ACTIVE AND PASSIVE TURBULENT DRAG REDUCTION

Pierre Ricco

Department of Mechanical Engineering
The University of Sheffield

Workshop on Turbulent Skin Friction Drag Reduction
Imperial College London - 4-5 December 2017

COLLABORATORS

- Synthetic jets: Ning Qin (Sheffield)
- Hydrophobic surfaces: Sohrab K. Aghdam (was at Sheffield, now gone...sailed for better unknown shores)
- Discs and rings: Stan Hahn (Honeywell), Claudia Alvarenga, Paolo Olivucci (Sheffield), Daniel Wise (Singapore)
- Laminar traveling waves: Peter Hicks (Aberdeen)

FINANCIAL SUPPORT

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- Department of Mechanical Engineering, University of Sheffield
- A* Star Singapore
- Iraqi goverment
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- H2020 EU

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PASSIVE: HYDROPHOBIC SURFACES

- Shear-dependent slip length model
- Power spent on the surface: passive-absorbing

ACTIVE: ROTATING RINGS

- Forced rotating discs
- Freely rotating discs
- Spinning rings

LAMINAR TRAVELING WAVES

- Numerical and asymptotic study of full parameter space
- Analytical solutions and explain an unexpected behaviour

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HYDROPHOBIC SURFACES

SLIP-LENGTH SURFACE MODEL

$$u(0) = l_s \left. \frac{\partial u}{\partial y} \right|_{y=0}$$

- Slip length l_s: *constant*
- It traces back a long time: Navier (1823)
- Used widely to model *laminar*, *transitional* and *turbulent* flows
- Limitation: no capturing of exact texture

SLIP/NO-SLIP SURFACE MODEL

$$u(0) = 0 \quad \text{over solid wall,} \quad \left. \frac{\partial u}{\partial y} \right|_{y=0} = 0 \quad \text{over air pockets}$$

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  - → wall-roughness effects, liquid-air interaction
  - → bubble deformation, *power spent is zero?!*

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SHEAR-DEPENDENT SLIP LENGTH

- Molecular dynamics simulations *Thompson, Troian* (1997)
- Mentioned in theoretical works *Lauga, Stone* (2003)
- Laminar-flow experiments *Choi et al.* (2003), *Choi, Kim* (2006)
  $\rightarrow$ l_s= 50 microns
- Recognized as relevant in DNS studies
  $\rightarrow$ *Min, Kim* (2004); *Busse, Sandham* (2012)
- Jung, Choi, Kim JFM (2016): DNS of water flowing on air pockets
  $\rightarrow$ air flow is simulated $\rightarrow$ cavity recirculation, for example
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Objective

- Turbulent channel flow
- Linear dependence of slip length on wall shear

$$l_s = \mathbf{a} \left. \frac{\partial u}{\partial y} \right|_{y=0} + \mathbf{b}$$

- Aghdam, Ricco Phys. Fluids (2016)

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Slip length shear-dependent along x, not along z

For this case → model works well **a finite, b small**

We compute very close drag reduction values

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$$\mathbf{a}\,(1-\mathcal{R}^\star)\,\frac{R_{\tau,r}^3}{R_p}+\mathbf{b}R_{\tau,r}=\left(\kappa^{-1}\ln R_{\tau,r}+F\right)\frac{1-\sqrt{1-\mathcal{R}^\star}}{1-\mathcal{R}^\star}-\frac{\ln(1-\mathcal{R}^\star)}{2\kappa\sqrt{1-\mathcal{R}^\star}}$$

Implicit relationship to find $\mathcal{R}^\star=\mathcal{R}^\star(a,b;R_{\tau,r})=\mathcal{R}/100$

$\mathcal{R}$ increases with a and b

Averaged slip length $\mathcal{L}$ can be introduced

$$\frac{a\,(1-\mathcal{R}^\star)\,R_{\tau,r}^2}{R_p}+b=a\,\frac{\mathrm{d}\mathcal{U}}{\mathrm{d}y}\bigg|_{y=0}+b=b-a\frac{\mathrm{d}P}{\mathrm{d}x}R_p=\mathcal{L}$$

This shows that $\boxed{\mathcal{R}=\mathcal{R}(\mathcal{L};R_{\tau,r})}$ irrespectively of a and b

