Turbulent flow over anisotropic porous media

Alfredo Pinelli

School of Mathematics, Computer Science and Engineering

City, University of London U.K.

Work done in collaboration with:

M. Omidyeganeh, A. Monti @ City, University of London, UK

M. Rosti, L. Brandt @ KTH Mechanics, Sweden

Workshop on Turbulent Skin Friction Drag Reduction.

Imperial College: 4th-5th December 2017

Motivations and Summary

- Turbulent flows bounded by a porous medium can be strongly affected by the properties of the solid matrix.
- These interactions can lead to better mixing of scalars or momentum that usually is associated with an increase on skin friction drag.
- Porous media: non smooth wall covered by a material with high protrusion heights: e.g., classical packed beds with certain porosity and thickness, flow over canopies, flow over bundles, flow over riblets….

We aim at characterising the impact of the porous material properties on the outer turbulent flow.

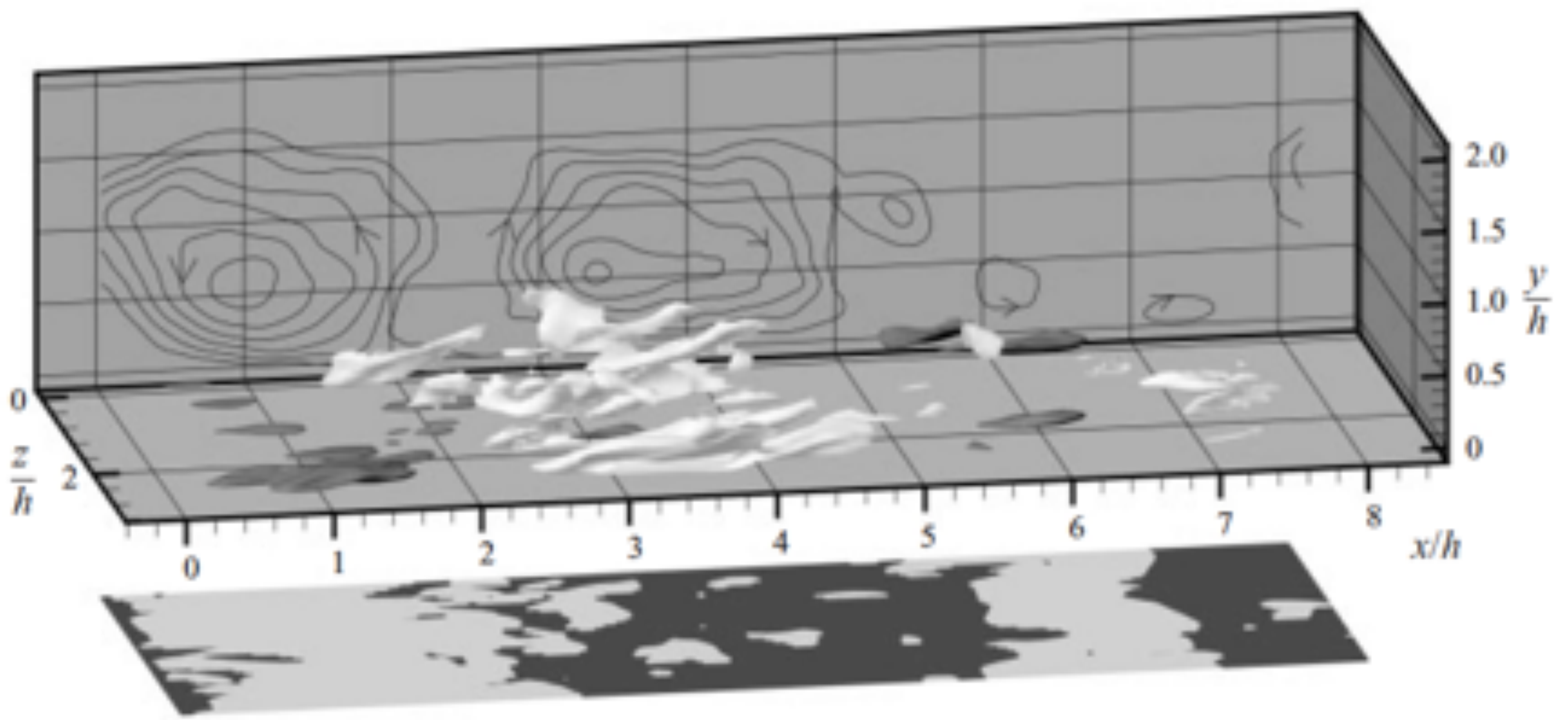
Summary

- Introductory examples showing the role of the porous substrate;
- Introduce an averaged formulation (i.e., VANS) that allows to model the flow within general porous media;
- Study the effect of permeability tensor on the outer flow.
- Conclusions and future work.

Example I: flow over an isotropic medium

Turbulent channel DNS (nominal $Re_\tau = 180$). One of the walls is permeable [J. Jiménez et al., Jou. Fluid Mech., 442 2001]. At the bottom wall:

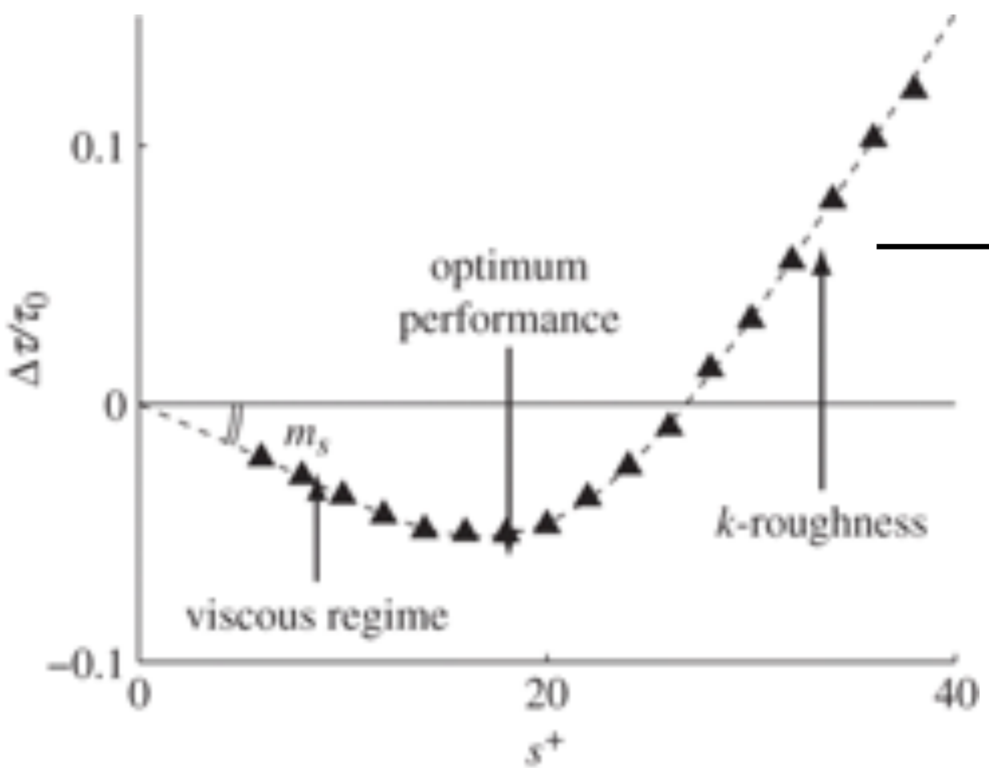
$$u(x,y,z,t) = w(x,y,z,t) = 0 \quad v(x,y,z,t) = \kappa \frac{\partial p}{\partial y}$$



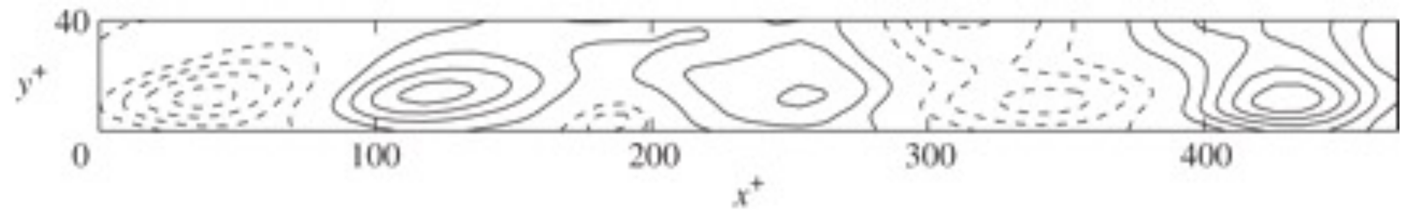
- Relaxing impermeability conditions leads to the appearance of an inflection point in the mean profile.
- Kelvin Helmholtz instability appears in the form of spanwise vorticity rollers.
- The rollers modulate the wall structures, increasing the Reynolds stresses and therefore drag [Breugem, W. P. et al. 2006 Jour. Fluid Mech. 566, Rosti, M.E: et al. 2015. Journal of Fluid Mechanics 784].

