A Thermodynamically-based Network Model for Intermittency during Multiphase Flow in Porous Media

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Darcy’s Law

Multiphase Darcy’s law:

\[ \bar{u}_i = \frac{K k_i^f}{\mu_i} (\Delta p_i - \rho_i g) \]

- Fluid flow through porous media is modelled using the two-phase extension of the Darcy’s law.
- There is a linear relationship between pressure gradient and flow velocity.
- Interface between fluid phases at a fixed saturation is invariant.
- The flow of a phase through the porous media occurs only through an established flow pathway.
However, recent experimental studies in relation to nonlinear intermittent flow behaviours during multiphase fluid flow in porous media established the following:

- At fixed average fluid saturations, the arrangement of fluid phases is dynamic (Tallakstad et al., 2009a & 2009b).

- There is a transition from a linear flow regime to a nonlinear flow regime as flow rate increases (Spurin et al., 2019; Gao et al., 2020).

- Relationship between capillary number and pressure gradient at certain range of flow rates becomes nonlinear (Zhang et al., 2021).
Fluid Intermittency

- Non-linear relationship between pressure gradient and capillary number is attributed to fluid intermittency.

- This is the periodic disconnecting and reconnecting of the non-wetting phase along flow pathways.

- Disconnection of the non-wetting phase occurs after a series of snap-off events along the flow pathways.

- Fluid intermittency is caused by the nonwetting phase periodically finding more conductive pathways through the pore space.

- The interplay of viscous and capillary forces largely determines the occurrence of intermittent flow.

- Intermittent pathway flow is controlled by:
  - Capillary number
  - Viscosity ratio
  - Pore geometry
  - Wettability
An analogy between thermodynamics and immiscible fluid flow in porous media could be used to study nonlinear flow behaviours (Hansen et al., 2022).

Insights will be taken from the thermodynamic formulation of multiphase flow proposed by Hansen and colleagues.

A new traditional quasi-static pore-scale network model will be first developed.

Modification to a probabilistic dynamic pore-scale network model will then be done.

\[ P_{filling} = z \exp\left(-\frac{\Delta E}{c}\right) \]

\[ \Delta E_{Drainage} = P^D_c - P^*_c \]

\[ \Delta E_{Imb} = P^*_c - P^I_c \]

\[ z = \frac{1}{\exp\left(-\frac{P^*_c - P^I_c}{c}\right) + \exp\left(-\frac{P^D_c - P^*_c}{c}\right)} \]
Capillary-dominated displacement (Drainage)

Circular:

\[ P_c = \frac{2\sigma \cos \theta_r}{r} \]

Angular:

\[ P_c = \frac{\sigma(1 + 2\sqrt{\pi G}) \cos \theta_r F_d(\theta_r, G)}{r} \]

\[ F_d(\theta_r, G) = \frac{1 + \sqrt{1 + 4GD/cos^2\theta_r}}{1 + 2\sqrt{\pi G}} \]

Capillary-dominated displacement (Imbibition)

Piston-like:

\[ P_c = \frac{\sigma(1 + 2\sqrt{\pi G}) \cos \theta_A F_d(\theta_A, G)}{r} \equiv \frac{\sigma \cos \theta_A}{r} C_{It} \]

Snap-off:

\[ P_c = \frac{\sigma \cos \theta_A}{r} (1 - \tan \theta_A \tan \beta) \]

Pore-body filling:

\[ P_c(I_n) = \frac{2\sigma \cos \theta_A}{r_p} - \sigma \sum_{i=1}^{n} b_i x_i \]
In traditional pore-scale network model, all elements either have a probability of 0 (not filled) or 1 (filled).

In this proposed model, area occupied by each phase and the conductance of each phase in each element will depend on the probability of filling.

In this proposed model, the probability is $[0, 1]$. The model should agree with the traditional model where there is no intermittency.

\[
A_w = P_{\text{filling}} \times A_{w,\text{max}} + (1 - P_{\text{filling}}) \times A_{w,\text{min}}
\]

\[
A_{nw} = (1 - P_{\text{filling}}) \times A_{nw,\text{max}} + P_{\text{filling}} \times A_{nw,\text{min}}
\]

\[
 g_w = P_{\text{filling}} \times g_{w,\text{max}} + (1 - P_{\text{filling}}) \times g_{w,\text{min}}
\]

\[
 g_{nw} = (1 - P_{\text{filling}}) \times g_{nw,\text{max}} + P_{\text{filling}} \times g_{nw,\text{min}}
\]
Quasi-static Network Model

Major milestones

Pore network extraction from x-ray images

Single phase flow computations

Two phase Drainage Simulation

Two phase Imbibition Simulation

Single phase computations

Element area

Conductance per unit length

Pressure distribution

Absolute permeability

Two phase computations

Half angles

Capillary pressure

Filling

Phase area

Conductance per unit length

Pressure distribution

Relative permeability
Primary drainage results

Secondary imbibition results

Simulation results
quasi-static network model
Simulation results

Probability filling when $c = 250$ Pa

Plots of $P_c$, $k_r$ and $f_w$ against $S_w$

Probability distribution for filling at the different values of $P_c^*$
Simulation results

Probability filling when $c = 500$ Pa

Plots of $P_c$, $k_r$ and $f_w$ against $S_w$

Probability distribution for filling at the different values of $P_c^*$
Plots of $P_c$, $k_r$, and $f_w$ against $S_w$.

Simulation results

Probability filling when $c = 1000$ Pa
Probability distribution for filling at the different values of $P_c^*$

Simulation results
Probability filling when $c = 2000$ Pa
Simulation results

Probability filling when $c = 3000$ Pa

Plots of $P_c$, $k_r$ and $f_w$ against $S_w$
Simulation results
Probability filling when $c = 5000$ Pa

Probability distribution for filling at the different values of $P_c^*$

Plots of $P_c$, $k_r$ and $f_w$ against $S_w$
Probability distribution for filling at the different values of $P_c^*$

Simulation results

Probability filling when $c = 7500$ Pa

Plots of $P_c$, $k_r$ and $f_w$ against $S_w$
Probability distribution for filling at the different values of $P_c^*$

Simulation results
Probability filling when $c = 10000 \text{ Pa}$

Plots of $P_c$, $k_r$ and $f_w$ against $S_w$
Probability distribution for filling at the different values of $P_c^*$

Simulation results

Probability filling when $c = 15000$ Pa

Plots of $P_c$, $k_r$ and $f_w$ against $S_w$
Simulation results
Probability filling when $c = 20000 \text{ Pa}$

Plots of $P_c$, $k_r$ and $f_w$ against $S_w$

Probability distribution for filling at the different values of $P_c^*$


