f(0)\|_{L^2(0,1)}$$

$C_1 \geq 1$ overshoot coefficient - $C_2 > 0$ decay rate
- Find conditions for stability → not necessarily nonlinear
**Previous work:** Control law in 2D channel flow $\rightarrow$ based on wall-tangential actuation (Balogh et al. [3]):

$$u(x, y = \pm 1, t) = \mp k \frac{\partial u}{\partial y}(x, \pm 1, t)$$  \hspace{1cm} (4)

- Extension to 3D channel flow carried out
- Link the mathematical formulation with a physical problem $\Rightarrow$ hydrophobic surfaces $\Rightarrow$ modification of no-slip condition:

$$u = L_s \frac{\partial u}{\partial y}|_{\text{wall}}$$  \hspace{1cm} (5)

$\Rightarrow$ Mathematical parameter in [3] $\Leftrightarrow$ slip-length

- Relevant scales for MEMS $\rightarrow$ embedded sensors and actuators in the walls to measure local shear

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Objective ⇒ stabilize a parabolic profile

Boundary control laws ⇒ decaying kinetic energy w.r.t time

Lyapunov-based approach using Lyapunov function:

\[ E(w) = \|w\|^2_{L^2(\Omega)} = \int_{-1}^{1} \int_{0}^{L_x} \int_{0}^{L_z} (u^2 + v^2 + w^2) \, dx \, dy \, dz \]  

\( \bigotimes \) translates as \( \|w(t)\|_{L^2(\Omega)} \leq C_1 e^{-C_2(t-t_0)} \|w(t_0)\|_{L^2(\Omega)} \)

Procedure: (a) take time derivative of Eq.(6) - (b) apply control - (c) prove regularity of solutions (involving Sobolev spaces)
<table>
<thead>
<tr>
<th>$L_x$</th>
<th>$L_y$</th>
<th>$L_z$</th>
<th>$Re_p$</th>
<th>$\Delta t$</th>
<th>time scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4\pi$</td>
<td>2.0</td>
<td>$4\pi/3$</td>
<td>4200.0</td>
<td>$2.5 \times 10^{-3}$</td>
<td>AB2</td>
</tr>
</tbody>
</table>

\[
L^+ = L \times Re_\tau \quad U^+ = U \times \frac{Re_p}{Re_\tau} \quad T^+ = T \times \frac{Re^2_\tau}{Re_p}
\]

Parabolic profile, constant mass flow rate, stretched wall-normal
Database of [4] used for comparison at $Re_\tau = 180$

<table>
<thead>
<tr>
<th># processors</th>
<th>256</th>
<th>512</th>
<th>1024</th>
<th>2048</th>
</tr>
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<tr>
<td>runtime (s)</td>
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<td>7258</td>
<td>6561</td>
<td>6921</td>
</tr>
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</table>

\[ \left\langle \frac{\partial^2 u}{\partial y^2} \right\rangle_{x,z}(y = 0), \left\langle -\frac{\partial u}{\partial y} \right\rangle_{x,z}(y = 2) \]

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$^4$ R. Moser, J. Kim, N. Mansour, Direct Numerical Simulations of turbulent channel flow up to $Re_\tau = 590$, Phys. of Fluids, 1999
Benchmark

<table>
<thead>
<tr>
<th>( \langle \frac{\partial u}{\partial y} \rangle_{x,z} ) walls</th>
<th>Re( \tau )</th>
<th>( C_{f,0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.64</td>
<td>179.1</td>
<td>8.18 \times 10^{-3}</td>
</tr>
</tbody>
</table>

\[ \bar{u} \]

\[ u_{\text{rms}}, v_{\text{rms}}, w_{\text{rms}} \]
Application of wall forcing methods in a turbulent channel flow using

- Incompact3d
- kmm

- $p_{rms}$ vs $y^+$
- $\omega_x$, $\omega_y$, $\omega_z$ vs $y^+$
Application of wall forcing methods in a turbulent channel flow using

