Department of Civil & Environmental Engineering
MSc courses in Advanced Structural Engineering

Standardizing the theoretical background of entrants

The Advanced Structural Engineering Cluster of MSc courses comprises the following titles:

- Concrete Structures,
- Earthquake Engineering,
- General Structural Engineering,
- Structural Steel Design.

These courses attract entrants with a considerable variety of first degree backgrounds. It is clearly desirable to establish an acceptable common level of attainment at the outset of the course, and this document tries to achieve this purpose.

It is hoped that, if such a clarification can be achieved, you will be reassured by the removal of any uncertainty concerning your own background, and the optimum use can be made of the time available on the course.

This document is not intended to be comprehensive but presumes a general background in structural engineering and therefore only discusses those topics that could conceivably have been omitted from any particular syllabus.

The topics are covered under various headings and each section gives suitable references by which a student who lacks a particular prerequisite may make good the omission. To quantify the standard required, where appropriate, a section is followed by a series of questions that you are encouraged to attempt. These satisfy the additional useful function of introducing the sign convention and notation used on the course.

Ahmer Wadee

Coordinator
Advanced Structural Engineering Cluster

ELASTIC MATERIAL & STRUCTURAL BEHAVIOUR

1. Isotropic stress/strain relationships (Hooke's law) in 1, 2 or 3 dimensions, Mohr's Circle.
2. Torsion of non-circular sections.
3. Computation of Strain Energy.
4. The stiffness and flexibility approaches to structural analysis.
   - The use of stiffness and flexibility matrix methods.
   - Familiarity with one method of analysis suitable for solution by hand.
5. Determination of deflections by virtual work.

References

- S. Timoshenko. Strength of Materials Part I, Chapter 2. Van Nostrand Reinhold.
- S. Timoshenko and J. Goodier, Theory of Elasticity, Chapter 10. McGraw-Hill. Torsion pp. 291–353.
- S. Timoshenko. Strength of Materials, Part I, Chapter 11.
- R. C. Coates, M. G. Coutie and F. K. Kong. Structural Analysis, Van Nostrand Reinhold, Chapters 4–8.
- See attached sheets.

Examples

1. An elastic layer is bonded between two perfectly rigid plates. The layer is compressed between the plates, the direct stress being σ_z. Supposing that the attachment to the plates prevents lateral strain ε_x and ε_y completely, find the apparent Young's modulus:

   $$\left(\frac{\sigma_z}{\varepsilon_z}\right)$$

   in terms of E and ν. Show that it is many times E if the material of the layer has a Poisson's ratio ν only slightly less than 0.5, e.g. rubber.

2. A closed circular thin-walled tube has a perimeter l and a uniform wall thickness δ. An open tube is made by making a longitudinal cut in it. Show that when the maximum shear stress is the same in both open and closed tubes.

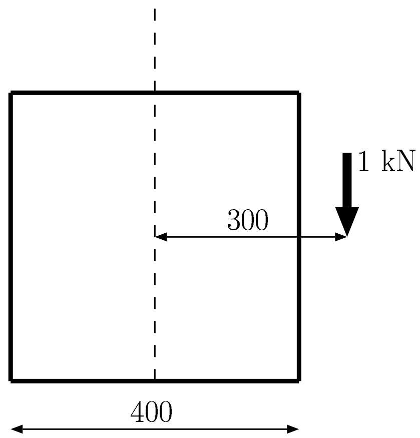
   $$\frac{T_{\text{open}}}{T_{\text{closed}}} = \frac{l\delta}{6A}, \quad \left(\frac{\mathrm{d}\theta}{\mathrm{d}x_{\text{open}}}\right) / \left(\frac{\mathrm{d}\theta}{\mathrm{d}x_{\text{closed}}}\right) = \frac{2A}{l\delta}, \quad \frac{K_{\text{open}}}{K_{\text{closed}}} = \frac{l^2\delta^2}{12A^2}$$

   where T is the applied torque, A is the area enclosed by the mid-thickness line of the tube wall, θ is the twist about the centre of twist and K is the torsional rigidity.

   Where are the centres of twist of the two sections?

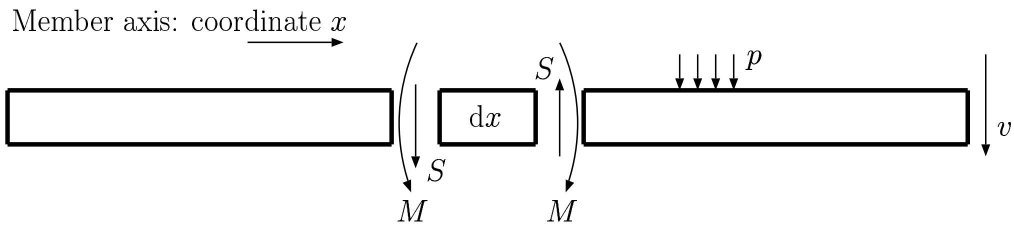
   What effect would it have on the ratios given above if the tube had a rectangular cross-section of sides $l/12$ and $5l/12$?