Flows with same $\mathcal{R}$ have the same averaged slip velocity $\mathcal{U}(0)$

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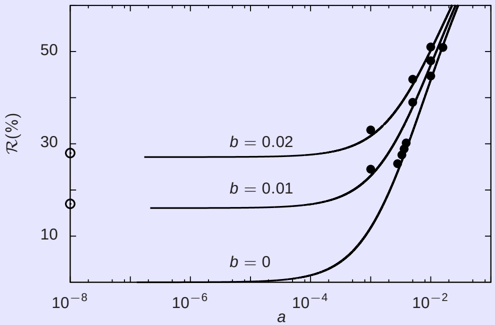
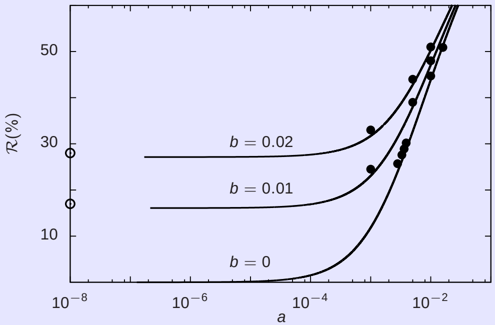


COMPARISON FUKAGATA - DNS

- Very good agreement between FKK theory and DNS
- Flow parameters in the laboratory: water channel
  - Channel height $2h^* = 3.4$mm
  - Bulk velocity $\mathcal{U}_b^* = 1.6$m/s
  - $b^* = 35\ \mu$m, $a^* = 0.01\mu$m s
  - a^* value is 10 times smaller than in Choi Kim PRL (2006)!
  - Shear-dependence may play a significant role → high wall-shear stress



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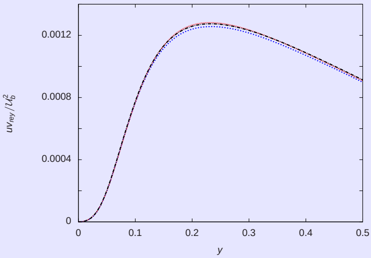
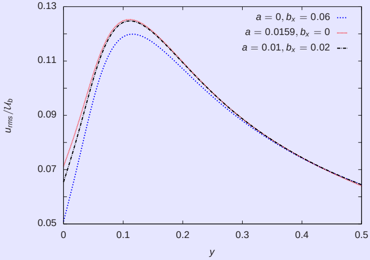
$$C_f = \frac{6}{\mathcal{U}_b R_p} - \frac{6}{\mathcal{U}_b^2}\int_0^1 (1-y)uv_{rey}\,\mathrm{d}y - \frac{6\,\mathcal{U}(0)}{R_p \mathcal{U}_b^2}$$

FLOWS WITH THE SAME AVERAGED SLIP LENGTH $\mathcal{L}$

- Same drag reduction $\mathcal{R}$
- Same averaged slip velocity $\mathcal{U}(0)$
- Same weighted integrated Reynolds stress profiles





Reynolds stresses agree throught the whole channel

Rms of streamwise velocity do not overlap

$$\mathcal{P}_x + \mathcal{W} + \mathcal{D} = 0$$

Power for x motion $\quad \mathcal{P}_x = 2\mathcal{U}_b L_x L_z \left((R_\tau / R_p)\right)^2$

Power spent at the wall $\quad \mathcal{W} = -\frac{2}{R_p}\left[\left\langle U(0)\left.\frac{\partial U}{\partial y}\right|\right\rangle_{y=0}\right]_{\mathcal{I}xz}$

Dissipation into heat $\quad \mathcal{D} = -\frac{1}{R_p}\left[\left\langle\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j}\right\rangle\right]_{\mathcal{I}xyz}$

POWER AT THE WALL

- Non-negligible: if $\mathcal{R} = 40\%$, $\mathcal{W} = 15\%$
- $\mathcal{W} = 0$ for slip/no-slip modelling $\rightarrow$ serious issue
- Physics
  - Lotus-leaf surface: power exerted by liquid on air pockets
    $\rightarrow$ air motion, stretching, detachment $\rightarrow$ drag increase
  - Pitcher-plant leaf surface: power exerted by water on infused oil