Profiles for $\overline{uv}$ and $\frac{\overline{uv}}{u_{rms}v_{rms}}$
DNS of channel flow with this forcing \[5\] \implies Drag reduction

- **Structure of forcing** → \[w = W_m \sin\left(\frac{2\pi}{T}\right)\]

- Dependent on magnitude and period of forcing

- Maximum DR of 40% for \[T_{\text{opt}}^+ = 100\]

- Experimentally \[6\] found DR \(\sim 35\%\)

\[5\] Jung *et. al*, Physics of Fluids, 4, pp 1605–1607 - 1992

\[6\] Laadhari *et. al*, Physics of Fluids, 6, pp 3218–3220 - 1994
Flow control  Drag reduction  Maths framework  Simulations

\[ \begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
L_x & L_y & L_z & n_x & n_y & n_z & Re_{\tau} & W_m^+ & T^+ \\
\hline
4\pi & 2.0 & 4\pi/3 & 256 & 129 & 128 & 200 & 27.0 & 125.0 \\
\hline
\end{array} \]

\[ DR_{[7]} = 44.5\% \text{ vs } DR_{\text{Incompact3d}} = 44.8\% \]

\[ \partial u / \partial y \text{ at } y = 0, \quad -\partial u / \partial y \text{ at } y = 2 \]

\[ t \]

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\[ ^7 \text{M. Quadrio, P. Ricco, J. Fluid Mech., 521, pp 251–271 - 2004} \]
Vorticity map at the wall

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Application of wall forcing methods in a turbulent channel flow using...
Active method for DR ⇒ injecting vorticity

Proposed as part of a patent by Keefe. Numerical study in [8]

Relevant parameters → \((D, W)\), diameter and maximum tip velocity of the disc

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Steady discs rotation: implementation

- **disk_phys.dat** (text file) from cpp program
  - → **scr.sh**
    - → **disk_phys_x.dat** (text file)
    - → **disk_phys_z.dat** (text file)
  - → **rfile.f90**
    - → **disk_phys_opacx.dat** (binary direct open-access)
    - → **disk_phys_opacz.dat** (binary direct open-access)

- **decomp2d_read_var** interface reads
  - **disk_phys_opacx.dat** and **disk_phys_opacz.dat**

- **voir_visu.f90** check process with ParaView
  - → **tampon_opac_x.vtr**
  - → **tampon_opac_z.vtr**
  - → **tampon_opac_sqr.vtr**

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Application of wall forcing methods in a turbulent channel flow using...
Steady discs rotation: simulations

\[ L_x \times L_y \times L_z = 6.79\pi \times 2.0 \times 2.26\pi - Re_p = 4200 - Nd_x \times Nd_z = 6 \times 2 - \Delta t = 0.0025 - D^+ = 640 - W^+ = 9 \]

\[ KMM \rightarrow C_f.10^3 = 8.18 \]

**BASE CASE** → \( nx \times ny \times nz = 384 \times 129 \times 256 \)

<table>
<thead>
<tr>
<th></th>
<th>( C_{f,0}.10^3 )</th>
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<tr>
<td>Ricco-Hahn</td>
<td><strong>Incompact3d</strong></td>
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<td><strong>Incompact3d</strong></td>
</tr>
<tr>
<td>8.25</td>
<td>8.15</td>
<td>6.64</td>
<td>6.62</td>
<td></td>
</tr>
</tbody>
</table>

**HIGH RESOLUTION IN** \( x \) → \( nx \times ny \times nz = 480 \times 129 \times 224 \)

<table>
<thead>
<tr>
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<td><strong>Incompact3d</strong></td>
</tr>
<tr>
<td>8.25</td>
<td>8.13</td>
<td>6.65</td>
<td>6.62</td>
<td></td>
</tr>
</tbody>
</table>