Example II: flow over an anisotropic medium

Flow over riblets in non linear regime [García-Mayoral R and and J. Jiménez J Phil. Trans. R. Soc. 369, 2011].

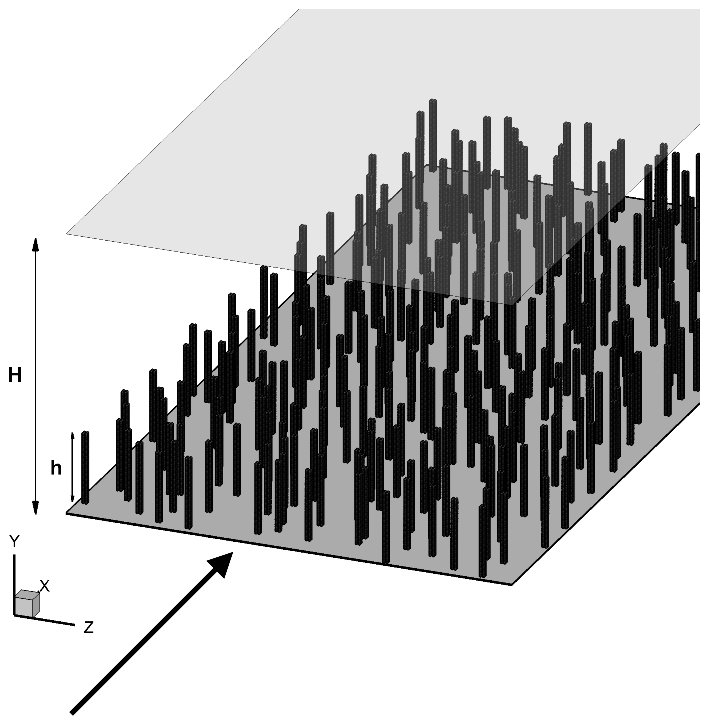


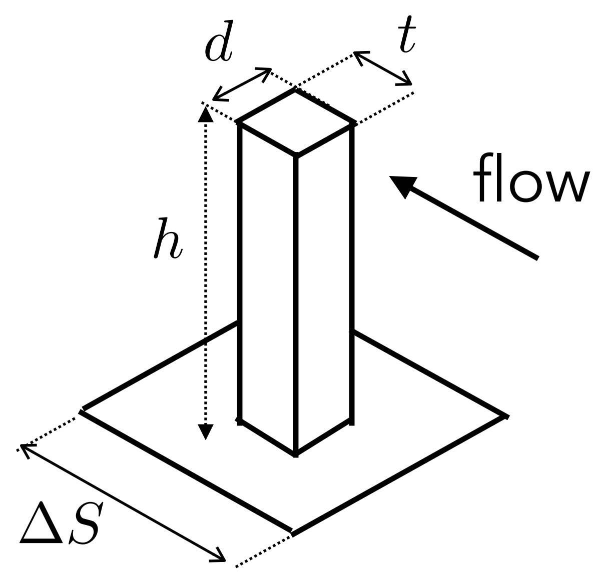
Definition of the drag-reduction regimes ($\Delta\tau/\tau_0$:)



- Riblets impose two different virtual origins for the wall: one seen from the stream-wise component of the velocity Δx and the other from the span-wise component Δz.
- The difference between the two $\Delta h = \Delta z - \Delta x$, termed as protrusion height is responsible for the attenuation of the quasi-stream-wise-vortices
- Debate still ongoing on the viscous break down and the drag increase in the non-linear regime.

Example III: flow over an anisotropic medium





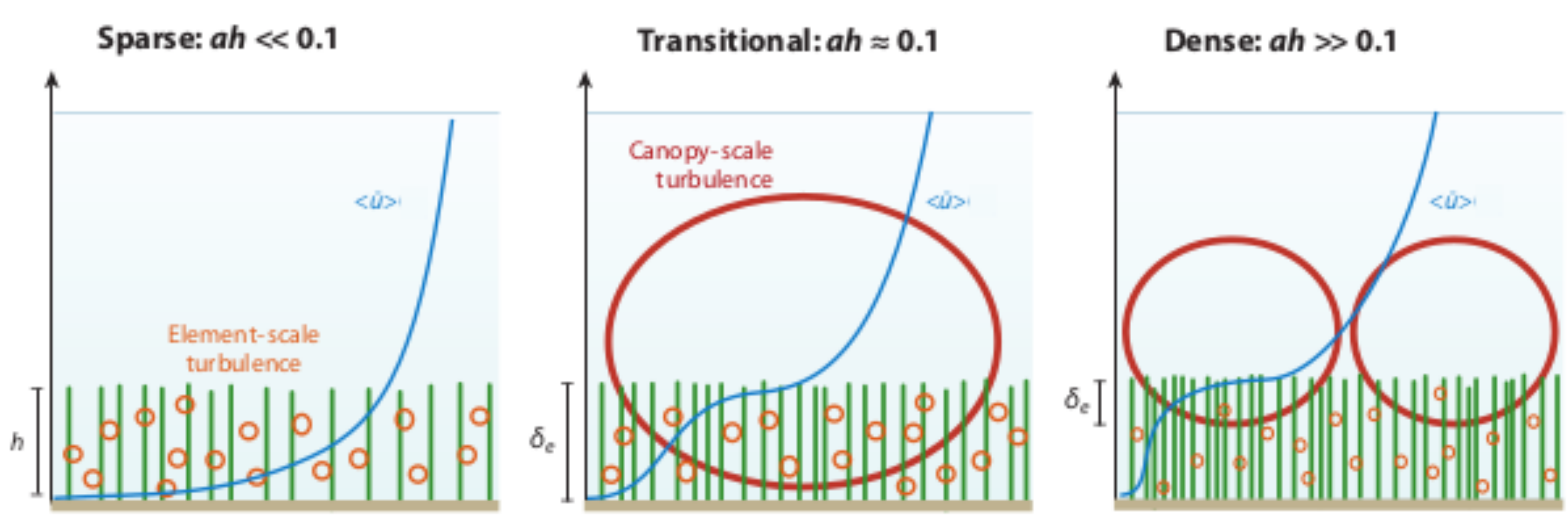
Let the frontal area per canopy volume to be $a = \dfrac{d}{\Delta S^2}$ then