PASSIVE ABSORBING

- Oscillating-wall, traveling waves: active $\rightarrow$ power injected into the fluid
- Riblets, wavy wall: passive-neutral $\rightarrow$ no power exchanged by fluid and surface
- Hydrophobic surfaces: **passive-absorbing** $\rightarrow$ power from fluid, absorbed by surface

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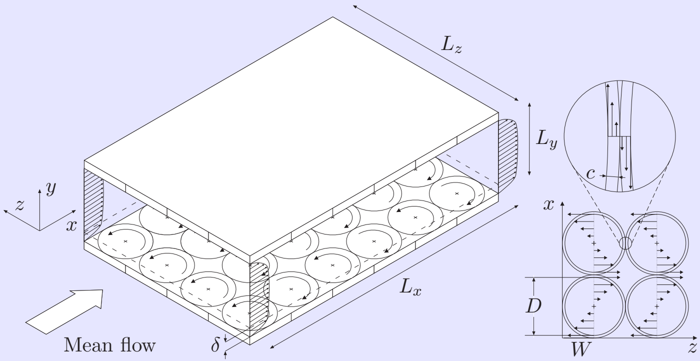
SPINNING DISCS



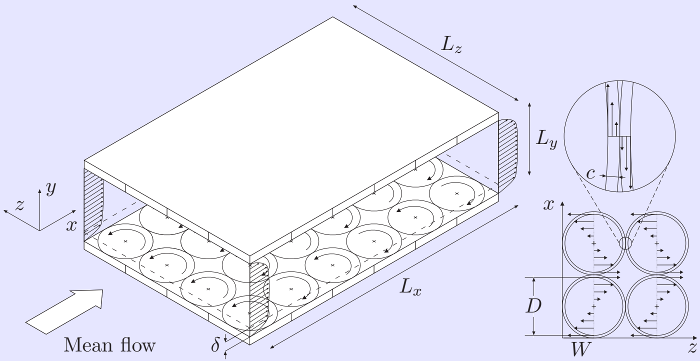
First proposed by L. Keefe (1998), but no results

Fully-developed turbulent channel flow $R_\tau = 180$ - DNS: x, z Fourier, y Chebyshev

Parameters: D diameter, W maximum tip velocity

Discs neighbouring along z: same sense of rotation

Discs neighbouring along x: opposite sense of rotation $\rightarrow$ *triangular steady wave*



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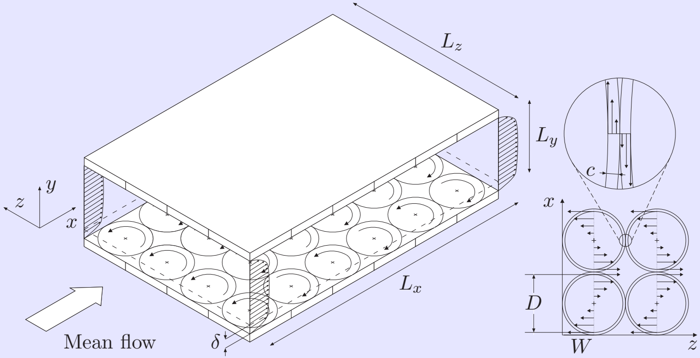

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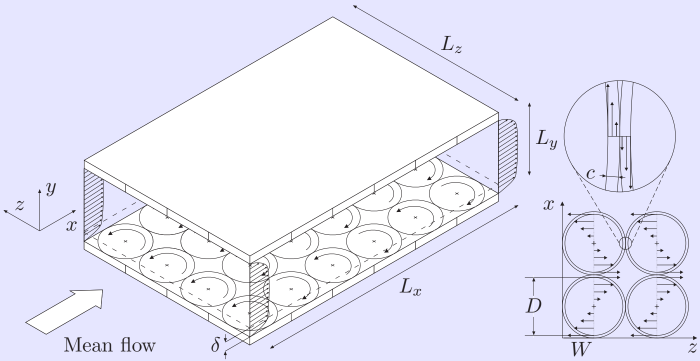