**HIGH RESOLUTION IN** \( z \) → \( nx \times ny \times nz = 384 \times 129 \times 320 \)

<table>
<thead>
<tr>
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<th>( C_{f,0}.10^3 )</th>
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<th>( C_{f}.10^3 )</th>
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<td><strong>Incompact3d</strong></td>
<td><strong>Incompact3d</strong></td>
</tr>
<tr>
<td>8.24</td>
<td>8.13</td>
<td>6.63</td>
<td>6.61</td>
<td></td>
</tr>
</tbody>
</table>

**HIGH RESOLUTION IN** \( x,y,z \) → \( nx \times ny \times nz = 512 \times 257 \times 320 \)

<table>
<thead>
<tr>
<th></th>
<th>( C_{f,0}.10^3 )</th>
<th>( C_{f,0}.10^3 )</th>
<th>( C_{f}.10^3 )</th>
<th>( C_{f}.10^3 )</th>
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<tr>
<td>Ricco-Hahn</td>
<td><strong>Incompact3d</strong></td>
<td><strong>Incompact3d</strong></td>
<td><strong>Incompact3d</strong></td>
<td><strong>Incompact3d</strong></td>
</tr>
<tr>
<td>N/A</td>
<td>8.13</td>
<td>N/A</td>
<td>6.63</td>
<td></td>
</tr>
</tbody>
</table>

S. Khosh Aghdam  Application of wall forcing methods in a turbulent channel flow using
Steady discs rotation: flow visualisations

- \( \mathbf{u} = \mathbf{u}_m + \mathbf{u}_d + \mathbf{u}_t \Rightarrow \text{Disc flow:} \ \mathbf{u}_d = (u_d, v_d, w_d) = \mathbf{u} - \mathbf{u}_m \)

- Mean flow: \( \mathbf{u}_m(y) = \langle \mathbf{u} \rangle \) with \( \bar{f} \triangleq \frac{1}{t_f-t_i} \int_{t_i}^{t_f} f \, dt \) and \( \langle f \rangle \triangleq \frac{1}{L_x L_z} \int_0^{L_x} \int_0^{L_z} f \, dx \, dz \)

- Compute 3D \( \sqrt{u_d^2 + w_d^2} \) in diagnostic tool + ParaView
Steady discs rotation: flow visualisations

Isosurface representation $\sqrt{\frac{u^2}{d} + \frac{w^2}{d}} = 0.09$
Time average of the streamwise wall friction \( \frac{\partial u}{\partial y} \bigg|_{y=0} \) in time window \([t_i h/U_P; t_f h/U_P] = [750; 2250] \)

- Large regions of negative wall-shear stress
Rotating discs subject to an oscillatory motion

Disc tip velocity \( W = W_m \cos \left( \frac{2\pi t}{T} \right) \)

Case giving optimal drag reduction

<table>
<thead>
<tr>
<th>xlx</th>
<th>yly</th>
<th>zlz</th>
<th>nx</th>
<th>ny</th>
<th>nz</th>
<th>( \Delta t )</th>
<th>( Re_p )</th>
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<tbody>
<tr>
<td>6.79(\pi)</td>
<td>2.0</td>
<td>3.39(\pi)</td>
<td>384</td>
<td>129</td>
<td>384</td>
<td>2.5 \times 10^{-3}</td>
<td>4200.0</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Nd_x</th>
<th>Nd_z</th>
<th>D^+</th>
<th>W^+</th>
<th>T^+</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
<td>960</td>
<td>12.0</td>
<td>10^3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ricco (Conf.)</th>
<th>Incompact3d</th>
<th>Ricco (Conf.)</th>
<th>Incompact3d</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{f,0} \times 10^3 )</td>
<td>( C_{f,0} \times 10^3 )</td>
<td>( C_f \times 10^3 )</td>
<td>( C_f \times 10^3 )</td>
</tr>
<tr>
<td>8.18</td>
<td>8.13</td>
<td>6.54</td>
<td>6.52</td>
</tr>
</tbody>
</table>
Oscillating discs