$$\lambda = ah = \frac{dh}{\Delta S^2}$$

is the ratio frontal area to canopy base area

Uniform random distribution in each elemental volume $\Delta S^2 h$

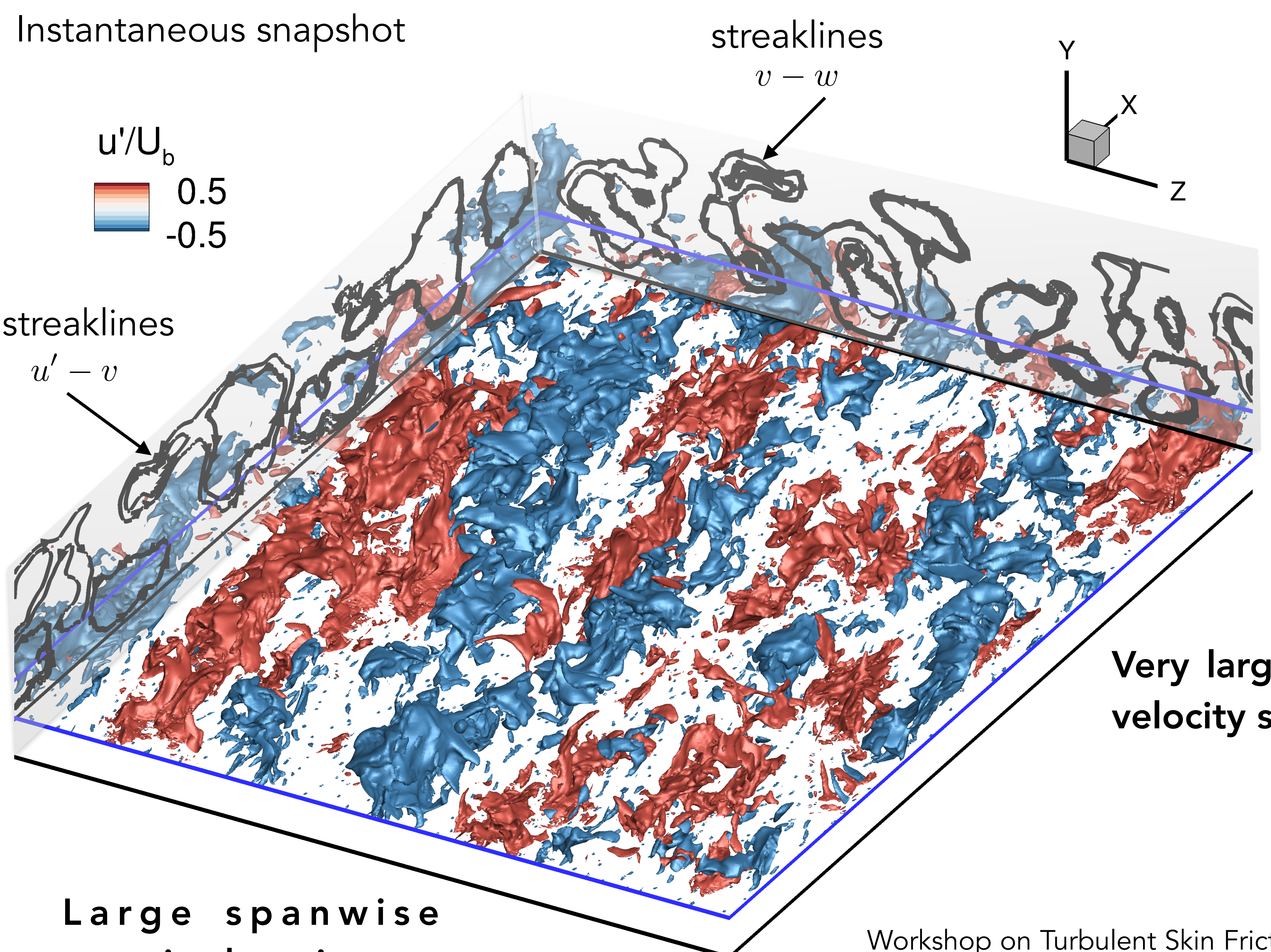
According to Nepf [Ann. Rev. Fluid Mech. 2012], canopies can be classified with the value of λ



$\lambda = 0.35$

is in the transitional/dense regime

Example III: flow over an anisotropic medium



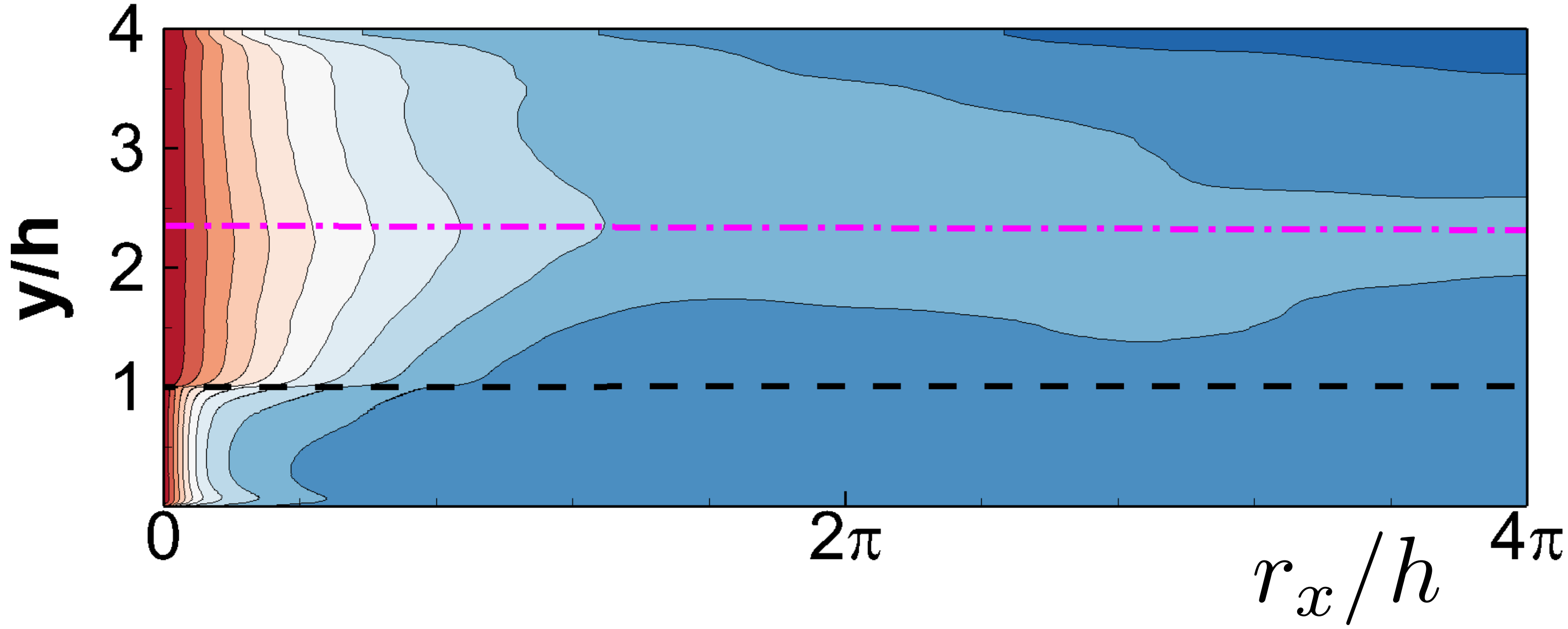
Very large elongated velocity streaks

Large spanwise organised vortices

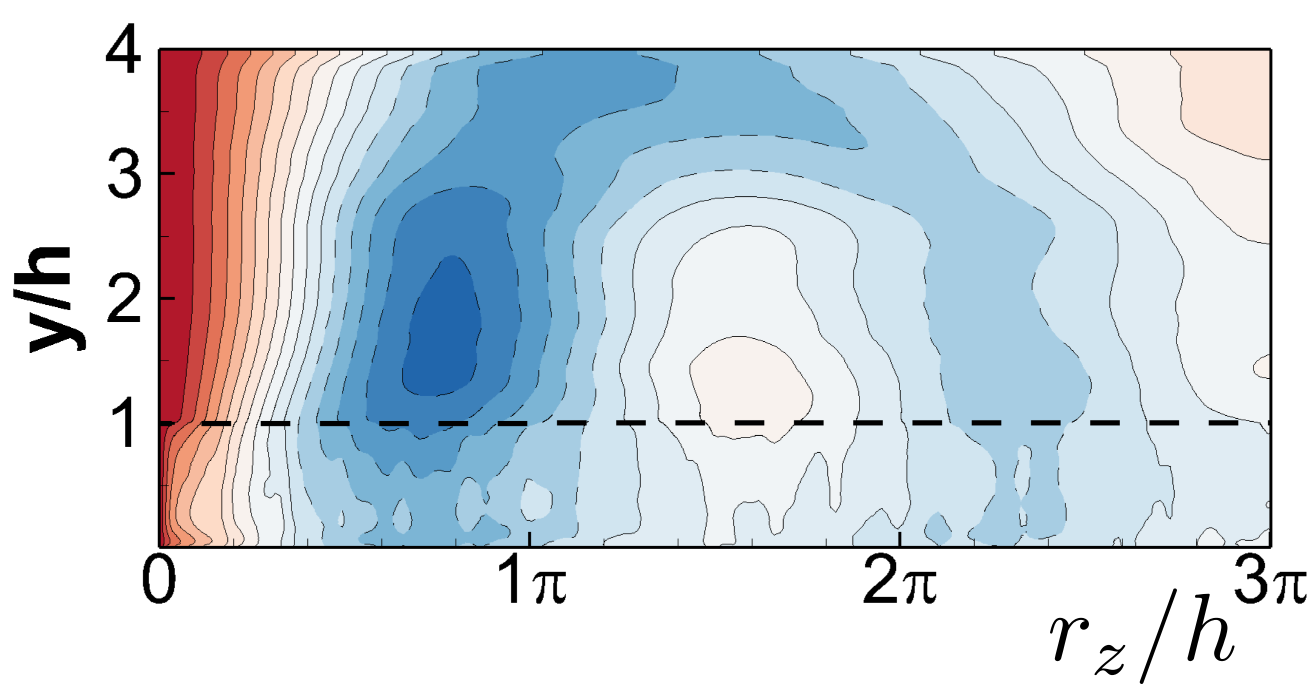
Example III: flow over an anisotropic medium