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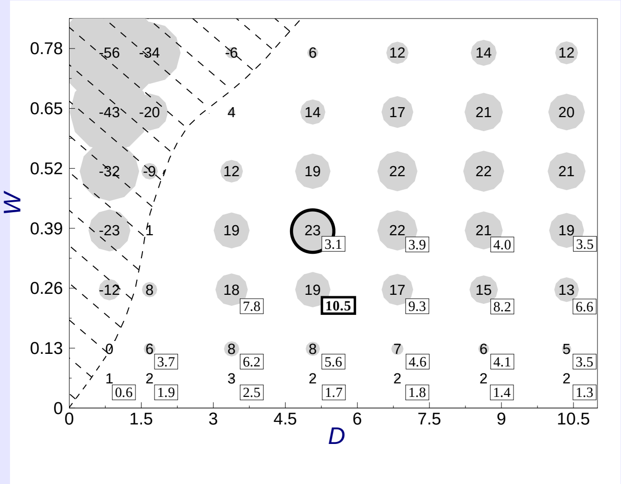
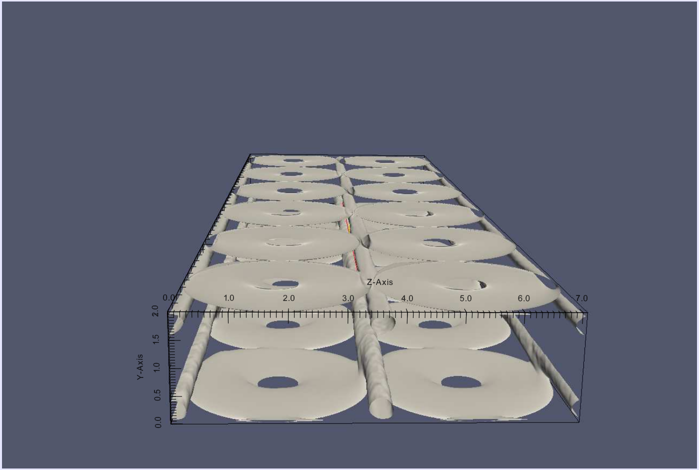


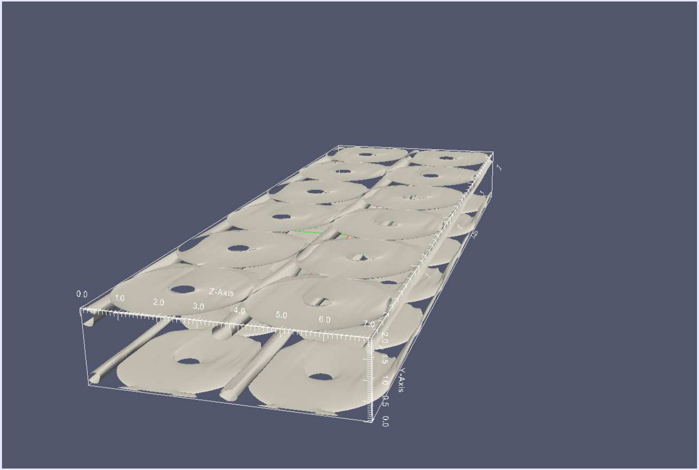
Drag *increase* may occur at small D, large W

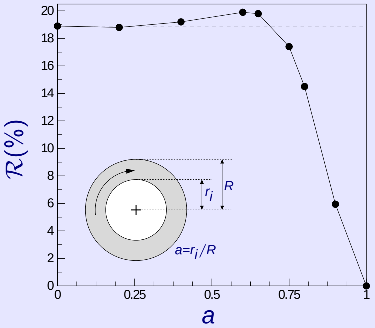
Maximum $\mathcal{R}$=23%, maximum $\mathcal{P}_{net}$=10%

Optimum in wall units: $D^+ \approx 1000, W^+ \approx 10$

Flow vis




0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
Z-Axis
2.0
1.5
1.0
0.5
0.0
Y-Axis

