Adaptive Mesh Refinement for Seismic Wave Simulation in Devito

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Background

Devito is an embedded domain-specific language for high-performance finite-difference PDE solvers, originally developed for industrial-scale seismic applications such as full waveform inversion (FWI) and reverse time migration (RTM) (Louboutin *et al.*, 2019). In these workflows, wave equations are solved on large, structured Cartesian grids, often at very fine resolution to cope with strong velocity contrasts, complex salt bodies, and near-surface heterogeneity. Using such a fine grid everywhere is straightforward but extremely expensive: many regions of a typical seismic model are smooth and could be treated accurately on a much coarser mesh.

Adaptive mesh refinement (AMR) offers a way to concentrate resolution only where it is needed, for example in regions of short wavelength, strong velocity gradients or complex scattering. In the finite element world, and particularly in work built around the Firedrake ecosystem, there has been substantial progress on anisotropic and goal-oriented mesh adaptivity for geophysical flows and ice dynamics, using metric-based refinement driven by adjoint error indicators (Wallwork *et al.*, 2020; Dundovic *et al.*, 2025). These approaches are implemented as extension libraries and toolchains that interface with Firedrake. A key ingredient in such dynamically adapted simulations is the ability to transfer solution fields between successive meshes without introducing spurious imbalance or noise; for instance, Maddison *et al.* (2011) show how carefully designed mesh-to-mesh interpolation can preserve geostrophic balance exactly in adaptive shallow-water models.

However, AMR remains much less developed in high-order finite-difference frameworks, where the focus has been on optimising regular-grid stencil operators and adjoints. Bringing AMR ideas from unstructured finite elements into a structured-grid DSL raises new questions about how to design refinement strategies, interface stencils and Galerkin-like” projection operators that interface seamlessly with Devito s high-level symbolic interface and code generation framework whilst conserving the stability properties of wave-equation solvers.

Project overview

This project aims to design and analyse adaptive mesh refinement for high-order finite-difference wave simulations in Devito, with seismic imaging (FWI and RTM) as the primary application but with methods that can generalise to other wave and diffusion problems.

You will begin by characterising when and where high resolution is actually needed in seismic models. Using canonical 2D and 3D benchmarks, you will explore refinement indicators based on local wavelength, heterogeneity in the velocity model, and possibly adjoint-based measures linked to the imaging objective, taking inspiration from metric-based strategies developed around Firedrake for shallow-water and ice-flow problems (Wallwork *et al.*, 2020; Dundovic *et al.*, 2025). The goal is to define simple, robust criteria that decide when a region of the domain should be refined or coarsened in a way that fits naturally with Devito s structured-grid setting.

A central scientific challenge is the transfer of wavefields and medium parameters between meshes as they adapt in time. The project will draw on ideas from Galerkin projection and balance-preserving interpolation in finite elements to design and study projection and restriction operators for finite-difference grids, aiming to minimise artificial reflections at refinement interfaces and preserve key properties of the wavefield as far as possible (Maddison *et al.*, 2011). You will analyse how these choices affect stability, phase accuracy and the behaviour of imaging functionals.

You will then evaluate the impact of adaptive meshes on realistic FWI/RTM test problems, comparing AMR-based simulations against high-resolution uniform-grid runs to quantify potential savings in runtime and memory for a given imaging quality, and to understand the trade-offs between refinement criteria, numerical error and computational cost. Although seismic imaging is the primary motivating case, the resulting AMR strategies should be generic enough to apply to other Devito applications, such as medical ultrasound, electromagnetic waves or fluid flows, that is, wherever multi-scale structure makes uniform meshes inefficient.

Impact

By adding AMR to Devito, this project aims to make high-fidelity seismic wave simulation substantially more efficient and flexible, directly benefiting industrial-scale FWI and RTM workflows that currently rely on uniformly fine meshes. Concentrating resolution where the physics demands it can lead to large reductions in computational cost and energy use, potentially making more ambitious imaging problems tractable within existing hardware budgets, and reducing environmental impact.

At the same time, the project will provide a concrete case study of how AMR concepts, metric-based refinement and structure-preserving mesh-to-mesh transfer, well established in the finite element literature, can be reinterpreted for a structured finite-difference DSL. The resulting methodology and software will be of interest not only to the Devito and seismic communities, but also to researchers working on other wave-based simulations who wish to combine the simplicity and performance of structured grids with the flexibility of adaptive resolution.

References

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