Long streaks characterisation

Streamwise autocorrelation of u'



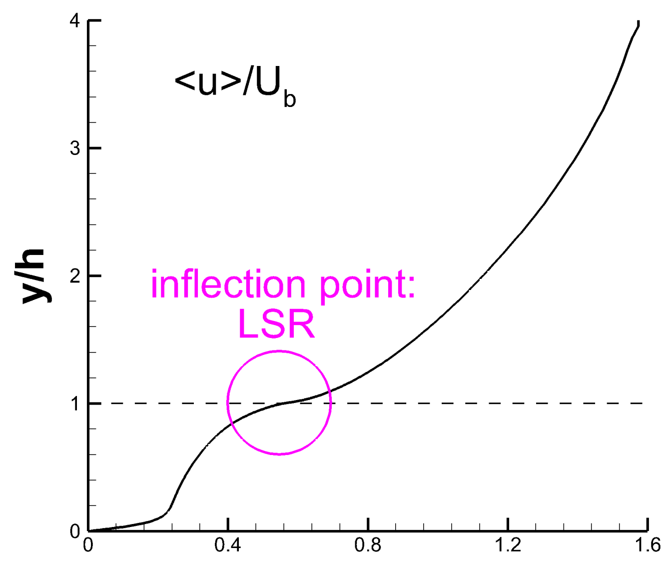
Spanwise autocorrelation of u'



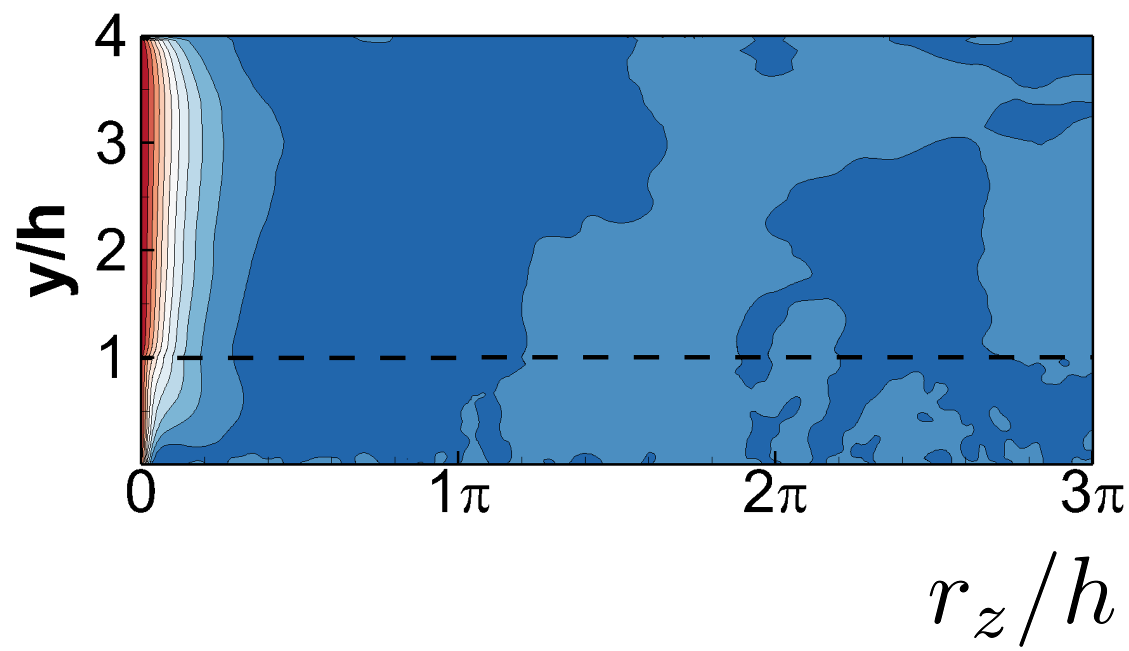
Log layer streaks, marginally penetrating in the canopy

Example III: flow over an anisotropic medium

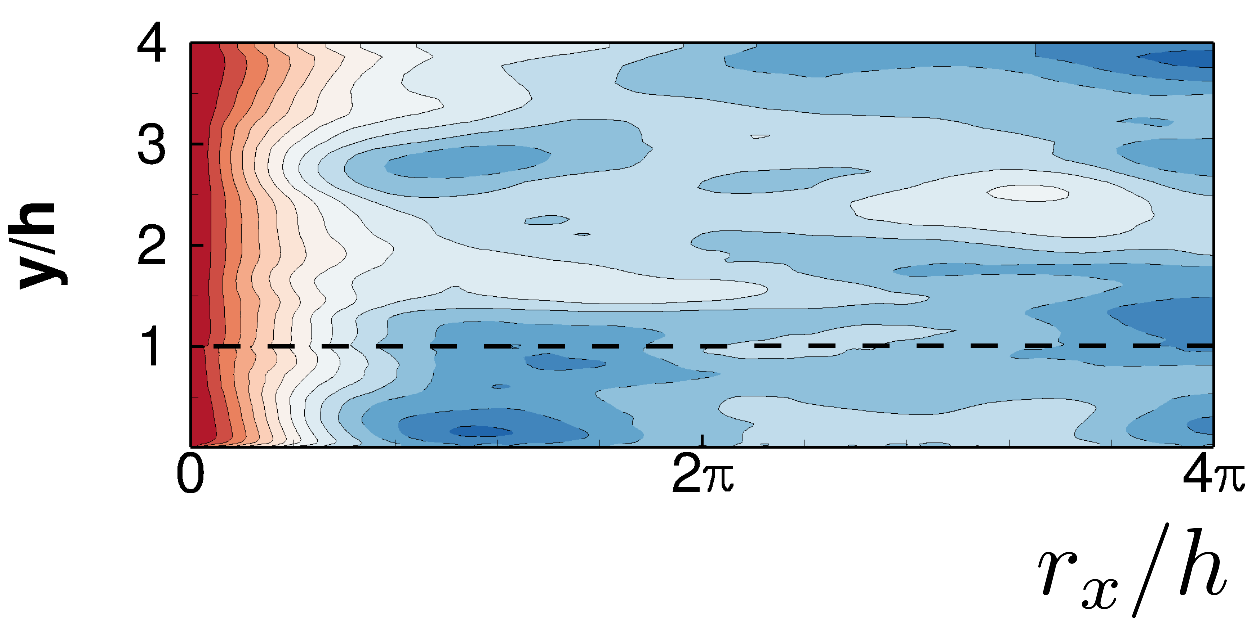
Spanwise vorticity characterisation



Spanwise autocorrelation of v'



Streamwise autocorrelation of v'



Spanwise oriented large coherent vortices

They do penetrate the canopy

Wrap up

We have seen examples that were covering scenarios in which the permeability was either:

- Isotropic

  or

- Anisotropic with large normal permeability and less cross-plane one.

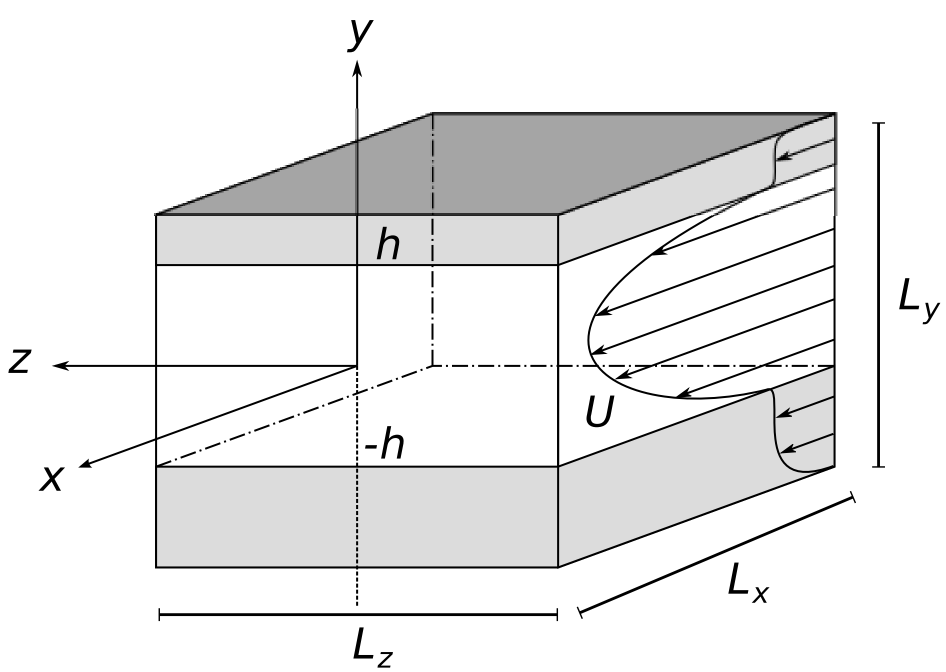
The examples taken from those two scenarios have some common features:

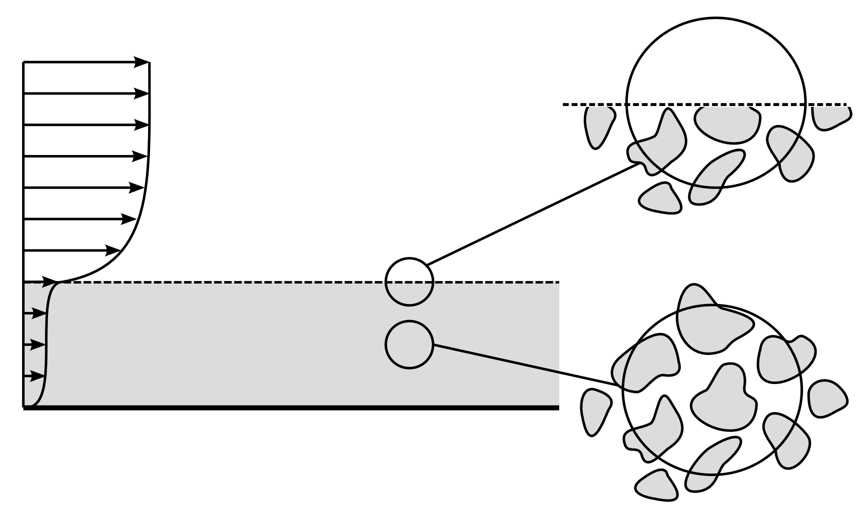
- Strong mixing in the vertical direction due to sweep and ejection events at the wall;
- Appearance of inflection point in the mean velocity profile;
- Appearance of coherent ''rollers'' of spanwise vorticity that largely contribute to increases Reynolds stresses and total drag increase.

What does it happen if we consider large ratios of cross plane permeability to normal permeability? Scenario similar to recent paper by N. Abderrahaman-Elena and R. García-Mayoral [PHYSICAL REVIEW FLUIDS 2, 114609 (2017)].

A generalised formulation for a systematic approach

Several flows over porous media characterised by isotropic distribution or anisotropic ones (where the in-plane permeability is low compared to the normal-to-the wall one) present many similarities. We tackle them with a generalised approach based on VANS and DNS to assess the impact of the permeability tensor on the outer flow.





ε

porosity

FORMULATION: VANS equations with linear Darcy approach for the closure.

$$\frac{\partial \langle \boldsymbol{u} \rangle^s}{\partial t} = -\varepsilon \boldsymbol{\nabla} \langle p \rangle^f + \frac{1}{Re} \nabla^2 \langle \boldsymbol{u} \rangle^s - \frac{\varepsilon}{Re} \left(\frac{\langle u \rangle^s \boldsymbol{i}}{\sigma_{xz}^2} + \frac{\langle v \rangle^s \boldsymbol{j}}{\sigma_y^2} + \frac{\langle w \rangle^s \boldsymbol{k}}{\sigma_{xz}^2} \right), \quad \boldsymbol{\nabla} \cdot \langle \boldsymbol{u} \rangle^s :$$

$$\varepsilon = \frac{\text{Volume of pores in CV}}{\text{Volume of CV}}$$

In plane and vertical permeabilities $\sigma_{xz}^2, \ \sigma_y^2$

$$\langle \phi \rangle = \varepsilon \langle \phi \rangle^f$$

We also define their ratio

$$\boxed{\psi = \frac{\sigma_{xz}}{\sigma_y}}$$

Parametric study

The bulk Reynolds number is fixed at $Re_b = 2800 \quad \rightarrow \quad Re_\tau \simeq 180$ for smooth impermeable walls

The porosity (i.e., ratio void fraction to whole volume) is fixed at $\varepsilon = 0.6$

The thickness of the porous slab is $0.2h$ (20% of the channel semi-height)

The code is a Fourier-Fourier-Padé sixth order in space, with a low storage RK in time. The formulation is the poloidal toroidal one (i.e., K.M.M.).

256×256 Fourier modes on a computational domain $4\pi \times 2\pi$. In the wall-normal $150 + 75$ nodes (outer and porous slab).

The permeability tensor is diagonal but with different diagonal entries. In particular:

Isotropic

Case	$10^3\ \sigma_{xz}$	$10^3\ \sigma_y$	ψ	Re_τ	$DR\%$	$\overline{u}_i$
$I\sigma \downarrow$	0.2500	0.2500	–	181	−3.37	0.0029
$I\sigma$ (green)	1.0000	1.0000	1	182	−5.55	0.0090
$I\sigma \uparrow$	4.0000	4.0000	–	188	−11.6	0.0384
(blue) $A\sigma_{xz} \downarrow \sigma_y \uparrow$	0.2500	4.0000	0.0625	198	−21.2	0.0034
(red) $A\sigma_{xz} \uparrow \sigma_y \downarrow$	4.0000	0.2500	16	177	1.10	0.0336
(orange) $A\sigma_{xz} \uparrow\uparrow \sigma_y \downarrow\downarrow$	8.0000	0.1250	64	171	7.63	0.0632
(pink) $A\sigma_{xz} \uparrow\uparrow\uparrow \sigma_y \downarrow\downarrow\downarrow$	16.0000	0.0625	256	164	17.7	0.1149