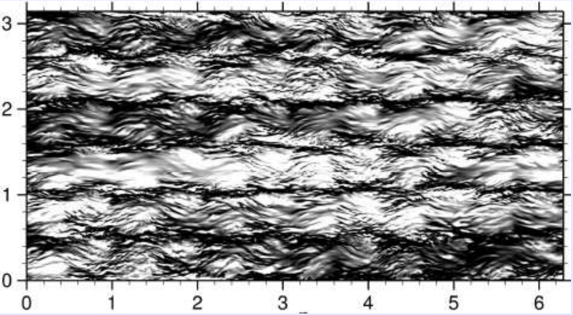


$a < 0.38$, $\mathcal{R}$ constant

$a = 0.6$, optimum $\mathcal{R}$

$\rightarrow$ higher than with full disc
$\rightarrow$ flow motion toward the wall is reduced
$\rightarrow$ von Kármán pump effect

$a \rightarrow 1$, $\mathcal{R} \rightarrow 0$



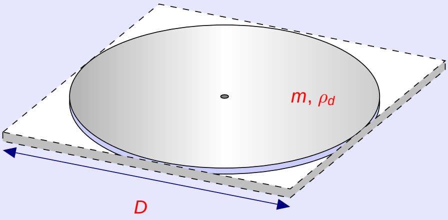
$R_b = 80,000$, $R_\tau = 2,000$ $\boxed{\mathcal{R} = 16\%}$

Contour of instantaneous u'^+ at $y^+ = 15$

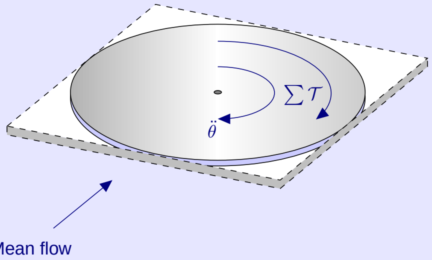
Streaks persist, but dragged by spinning disc motion

Large scales: combination of imprint from outer super-structures and disc rotation

FREELY ROTATING DISCS



- Discs on the channel walls: diameter D, thickness t, density ρ_d, mass m
- Fluid flow over discs causes turbulent torque, $\mathcal{T}_t$
- Rolling friction from the bearing $\mathcal{F}_m$
- Fluid friction from the disc housing, $\mathcal{F}_f$



EQUATION OF MOTION

$$J\frac{d^2\theta}{dt^2} = \sum \mathcal{T}$$

$\sum \mathcal{T} = \mathcal{T}_t - \mathcal{F}_m - \mathcal{F}_f$: sum of torques

$J = \frac{mD^2}{8}$: moment of inertia

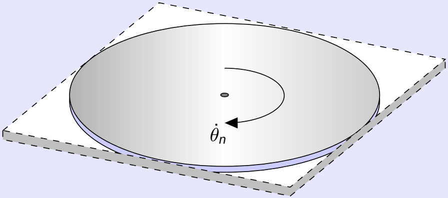
$\frac{d^2\theta}{dt^2}$: angular acceleration

First discretize in time: $\dot{\theta}_{n+1} = \dot{\theta}_n + (\Delta t/J) \sum \mathcal{T}$

Turbulent torque, $\mathcal{T}_t = \int_0^R \int_0^{2\pi} \tau_\theta r (r \mathrm{d}\theta) \mathrm{d}r$

Fluid friction torque from housing of disc: $\mathcal{F}_f = -\kappa |\dot{\theta}|^{3/2}$

Mechanical friction torque in bearing: $\mathcal{F}_m = -f(\mathcal{T}_t - \mathcal{F}_f)$; $f = 0.0015$



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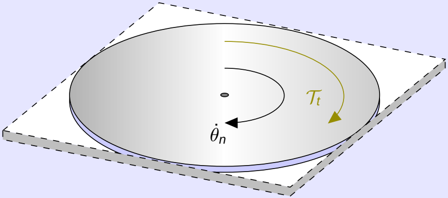

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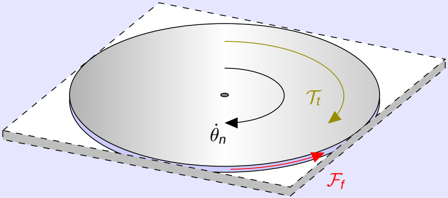

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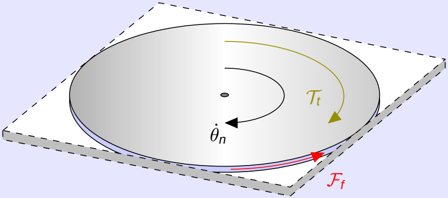
Fluid friction torque from housing of disc: $\mathcal{F}_f = -\kappa |\dot{\theta}|^{3/2}$