3. A box girder 400 mm square with a uniform wall thickness of $10\mathrm{mm}$ acts as a cantilever of length $2\ \mathrm{m}$. It is subject to an end load of $1\ \mathrm{kN}$ in the position shown below:



Determine the total strain energy in the cantilever. What proportion of the total deflection of the load is due to non-torsional shear strains and how would this proportion vary if the cantilever had a length of $1\ \mathrm{m}$?

4. The sign convention for bending of beams that is generally used in these courses is that the directions indicated are positive:



Displacement v, moment M, shear force S and load intensity p

Write the four successive first-order differential relations between the variables in accordance with this convention, for example

$$S = \frac{\mathrm{d}M}{\mathrm{d}x}$$

and the resulting fourth order equation relating to v to p.

(a) A cantilever beam has second moment of area

$$I_0 \frac{\pi x}{2L \sin(\pi x/2L)}$$

where x is measured from the free end (I_0 is thus the value at the free end), and is of length L. Derive an algebraic solution for deflection v (as a continuous function of x) caused by a point load P applied at the free end. Evaluate the deflection at the free end from this solution and also by an independent solution; the method of virtual work[1] is preferred.

(b) Derive solutions for bending moment and deflection in a simply-supported beam of uniform section and span L subject to a load of intensity.

$$p = p_0 \sin \frac{\pi x}{L}$$

where p_0 is the load intensity at mid-span. Evaluate the slope at the ends of the beam.

[1] see attached sheets "Application of virtual work to calculation of deflections".

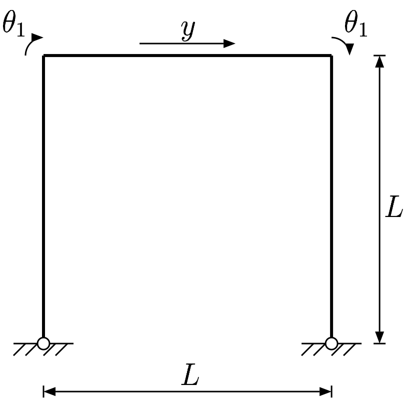
(c) Using the above result, evaluate the end moments in a fixed-ended beam of uniform section under the same loading.

(d) State the value of shear force arising in a uniform beam of length L with its ends fixed in direction when one end is given a transverse deflection Δ relative to the other end.

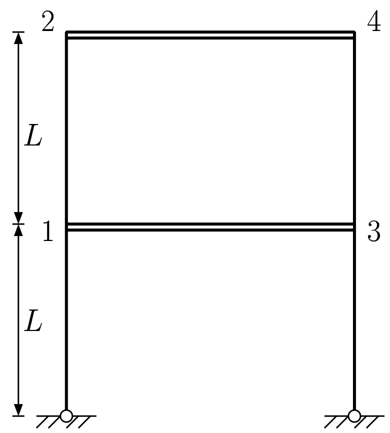
5. All members of the portal frames considered in this question are of uniform second moment of area and can be regarded as inextensible.

What is the physical interpretation of the matrix elements k_{ij} and f_{ij} defined in the sections that follow?

(a) Insert the values of the elements k_{ij} in the matrix equation $\mathbf{P} = \mathbf{K}\mathbf{y}$ where $\mathbf{y}$ is the vector of the nodal displacements $[y, \theta_1, \theta_2]$ and $\mathbf{P}$ is the vector of the corresponding external loads (namely clockwise moments at joints 1 and 2 and total horizontal force acting at roof level). The matrix $\mathbf{K}$ is the stiffness matrix of the structure. Hence find the value of the horizontal force p to give unit displacement if there is no constraint on joint rotation (i.e. externally applied bending moments are set to zero).