Anisotropic

easy y, hard x-z

hard y, easy x-z

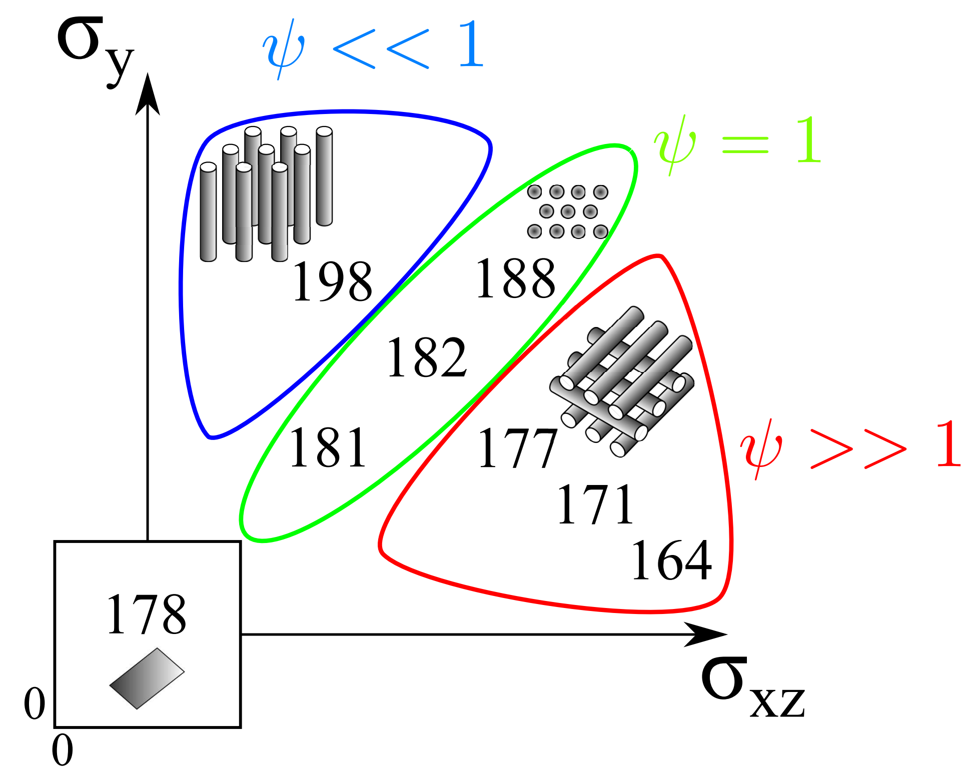
Parametric study

Isotropic ● $\psi = 1$

Anisotropic

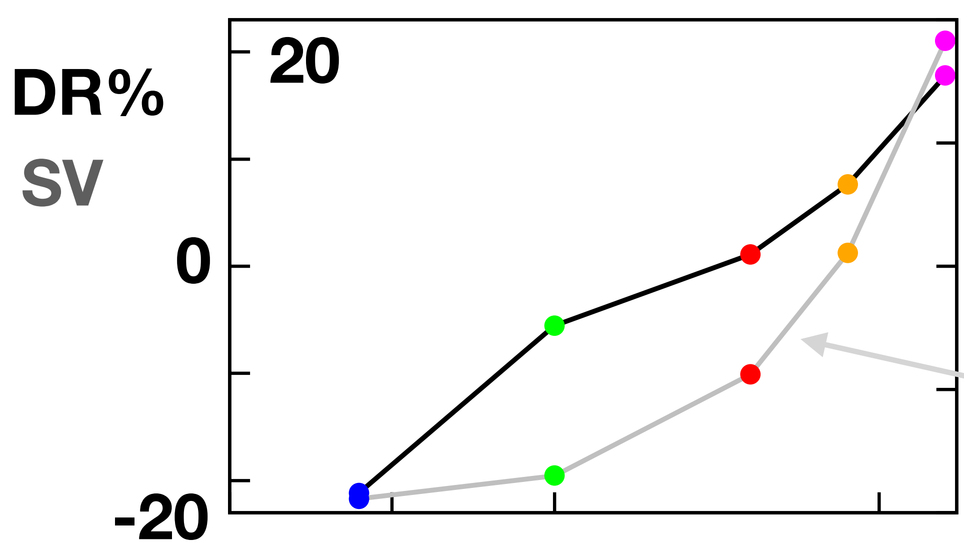
$\psi = 0.0625$ ●
$\psi = 16$ ●
$\psi = 64$ ●
$\psi = 256$ ●

$$\psi = \frac{\sigma_{xz}}{\sigma_y}$$



Skin friction Reynolds number computed with

$u_\tau = \sqrt{\tau_{tot}/\rho}$ $\quad \tau_{tot}$ is the total stress at the interface



Slip velocity

$$DR = \frac{\Delta G}{G_0}$$

Drag seems to decrease monotonically as ψ is increased without apparent saturation (as it happens for the riblets).

Limit of approximation

For isotropic media and media with higher normal to the wall permeability the drag increases.

For porous media with lower permeability in the wall normal direction, drag reduction increases without any apparent saturation.

However, for the closure (based on Darcy, linear formulation) to be valid the non dimensional coefficient of the Farchheimer contribution should be small compared to Darcy.

Darcy

Forchheimer

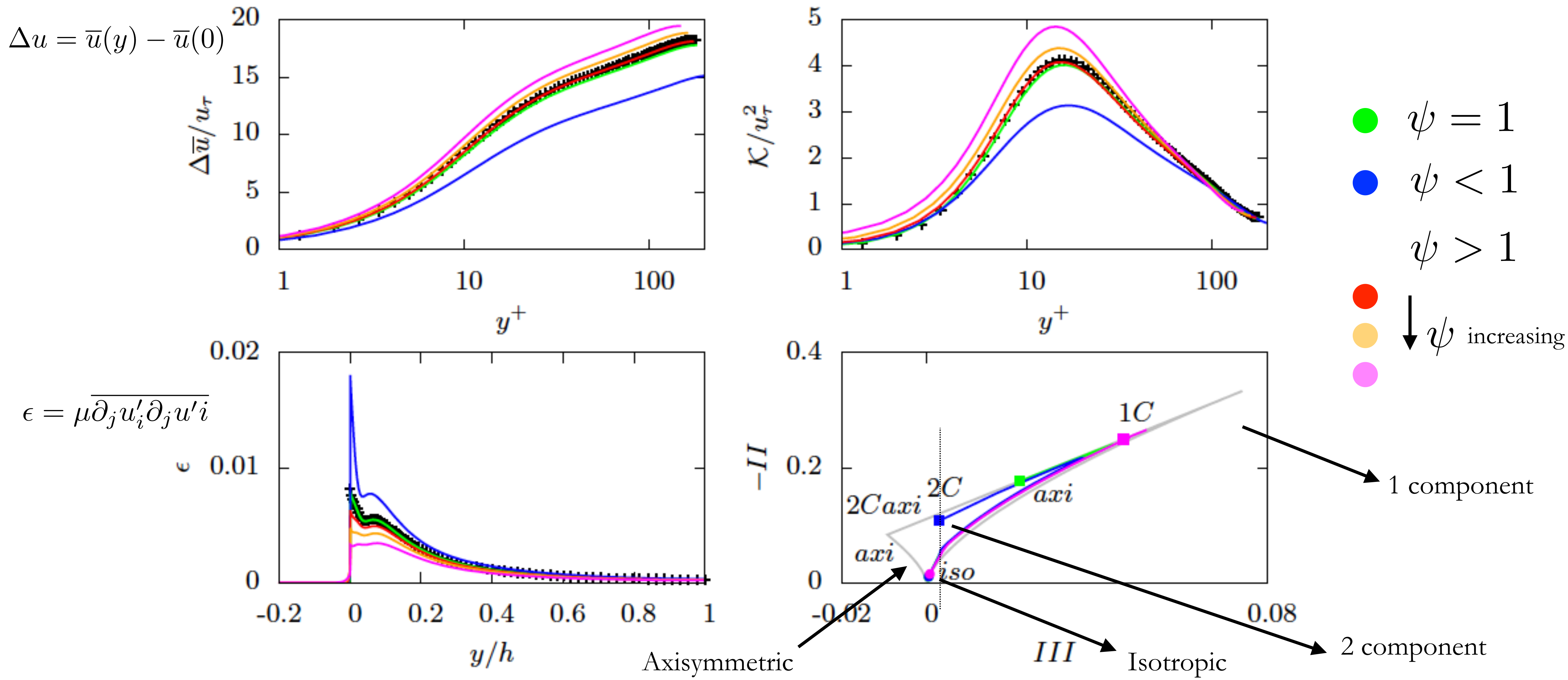
$$\vec{\nabla} p = -\frac{\mu}{\kappa}\vec{V} - \beta \rho_f |\vec{V}|\vec{V}$$

$$\beta = \frac{C_E}{\sqrt{\kappa}}$$

Ergun Coef.: constant in laminar regime (within PM)

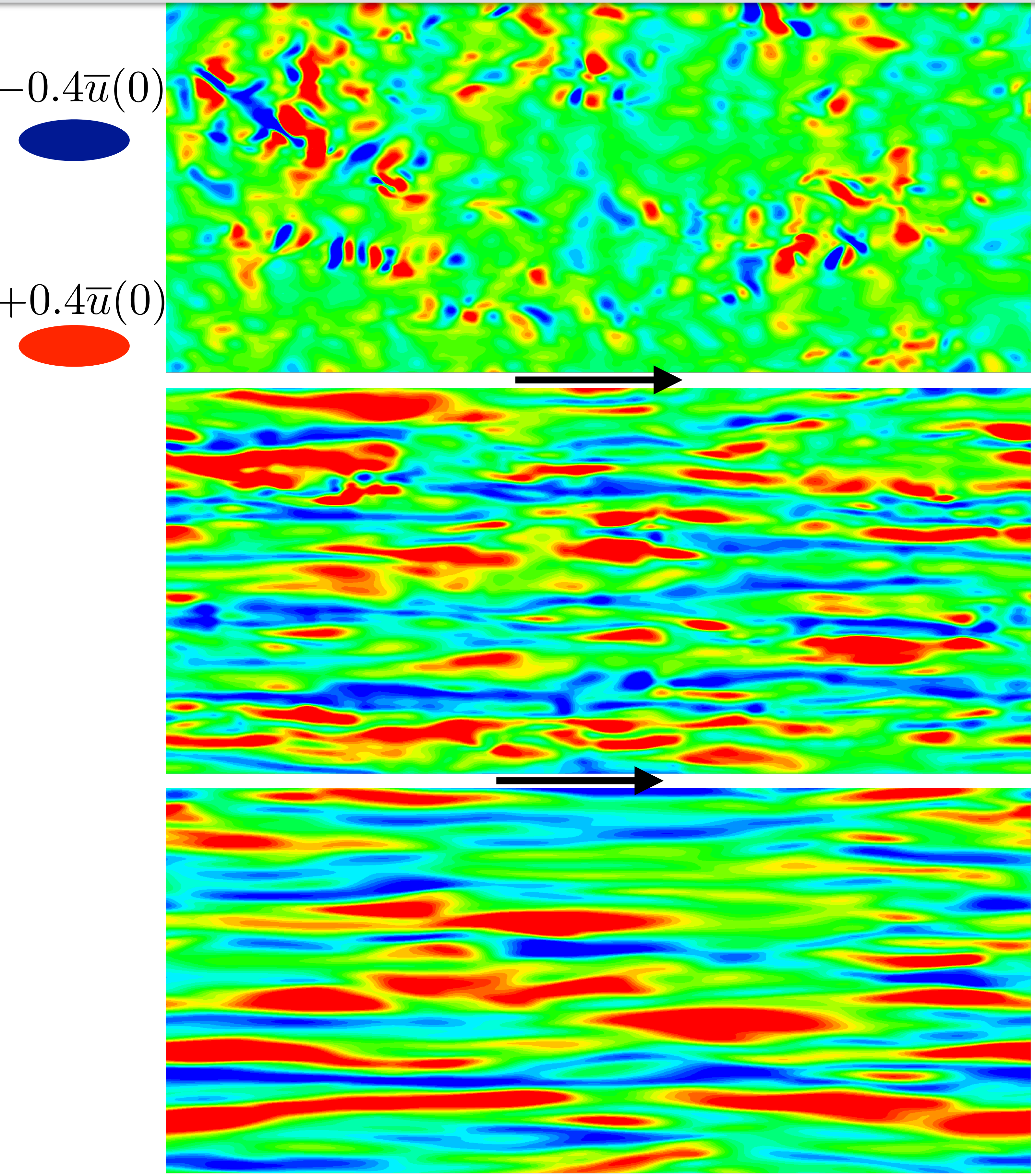
We did not consider cases that were outside the linear regime: the value of the permeability tensor kept at moderate value for not breaking the model hypothesis.