Mechanical friction torque in bearing: $\mathcal{F}_m = -f(\mathcal{T}_t - \mathcal{F}_f)$; $f = 0.0015$



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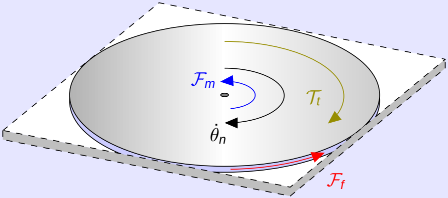

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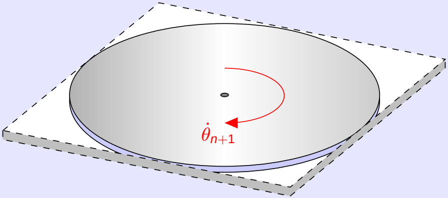
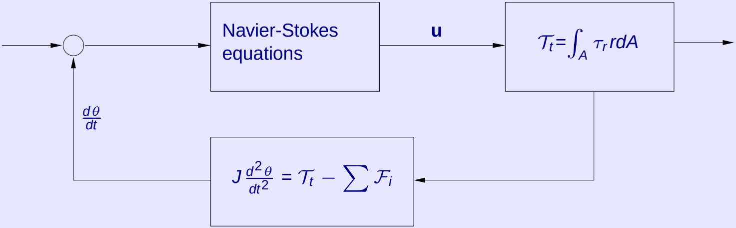




Freely rotating discs can be seen as a **feedback system**

Wall boundary conditions are not pre-determined

Each disc moves independently

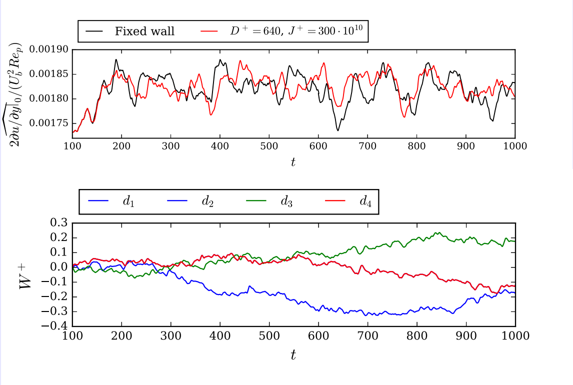
Disc dynamics is fully coupled with turbulent flow dynamics

Water channel: U_b=0.32ms^{-1}, h=0.1m $\rightarrow$ R_b=3160, R_τ=180

D^+=640 $\rightarrow$, J for Titanium, D=35mm

No drag reduction $\rightarrow$ maximum disc-tip velocity is W_m^+<0.5

J reduced to non-realistic values $\rightarrow$ drag reduction



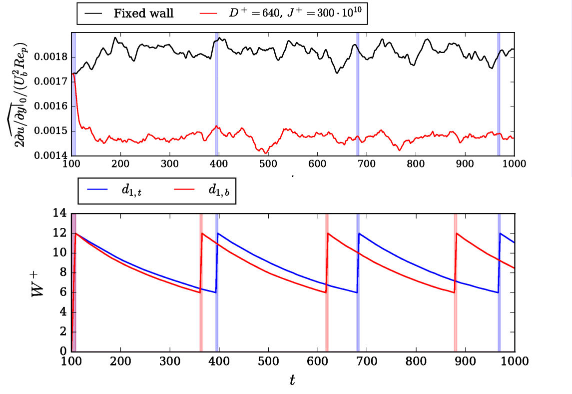
Switching discs off/on when threshold tip velocity is reached

D^+=640 $\rightarrow$ Lower threshold: W_l^+=6, Upper: W_u^+=12

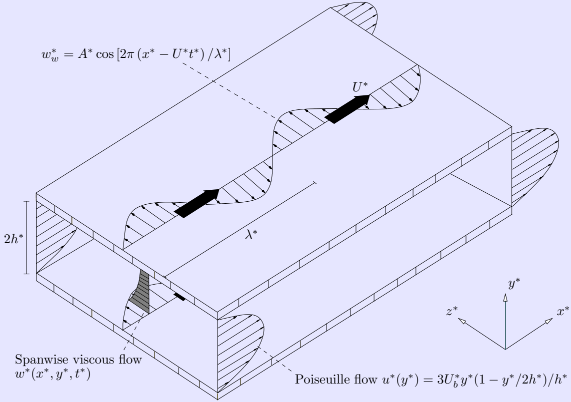
Drag reduction $\mathcal{R}$=19% Power spent $\mathcal{P}_{sp}$=10%

