

High-Reynolds-number asymptotics of steady separated flows

There are problems that are easy to pose but difficult to solve. Among them is the problem of high-Reynolds-number asymptotics of steady incompressible flow past a bluff body. Steady Navier-Stokes equation involves only one parameter, Re , and there are only two asymptotic limits: $Re \rightarrow 0$ and $Re \rightarrow \infty$. The limit $Re \rightarrow 0$ was found by George Gabriel Stokes in 1851. For streamlined bodies (like aerofoils) the limit $Re \rightarrow \infty$ was found by Ludwig Prandtl in 1904. For bluff bodies (like a cylinder) the $Re \rightarrow \infty$ limit was found in 1988 by S.I.Chernyshenko.

$$\mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}$$

$$Re \rightarrow \infty, \quad \mathbf{u} \rightarrow ?$$

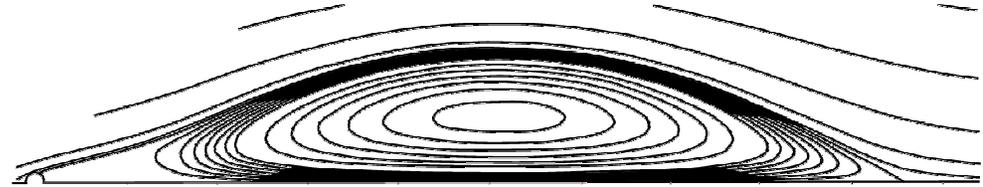
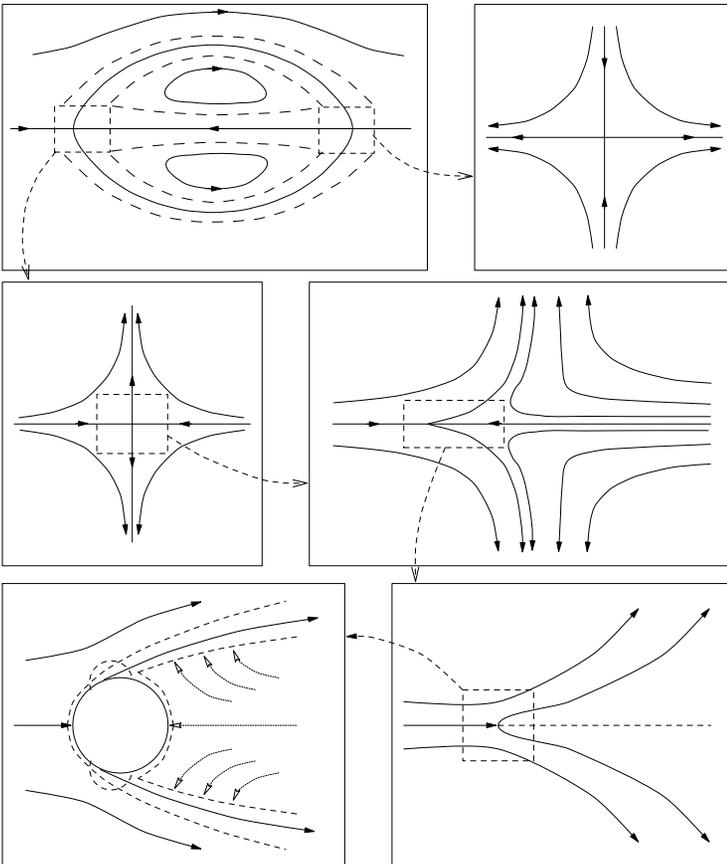


Fig.1 Steady flow past a circular cylinder at $Re_{radius}=300$ calculated numerically by B.Fornberg in 1985.

In contrast to the Stokes theory and the boundary layer theory, bluff body high- Re steady solutions never correspond to real flows quantitatively, since real separated flows at high Re are always turbulent. This is illustrated by the long and wide wake in Fig.1. In averaged turbulent flow the eddy is narrower and shorter. Nevertheless, the steady asymptotics of separated flows always arose great interest not only because of its beauty and difficulty but also because its solution throws light on general properties of the Navier-Stokes equations. Moreover, in spite of the differences, real and steady high- Re flows have enough in common for the steady high- Re asymptotics to be used (with caution) for qualitative analysis of real turbulent flows.

The theory proves that in steady separated flows the eddy length and width are both proportional to Re , while the drag coefficient tends to zero as $1/Re$. The asymptotic flow structure consists of many distinguished limits. Following the arrows in Fig.2 one can go step-by-step from the Sadovskiy flow on the $Re \times Re$ eddy scale to the Kirchhoff flow on the 1×1 body scale.

Fig.2. Asymptotic structure of steady separated flow

More information: S.I.Chernyshenko. Asymptotic theory of global separation, Applied Mechanics Reviews, 1998, Vol:51, pp. 523-536