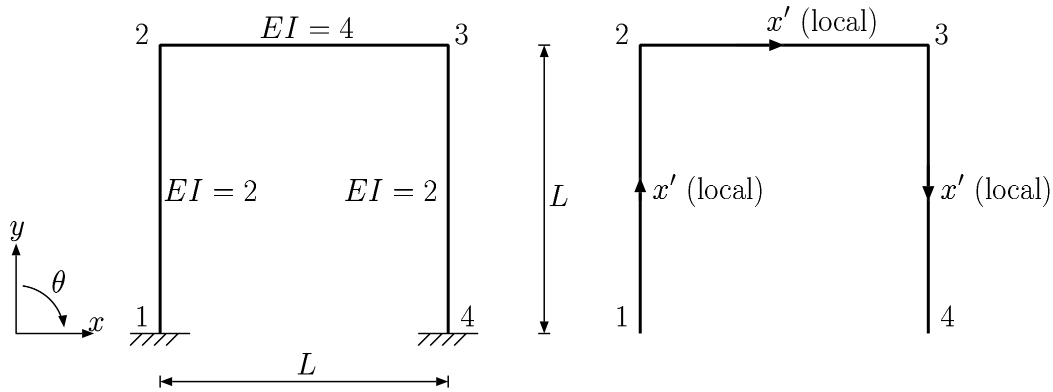


(b) The beams 1–3 and 2–4 are rigid, the remaining members of equal second moment area. Insert values of the elements k_{ij} in the equation $\mathbf{P} = \mathbf{K}\mathbf{y}$, where y is the vector of node translational displacements $[y_1, y_2]$ and $\mathbf{P}$ the corresponding horizontal forces.



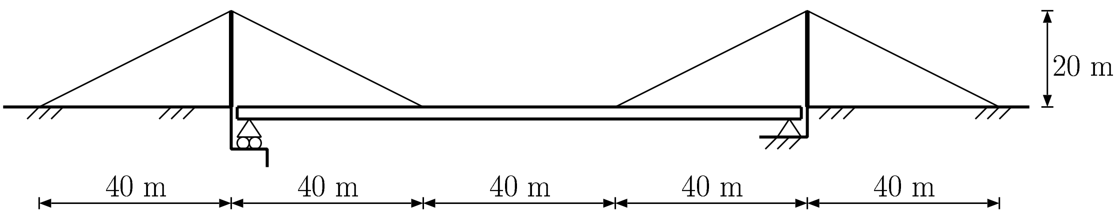
(c) Repeat question 5(b), but with beams 1–3 and 2–4 having the same length and second moment of area as the other members. Note that this can be solved by assembling the complete stiffness matrix including joint rotations and moments, followed by elimination of these terms by setting the external rotating constraints to zero as in question 2(a); but that it is likely to be much quicker to aim directly to the required result, for example by moment distribution. The student may find it helpful to confirm that he or she can use both methods, but this is not essential.

(d) Returning to the simpler structure defined in 5(b), evaluate the elements f_{ij} in the equation $\mathbf{y} = \mathbf{Fp}$ (do not obtain $\mathbf{F}$ by inversion of $\mathbf{K}$). The matrix $\mathbf{F}$ is the flexibility matrix of the structure. Confirm that $\mathbf{FK} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix.



6. The plane portal frame shown has 3 degrees of freedom $[\delta x, \delta y, \theta]$ at each of the 4 joints. Assemble the structure stiffness matrix $\mathbf{K}$ beginning with the beam stiffness matrices in terms of the local coordinate systems shown, and using transformation matrices $\mathbf{T}$ to transform from the local to the global coordinates. $\mathbf{K}$, the (12×12) matrix, is singular— why? Subsequently apply the boundary conditions of zero displacement at joints 1 and 4 to give a (6×6) stiffness matrix. Finally apply constraints of inextensibility of members to give a (3×3) matrix in terms of θ_2 and θ_3 and the horizontal displacement of the beam. [2]

7. Consider the structure shown: the towers are pinned at the base, and the dead load tension in the cables is sufficiently high that they can be regarded as straight tension members having $E = 160\ \mathrm{kN/mm^2}$. The area of the cable is $12 \times 10^3 \mathrm{mm}^2$ and the cross-section of the tower is sufficiently large to be regarded as relatively stiff. The deck is a steel box $4\ \mathrm{m}$ wide and $2.4\ \mathrm{m}$ deep, made of stiffened steel plate of effective thickness $24\mathrm{mm}$ ($16\mathrm{mm}$ plate plus allowance for stiffeners).



Evaluate the deflection at mid-span and at the points of attachment of the cables to the deck, due to a load of intensity

$$p = p_0 \sin \frac{\pi x}{L}$$

Where $p_0 = 1kN/m$, $L = 120m$, and x is the distance from the left-hand support.

[2]This aspect of stiffness is not a pre-requisite for the course—the question is included as a useful refresher for those familiar with the subject.

