Mixed-Precision Algorithms for High-Performance Physics Simulations

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Background

Modern CPUs and GPUs now deliver dramatically higher throughput for low-precision arithmetic (FP32, FP16, BF16, FP8) than for traditional FP64, driven largely by machine-learning workloads. Mixed-precision algorithms, where different parts of a computation use different numeric precisions, offer a way to tap into this performance while still achieving full-precision" scientific accuracy. This strategy has been studied extensively in numerical linear algebra, where mixed-precision iterative refinement and related techniques can give large speedups without sacrificing stability (Higham and Mary, 2022).

Recent surveys argue that similar opportunities exist across many areas of scientific computing, but that domain-specific insight is needed to decide where precision can be safely reduced (Kashi *et al.*, 2024). For PDE solvers and wave propagation, the picture is starting to emerge: mixed-precision time-stepping has been explored in high-order discontinuous Galerkin methods for hyperbolic systems, showing that some kernels can run in very low precision while others must remain in higher precision to preserve convergence and stability (Marot-Lassauzaie and Bader, 2025). In seismic exploration, half-precision modelling and inversion studies indicate that carefully scaled acoustic wave solvers can run in FP16 with acceptable error, offering substantial performance gains (Fabien-Ouellet, 2020). Kahan summation has been demonstrated as another applicable technique for accurate wave propagation with FP16 representation (Gao and Harms, 2025).

Devito is a domain-specific language and compiler for finite-difference PDE solvers, widely used for large-scale wave-equation modelling in seismic imaging and related applications (Louboutin *et al.*, 2019). Its symbolic Python front-end and automated code generation make it possible to rapidly experiment with different discretisations and optimisation strategies. This project will leverage Devito s high-level representation of the wave equation to explore where, in a realistic wave-propagation pipeline, reduced precision can be used safely, and how compiler-driven transformations might eventually help automate such choices. Tools like Herbie, which automatically rewrite floating-point expressions to reduce rounding error (Panchekha *et al.*, 2015), provide further inspiration for combining program analysis with domain knowledge.

Project overview

The core aim of this project is to understand how far mixed precision can be pushed in finite-difference wave-equation solvers expressed in Devito without compromising the accuracy and robustness required for applications such as full-waveform inversion (FWI) and reverse-time migration (RTM). The central scientific question is where high precision is genuinely needed in the wave-propagation and imaging pipeline, and where reduced precision can safely be used to save time and memory while keeping errors under control.

You will construct a set of well-controlled acoustic (and possibly elastic) wave-equation test problems in Devito, ranging from simple homogeneous and layered media to strongly heterogeneous and realistic seismic models. These will be used to map out how different patterns of precision choice affect phase and amplitude errors, numerical stability, and ultimately misfits and imaging or inversion objectives (Fabien-Ouellet, 2020). The emphasis is on developing quantitative error and robustness measures and using them to build precision maps" for representative workloads, rather than on any single discretisation tweak.

Guided by ideas from mixed-precision numerical linear algebra and high-order hyperbolic schemes (Higham and Mary, 2022; Marot-Lassauzaie and Bader, 2025), you will identify the main algorithmic components of Devito-based wave solvers, such as stencil updates, boundary treatments, imaging conditions and gradient calculations, and classify them according to their sensitivity to reduced precision. The goal is to develop simple models and numerical experiments that reveal which parts of the computation can be safely demoted to lower precision, which require higher-precision accumulation, and how scaling or reformulation can extend the useful range of low-precision formats.

A complementary strand of the project explores how high-level program analysis might support these decisions. Devito s symbolic representation of PDE operators exposes the underlying floating-point expressions, and tools such as Herbie, which automatically rewrite expressions to reduce round-off sensitivity, offer one way to make individual kernels more tolerant of reduced precision. The aim here is not to build a fully automatic precision-tuning compiler, but to take concrete steps towards combining symbolic PDE descriptions, error-aware expression analysis and mixed-precision experimentation in a coherent workflow.

Throughout, you will evaluate promising mixed-precision strategies on modern CPU and GPU hardware, balancing performance gains against accuracy and robustness. The intended outcome is a set of principled guidelines, grounded in both analysis and numerical evidence, for when and how mixed precision can be safely employed in wave-equation modelling and imaging, with clear potential for transfer to related applications such as medical ultrasound or electromagnetic wave propagation.

Impact

Forward and adjoint wave-equation solvers are the main computational bottleneck in seismic FWI and RTM. Even modest speedups translate directly into shorter turnaround times or more ambitious surveys. Furthermore, with increased real estate on modern architectures dedicated to reduced-precision operations, leveraging mixed precision where possible is essential to efficiently can cost-effectively harness the hardware. By systematically exploring mixed-precision strategies in a realistic, production-oriented framework like Devito, this project has the potential to deliver substantial reductions in runtime and memory use for industrial imaging workloads, while maintaining scientifically acceptable accuracy.

At a broader level, the project will help bridge the gap between abstract mixed-precision theory, largely developed in the context of linear algebra (Higham and Mary, 2022; Kashi *et al.*, 2024), and the realities of PDE-based simulation codes. It will demonstrate how a symbolic DSL can act as a laboratory for precision-aware algorithm design, and how ideas from floating-point program analysis can inform domain-specific decisions about where to spend" or save" precision. The resulting methods, benchmarks and guidelines will be of interest to the Devito community, to researchers working on wave-equation imaging, and to the wider scientific-computing community seeking principled ways to exploit emerging low-precision hardware for complex PDE solvers.

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