Better than standard rotating cases

- W^+=6: $\mathcal{R}$=17.6%, $\mathcal{P}_{sp}$=10.1%
- W^+=12: $\mathcal{R}$=12.3%, $\mathcal{P}_{sp}$=43.5%



LAMINAR TRAVELING WAVES



Spanwise laminar flow can be used to study drag-reduced turbulent flow

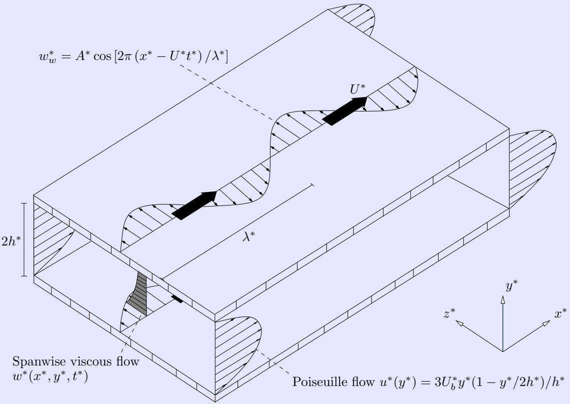
Space-averaged spanwise turbulent flow

Power spent by the waves

Scaling parameter

Stokes layer thickness

Modelling



We did some work on laminar wave flow $\rightarrow$ but Stokes layer was assumed **thin** $\delta \ll h$

But interesting flow features may occur when $\delta = \mathcal{O}(h)$

Thus we have carried out a full asymptotic and numerical study on laminar flow

$\rightarrow$ final objective is turbulent drag reduction modelling

To appear in J. Eng. Math.

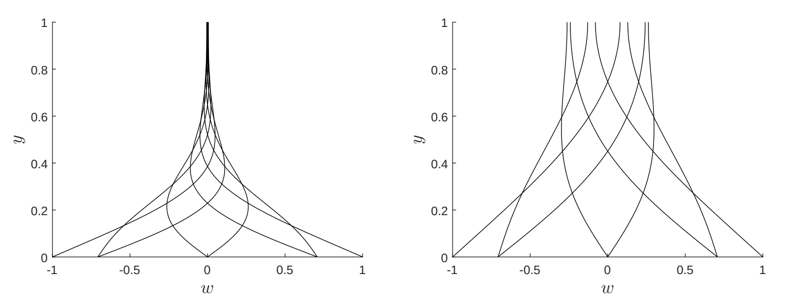
$$\underbrace{W''(y)}_{y\text{-diffusion}} + \Bigg(\underbrace{\frac{3\pi i R}{\lambda} y^2 - \frac{6\pi i R}{\lambda} y}_{\text{Poiseuille convection}} + \underbrace{\frac{2\pi i R U}{\lambda}}_{\text{wave convection}} \underbrace{- \frac{4\pi^2}{\lambda^2}}_{x\text{-diffusion}} \Bigg) Wy = 0,$$

$$W(0) = W(2) = 1$$

Parabolic cylinder function

$$w(x, y, t; R, U, \lambda) = \Re \left\{ \frac{\mathcal{D}_a[\phi(1-y)] + \mathcal{D}_a[\phi(y-1)]}{\mathcal{D}_a(\phi) + \mathcal{D}_a(-\phi)} e^{2\pi i (x-t)} \right\}$$

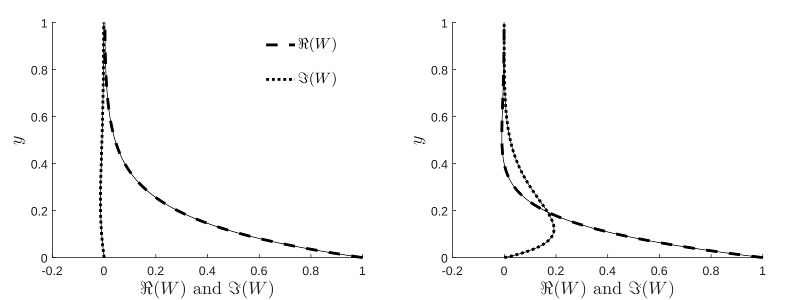
:-(But not that useful...