PLASTIC MATERIAL & STRUCTURAL BEHAVIOUR

1. Failure criteria for steel i.e. Von Mises & Tresca.
2. A knowledge of the elementary theory of plasticity as applied to beams and frames covering the following topics:
   - Plastic hinge assumption and the determination of the plastic moment to a section.
   - Calculation of plastic collapse load factor for continuous beams and simple frames.

References

- C. R. Calladine. Engineering Plasticity. Chapter 2. Pergamon.
- M. R. Horne. Plastic theory of Structures. Chapters 1–3. Nelson.
- B. G. Neal. The plastic methods of structural analysis. Chapters 1–4. Chapman & Hall, 1977.
- R. C. Coates, M. G. Coutie, F. K. Kong, Structural Analysis, Chapter 14, Van Norstrand Reinhold.

Examples

1. Demonstrate by algebraic manipulation the equivalence of the following formulae for the Mises yield condition in terms of the principal stresses:

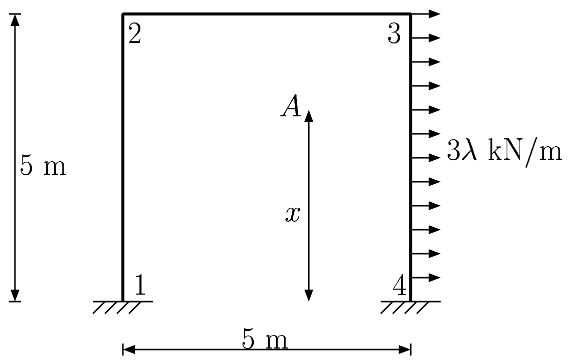
   $$-\left(\sigma_1'\sigma_2' + \sigma_2'\sigma_3' + \sigma_3'\sigma_1'\right) = \frac{1}{2}\left({\sigma_1'}^2 + {\sigma_2'}^2 + {\sigma_3'}^2\right) = k^2 = \sigma_{\mathrm{y}}^2/3.$$

   where $\sigma_0 = (\sigma_1 + \sigma_2 + \sigma_3)/3$ is the hydrostatic stress, $\sigma_1' = \sigma_1 - \sigma_0$, etc. are the deviatoric stress components, k is a constant, y is the uniaxial yield stress.

2. An I-section member is fabricated from two $400 \times 36\mathrm{mm}$ flange plates welded to a $1000mm \times 12\mathrm{mm}$ web plate. If the yield stress is $250\ \mathrm{N/mm^2}$, find the plastic moment for bending about the major axis.

   What would be the plastic moment if the bottom flange only was made of high tensile steel, with a yield stress of $360\ \mathrm{N/mm^2}$? Ans: 4480, 5091 kNm.

3. In the frame shown in Fig. 1 the plastic moment of each member is $12\ \mathrm{kNm}$. Assume first a collapse mechanism with plastic hinges at sections 1, 2, 3 and 4. Show that this gives $\lambda = 1.28$, and that in the corresponding bending moment distribution the maximum bending moment in the member 3–4 is of magnitude $15\ \mathrm{kNm}$ and occurs at the position A defined by $x = 3.75\ \mathrm{m}$.



   Now assume a mechanism with plastic hinges at sections 1, 2, 4 and A, retaining x as a variable. Show that in the actual collapse mechanism $x = 3.66\ \mathrm{m}$, and that $\lambda_c = 1.194$.

APPLICATION OF THE METHOD OF VIRTUAL WORK TO THE CALCULATION OF DEFLECTIONS

(See: Elastic Material & Structural Behaviour).

Consider a structure subject to various loadings, from which one can identify two "systems".

1. A system of external loads W_1, (see the comment headed "notation" that follows) which are in equilibrium with a set of internal bending moments M_1 within the structure. In general the VW equation is valid when applied to any set in equilibrium, not necessarily the actual set in a statically indeterminate structure; it is often convenient to construct the set M_1 by attributing arbitrary values (commonly zero) to a suitable number of quantities regarded as "the statical indeterminacies".
2. A system of curvatures (κ_2) of the elements (members) making up the structure, and the displacements (Δ_2) which are compatible with them—i.e. the displacements which would be found if the curvatures are integrated twice and account is taken of the compatibility conditions where elements join. In general, no statement is made about equilibrium in this system—for example, the bending moments corresponding to κ_2 (assuming behaviour in the linear range and knowing EI) in the members meeting at a node need not be in equilibrium with each other.