Outer flow characterisation



The Lumley's triangle of the II and III invariants of the Reynolds stress anisotropy tensor. Squares: interface; bullets centreline. Close to the interface turbulence is strongly one-component in the drag reducing case, while two-components turbulence characterises the other two cases.

Flow structures



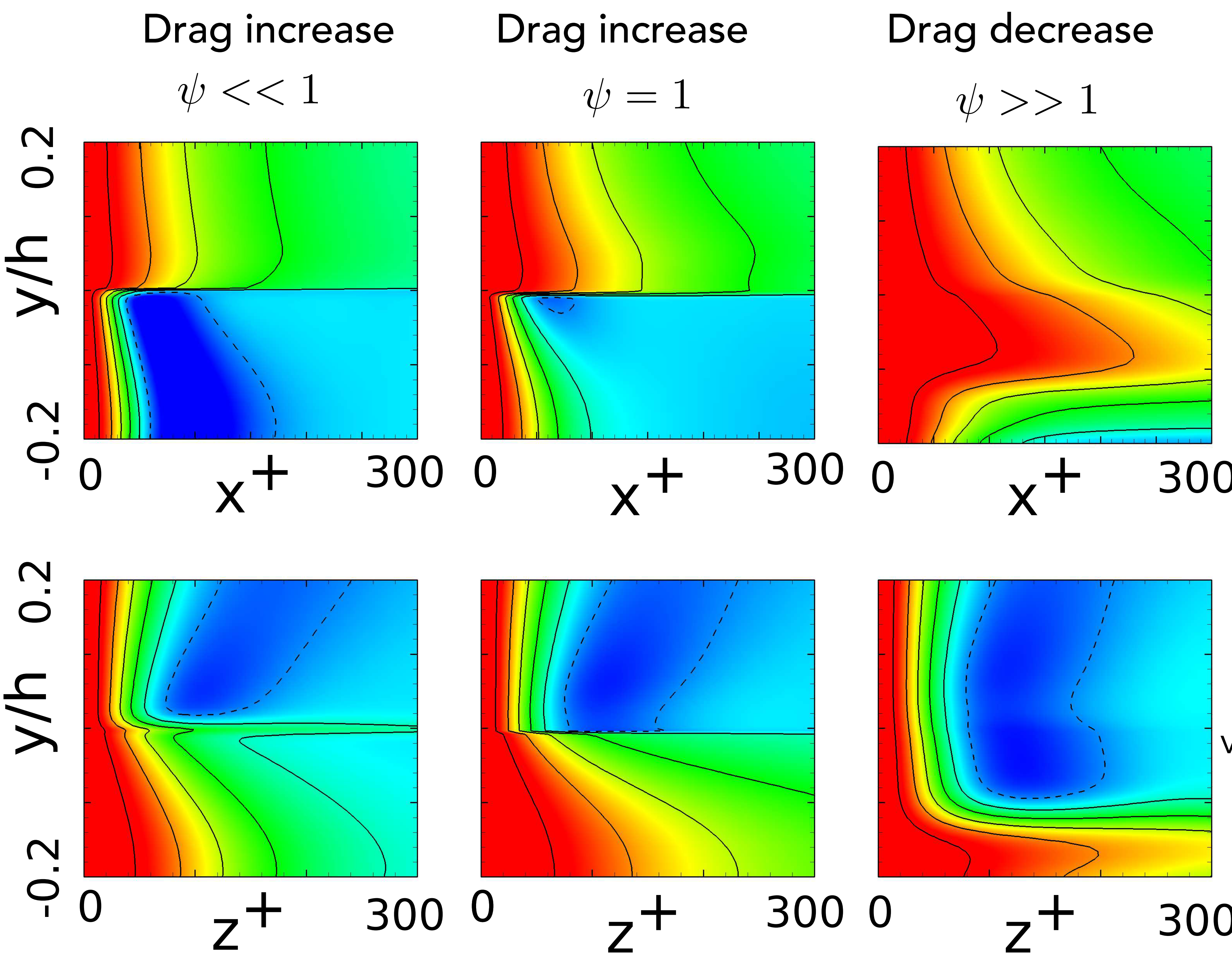
Contours of instantaneous streamwise velocity fluctuation at the interface.

$\psi << 1$ **Drag increase**

$\psi = 1$ **Drag increase**

$\psi >> 1$ **Drag decrease**

Flow structures



Streamwise velocity autocorrelation

Normal to the wall velocity autocorrelation

Solid positive
Dashed negative

$[-0.1, 0.2, 0.9]$

Conclusions

- Flow over porous walls always studied on either isotropic or anisotropic medium with a wall normal permeability much lower that the in-plane one.
- All these flows have as common feature: insurgence of large coherent structures that are probably the result of an inherent linear instability kicking in when the impermeability condition at the wall is relaxed.
- The insurgence of large scale motions seems to break the coherence of buffer layer structures increasing the Reynolds stress intensity and therefore the total drag.
- In this study we have considered a sort of *mille-feuille* porous medium with a strong ability to shield the layer from vertical flow inrushes but that preserves the shear at the interface allowing large velocity magnitudes in the streamwise direction within the porous layer.
- A naive explanation of the obtained drag reduction is: increased slip velocity with a prohibited appearance of linearly unstable modes (i.e., $v=0$ at the interface) and therefore Reynolds stresses are also forced to go to zero at the interface.
- The wall cycle seems to be still in place but the structures are quite larger (reduced drag).
- Inducing this type of anisotropy (i.e., $\psi >> 1$) seems to be a winning ingredient for drag reduction

Outlook

Reynolds scaling: Rosti et al. [Jou. Fluid Mech. 2015] claim that if the outer *Re* is increased the same flow geometry is preserved is the porous Reynolds number is scaled in a proportional way. However this has been verified only for isotropic porous media and low to moderate Reynolds number (no scale separation). ⟶ It needs to be verified

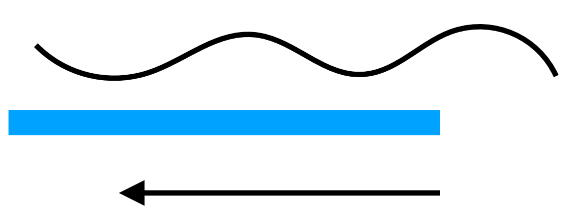
$$Re_p = \frac{\sqrt{K}v}{\nu}$$

Generalising what it has been observed one can imagine to introduce a forcing term into the RHS of the Navier-Stokes equations

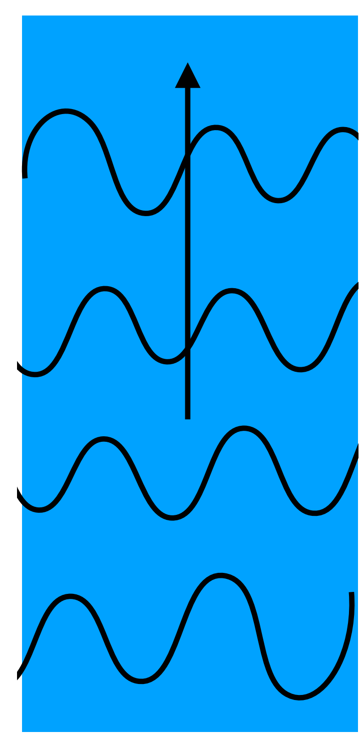
$$\frac{D\mathbf{u}}{Dt} = -\nabla p + \frac{1}{Re}\nabla^2\mathbf{u} + \nabla \cdot \overline{\overline{F}}$$

- Can the anisotropy of the forcing tensor applied to the close to the wall region a universal key for wall turbulence modulation?