Steady waves

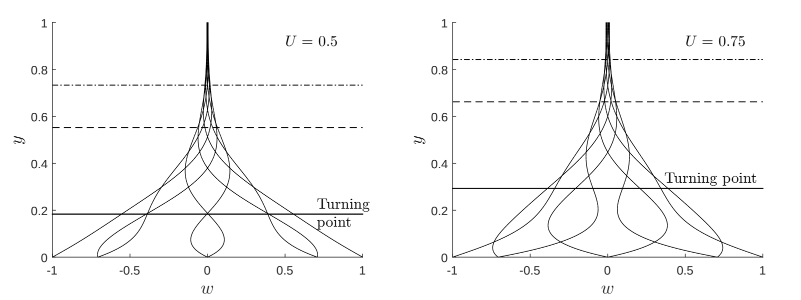
$$W(y) = \frac{P_0^{1/4}(1-\mathrm{e}^{-\psi})}{2P(y)^{1/4}\sinh\psi}\left(\frac{y-1+\alpha(y)}{\sqrt{b}-1}\right)^{\gamma(1-b)}\exp\left[\alpha(y)\gamma(1-y)-\gamma\sqrt{b}\right]+\frac{P_0^{1/4}(\mathrm{e}^{\psi}-1)}{2P(y)^{1/4}\sinh\psi}\left(\frac{\sqrt{b}-1}{y-1+\alpha(y)}\right)^{\gamma(1-b)}\exp\left[\alpha(y)\gamma(y-1)+\gamma\sqrt{b}\right],$$

WKBJ solution valid for $\delta = \mathcal{O}(h)$



WKBJ asymptotic solution and numerical solution match

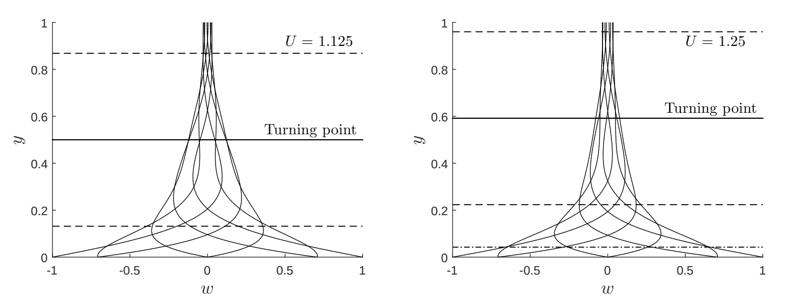
...but NOT valid for U_{waves} close to U_{bulk}



Traveling waves with U_{wave} comparable with U_{bulk}

Unexpected wiggly behaviour

...not a Stokes layer



Traveling waves with U_{wave} comparable with U_{bulk}

Unexpected wiggly behaviour

...not a Stokes layer

WKBJ composite solution valid for U_{waves} close to U_{bulk}

...but too complicated

Thanks to asymptotic analysis $\rightarrow$ wiggly behaviour occurs when

convection due to waves balances convection due to wave transport

$$W(y) = \left[-\frac{i\eta(y)}{\overline{P}(y)}\right]^{1/4} \left\{a_L \mathrm{Ai}\left[\epsilon^{-1/3}\eta(y)\right] + b_L \mathrm{Bi}\left[\epsilon^{-1/3}\eta(y)\right]\right\},$$

$$\eta(y) = \begin{cases} \left[\frac{3}{2}\sqrt{i}\kappa(y)\right]^{2/3}, & \text{for} \quad y > y_0, \\ -\left[-\frac{3}{2}\sqrt{i}\kappa(y)\right]^{2/3}, & \text{for} \quad y < y_0, \end{cases}$$

Asymptotic Langer-method solution for wiggly case

Next step: turbulence modeling...

WKBJ composite solution valid for U_{waves} close to U_{bulk}

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THANKS!