NOTATION

The shorthand W_1 (and so on) is used to identify the external load actions in System 1; there may be a number of points loads, point couples, and/or distributed loadings. The VW equation calls for summations or integrals of products of a quantity from System 1 with a quantity from System 2; common sense is sufficient for the pairing—for example, force times translational (i.e. linear) displacement, couple times rotational displacement; the summation of products for all the point actions is added to integrals of the distributed actions, etc. Similarly the shorthand M_1 covers all internal actions in System 1, and common sense and engineering judgment shows where axial forces, torques, shears have to be included. In System 2, κ_2 likewise covers the deformations of the structure corresponding to the relevant internal actions in System 2; axial strain, twist, etc., as well as curvatures.

EQUATION

The corresponding shorthand for the VW equation is

$$\sum W_1 \Delta_2 = \int M_1 \kappa_2 \, \mathrm{d}s$$

where the summation covers all external actions in System 1, and the $\mathrm{d}s$ covers integration along the axis of every member. All the actions mentioned above are deemed to be included; in particular, note that the right hand side covers all internal actions and judgment must be used to determine where the corresponding integral of axial force times strain (or summation over the members of axial force times linear extension of each member) must be included.

For a structure in the linear range one can write

$$\kappa_2 = \frac{M_2}{EI} \text{ whence } \sum W_1 \Delta_2 = \int \frac{M_1 M_2}{EI} \, \mathrm{d}s$$

To apply this equation to calculate the deflection under a given loading, consider the left hand side if System 1 is deliberately set up with a *single unit external load only, acting at the point where the deflection is required* (and in the required direction). Clearly the required deflection is given directly, as Δ_2; i.e. System 2 is defined as the real system of curvatures and deflections under consideration, and M_2 must be the real internal moments (etc. The latter requires proper solution of the compatibility conditions (whether by a stiffness or by a flexibility method) if the real structure is statistically indeterminate. System 1 is only restricted by the conditions already set out, however; thus the system M_1 can be arbitrarily set up in equilibrium to the load W_1 without regard to compatibility. Note that although "statical indeterminacies" can be arbitrarily set at will in this system (to zero or otherwise) it is not valid to postulate statically impossible values—for example, the moment must pass to zero at pin joints and satisfy overall conditions imposed by roller supports.

MATHEMATICS

1. Simple Matrix Algebra, including eigenvalues and eigenvectors.
2. Fourier Series
3. Partial Differentiation and Partial Differential Equations including 2nd and 4th order homogeneous and non-homogeneous equations.

References

- G. Stephenson. An introduction to matrices, sets and groups, Longmans, 1965. Chapters 2–5.
- L. A. Pipes. Applied Mathematics for Engineers and Physicists. McGraw-Hill, Chapter 4.
- L. A. Pipes. Applied Mathematics for Engineers and Physicists, McGraw-Hill, Chapter 3.
- G. Stephenson. Mathematical Methods for Science Students, Longmans. Chapter 15.
- L. A. Pipes. Applied Mathematics for Engineers and Physicists. McGraw-Hill. Chapters 11 and 17.
- G. Stephenson. Mathematical Methods for Science Students. Longmans. Chapters 9 and 24.

Examples

1. Determine the matrix M so that $AMB = C$ where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 1 & 1 \end{bmatrix}$$

2. Attempt to solve for x, y, z, the equations

$$-2x + y + z = a, \quad x - 2y + z = b, \quad x + y - 2z = c$$

   and explain, in terms of either vectors or geometry, the meaning of result you obtain.

3. Calculate the eigenvalues and eigenvectors of

$$\begin{bmatrix} 5 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

4. Find the Fourier series of the function $f(x)$ where

$$f(x) = \begin{cases} \cos x & \text{for } \theta = (-\pi, 0) \\ -\cos x & \text{for } \theta = (0, \pi) \end{cases}$$

   and deduce that

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2n-1}{(4n-1)(4n-3)} = \frac{\pi}{8\sqrt{2}}$$

5. Compute:

$$\frac{\partial^2 z}{\partial x^2}, \quad \frac{\partial^2 z}{\partial x \partial y}, \quad \frac{\partial^2 z}{\partial y \partial x}, \quad \frac{\partial^2 z}{\partial y^2},$$

   For each of the following functions and hence show that

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$$

(a) $z = \left(x^2 + y^2\right)^{1/2}$

(b) $z = x \cos y + \cos x$

(c) $z = \arctan(y/x)$

6. Find the connection between the constants w and α so that

$$v = Ae^{\alpha x} \sin(wt + \alpha x) + Be^{-\alpha x} \sin(wt - \alpha x)$$

may be a solution of

$$\frac{\partial^2 v}{\partial x^2} = 2\frac{\partial v}{\partial t}$$

Find A and B when $v = 2 \sin t$ for $x = 0$, given that $\alpha > 0$, and $v \to 0$ as $x \to \infty$.