Isosurface representation $\sqrt{u_d^2 + w_d^2} = 0.09$
Studied by Min-Kim (2004) with BC forcing term: \( u = L_s \frac{\partial u}{\partial y} \)\text{wall} \]

- \( u, w \) and \((u, w)\) can be forced but \( u \) gives optimal DR

- \( u^{n+1} \)\text{wall} = \( u^n \)\text{wall} + \( L_s \frac{\partial u^n}{\partial y} \)\text{wall}

**Problem**: Enforce BC at each time step \( \Rightarrow \) Generation of a thin boundary layer \([10]\) \( \Rightarrow \) Numerical instability

**Solution in \([10]\) \( \rightarrow \)** (1) keep the same BC for several time steps - (2) continuous update

**Solution adopted:**

- compute \( \frac{\partial u}{\partial y} \) at 1\text{st} time step - pass it as BC \( \forall t \) (\( L_s = 10^{-3} \) (s), \( L_s = 2 \times 10^{-3} \) (s) and \( L_s = 10^{-2} \) (s))

- compute \( \frac{\partial u}{\partial y} \) at each time step - pass it as BC (\( L_s = 10^{-3} \) (s), \( L_s = 2 \times 10^{-3} \) (s) and \( L_s = 10^{-2} \) (s))

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Application of wall forcing methods in a turbulent channel flow using hydrophobic surface: preliminary results.

$\frac{\partial \bar{u}}{\partial y} (y = 0)$

$L_s = 0.001$ - update 1\textsuperscript{st} step
$L_s = 0.002$ - update 1\textsuperscript{st} step
$L_s = 0.01$ - update 1\textsuperscript{st} step
$L_s = 0.001$ - update for all t
$L_s = 0.002$ - update for all t
Hydrophobic surface: statistics

\[ u_{\text{rms}}, v_{\text{rms}}, w_{\text{rms}} \] vs. \( y^+ \) for different values of \( L_s \) (no forcing, \( L_s = 0.001 \), \( L_s = 0.002 \), \( L_s = 0.01 \)).

- \( u_{\text{rms}} \) for \( L_s = 0.001 \) is higher than for \( L_s = 0.002 \) and \( L_s = 0.01 \).
- \( v_{\text{rms}} \) for \( L_s = 0.002 \) is lower than for \( L_s = 0.001 \) and \( L_s = 0.01 \).
- \( w_{\text{rms}} \) for \( L_s = 0.001 \) is higher than for \( L_s = 0.002 \) and \( L_s = 0.01 \).

S. Khosh Aghdam

Application of wall forcing methods in a turbulent channel flow using...
### Hydrophobic surface: Summary

<table>
<thead>
<tr>
<th>$L_s$</th>
<th>test_1</th>
<th>test_2</th>
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</thead>
<tbody>
<tr>
<td>0.001</td>
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<td>constant</td>
</tr>
<tr>
<td>0.002</td>
<td>updated</td>
<td>constant</td>
</tr>
<tr>
<td>0.01</td>
<td>crashed</td>
<td>constant</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>$L_s$</th>
<th>$\frac{\partial u}{\partial y}$</th>
<th>DR</th>
<th>DR (Kim-Min 2004)</th>
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<tbody>
<tr>
<td>0.001</td>
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<td>2.1%</td>
<td>2%</td>
</tr>
<tr>
<td>0.001</td>
<td>constant</td>
<td>2.4%</td>
<td>2%</td>
</tr>
<tr>
<td>0.002</td>
<td>updated</td>
<td>4.9%</td>
<td>5%</td>
</tr>
<tr>
<td>0.002</td>
<td>constant</td>
<td>4.9%</td>
<td>5%</td>
</tr>
<tr>
<td>0.01</td>
<td>updated</td>
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</tr>
<tr>
<td>0.01</td>
<td>constant</td>
<td>17%</td>
<td>18%</td>
</tr>
</tbody>
</table>
## Summary

- Incompact3d efficiently dealing with various drag reduction methods
- High scalability allows for future control studies with larger Reynolds number