- Another example:



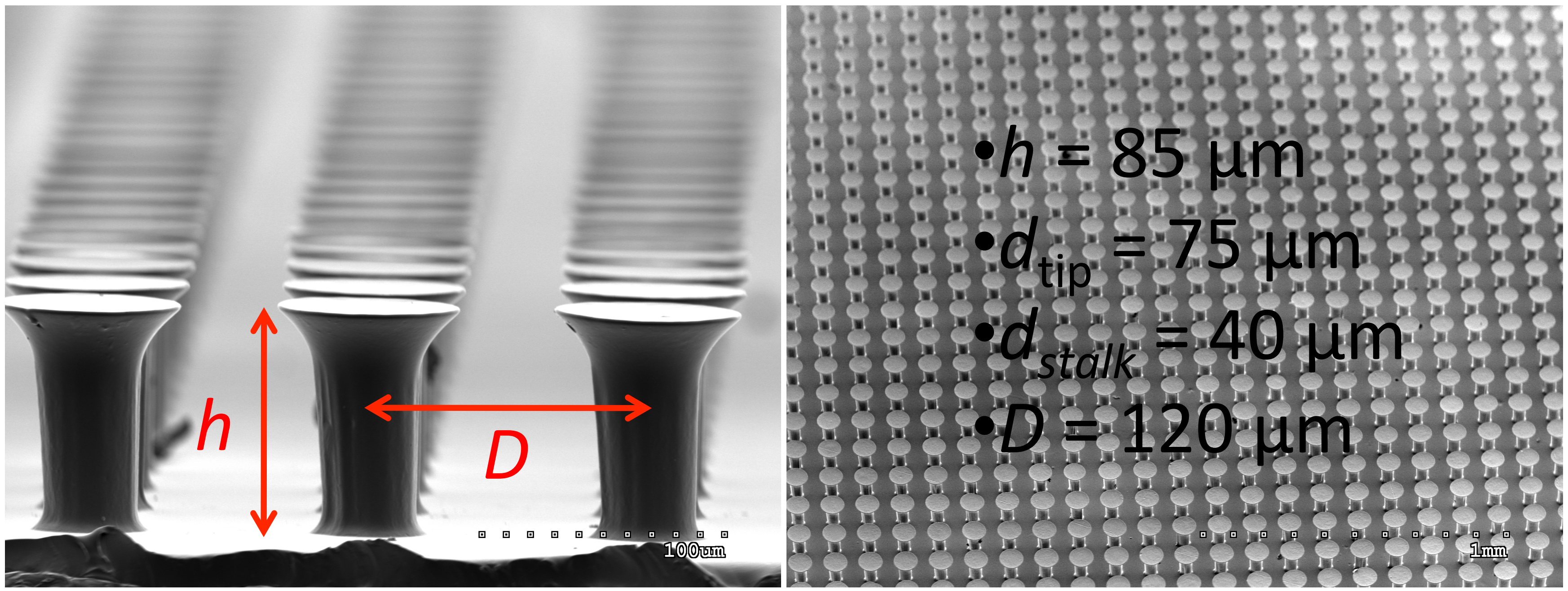
The mean flow sees an anisotropic Stokes layer



Oscillating wall

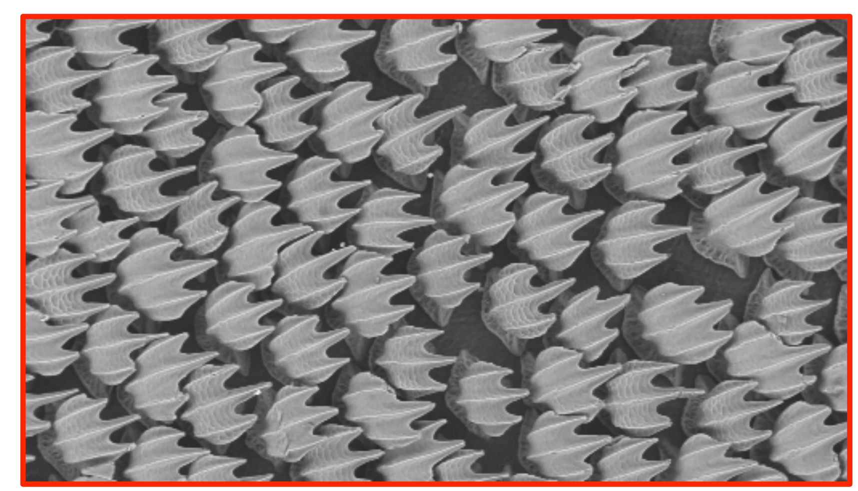
Outlook

Separation control and mitigation



Aksak, Murphy & Sitti (2007), Langmuir, **23**

Sharks lateral denticles



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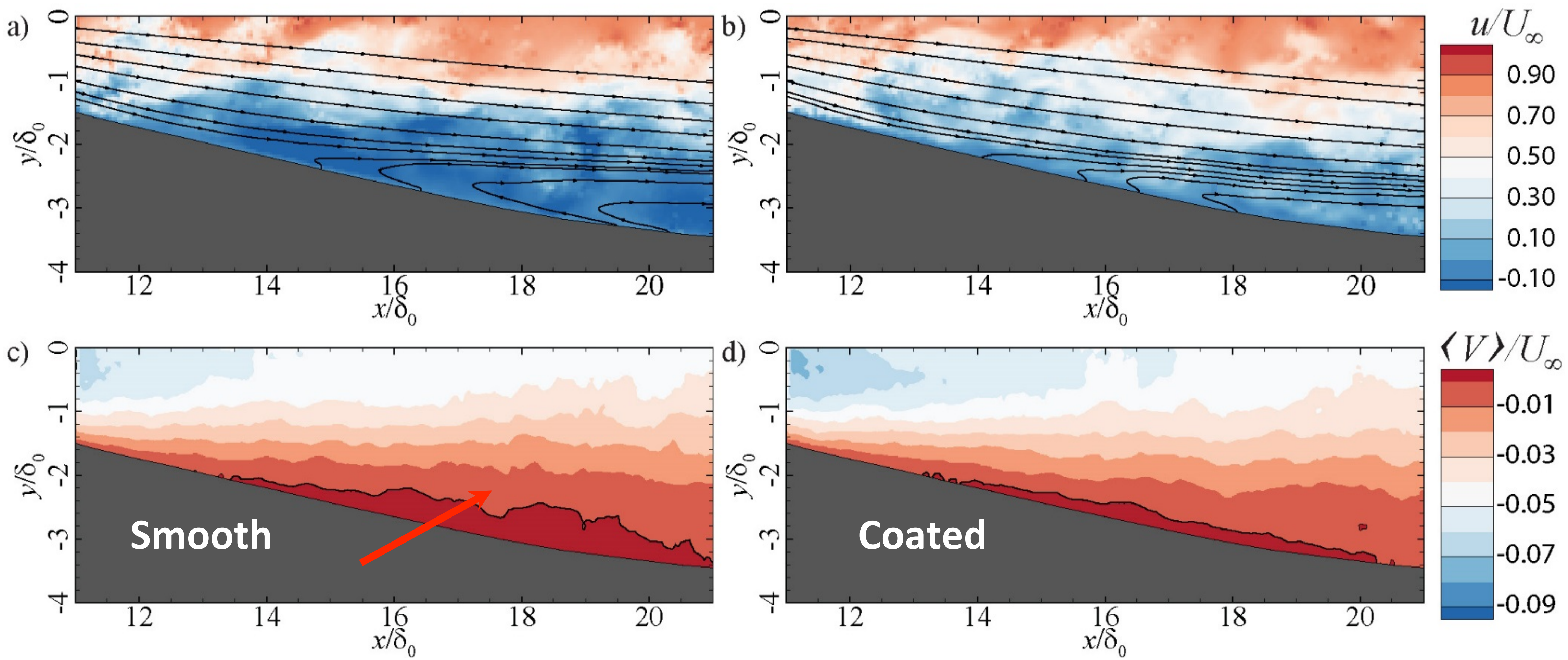
Outlook

Separation control and mitigation



60% reduction in area with reverse flow

Courtesy of Bocanegra & Castillo APS-DFD 2017