Reduced-Order and Sparse Reconstruction Methods for Cardiac Ultrasound FWI

Supervisor: Dr Rhodri Nelson, Department of Earth Science and Engineering, Imperial College London
Co-supervisors: Prof Gerard Gorman, Department of Earth Science and Engineering, Imperial College London; clinical and imaging partners TBC

Keywords: full waveform inversion (FWI), adaptive waveform inversion (AWI), ultrasound imaging, cardiac imaging, digital twins, uncertainty quantification, high-performance computing, inverse problems

---

Background

Clinical cardiac imaging currently relies on a combination of CT, cardiovascular magnetic resonance (CMR) and echocardiography, each with different trade-offs in resolution, contrast, portability, safety and cost. Patients with complex disease often require multiple tests across modalities to obtain a full picture of anatomy and function, which is inconvenient, expensive and not always feasible (for example, in the presence of pacemakers or metallic implants).

Full waveform inversion (FWI) offers a different route to medical imaging. Originally developed in geophysics, FWI reconstructs quantitative maps of acoustic properties by fitting the full recorded ultrasound wavefield with numerical solutions of the wave equation, rather than relying on ray-based or beamformed approximations. Recent work has shown that low-frequency ultrasound FWI can recover high-resolution, three-dimensional images of the human brain through the skull, overcoming strong scattering and heterogeneity using accurate physics and adaptive waveform inversion (Guasch *et al.*, 2020). This suggests that similar techniques could eventually be applied to the heart and torso.

In parallel, there is growing interest in "digital twins" and generative-AI–accelerated inversion and uncertainty quantification, where physics-based simulations are combined with probabilistic inference to track the evolving state of a system (Herrmann, 2023; Yin *et al.*, 2025). For cardiac imaging, this opens up the possibility of not only producing a single best image, but also quantifying uncertainty and integrating imaging with personalised computational models of the torso and heart.

Project overview

The aim of this project is to develop and test full waveform inversion methods for ultrasound imaging of the heart, with a particular focus on reduced-order and sparse reconstruction techniques. The long-term goal is to create a modality that combines key attributes of CT, CMR and echo within a single examination, potentially reducing the need for multiple tests while improving quantitative image quality.

You will start from open-access anatomical and acoustic models of the torso to build realistic numerical "digital phantoms" and generate synthetic ultrasound data under different acquisition scenarios, such as limited transducer apertures, low frequencies and the presence of a coarse CT-derived prior (Guasch *et al.*, 2020). On this synthetic test bed, you will formulate and implement three-dimensional time-domain (or frequency-domain) FWI and AWI workflows for cardiac imaging, investigate how to design robust objective functions and regularisation, and study their sensitivity to noise, modelling errors and uncertain priors.

A central theme will be the use of reduced-order and sparse representations to make these inversions practical. You will explore ideas inspired by recent "WISER-like" variational inference frameworks, where generative models approximate the posterior over torso and cardiac properties and are then refined with a limited number of wave-equation solves (Herrmann, 2023; Yin *et al.*, 2025). The aim is to obtain computationally feasible uncertainty estimates and sparse, interpretable reconstructions rather than just a single best-fit image.

Using national and Imperial supercomputing facilities, you will quantify the cost of high-fidelity 3D inversion and explore routes to deployment, from dedicated in-hospital clusters to cloud-based services. Within the modelling framework, you will test strategies for placing transducers around the torso, compare reflection-based approaches such as reverse time migration with transmission FWI, and assess the potential of low-frequency or novel sensor designs to see through the lungs and ribs while retaining clinically useful resolution. Over time, the same framework could be adapted towards more ambitious concepts such as continuous monitoring or wearable devices with a 5G uplink, if the physics and computational constraints look favourable.

Impact

If successful, this project would take a significant step towards a new, quantitative ultrasound-based modality for cardiac imaging, capable of delivering CT- or CMR-like structural information using safe, portable ultrasound hardware and advanced inversion algorithms. It would provide a rigorous numerical assessment of what FWI and related methods can realistically deliver for the heart, clarify the trade-offs between acquisition design, model complexity, sparsity assumptions and computational cost, and establish a path towards clinical pilot studies.

Beyond the specific cardiac application, the project will contribute to the broader development of FWI-based medical imaging, reduced-order and sparse inverse methods, and generative-AI–accelerated uncertainty quantification, with potential impact across neurology, cardiology and other areas where fast, bedside, high-quality imaging could transform patient care.

References

Guasch, L., Calderón Agudo, O., Tang, M.-X., Nachev, P. and Warner, M. (2020) Full-waveform inversion imaging of the human brain. *npj Digital Medicine*, 3, 28. https://doi.org/10.1038/s41746-020-0240-8

Herrmann, F.J. (2023) Digital twins in the era of generative AI. *The Leading Edge*, 42(11), 730–732. https://doi.org/10.1190/tle42110730.1

Yin, Z., Orozco, R. and Herrmann, F.J. (2025) WISER: Multimodal variational inference for full-waveform inversion without dimensionality reduction. *Geophysics*, 90(2), A1–A7. https://doi.org/10.1190/geo2024-0483.1