Growing waves in drifted plasmas

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Abstract

Self-consistent wave solutions for structures consisting of drifting plasmas and dielectrics are presented and the appearance of growing waves in one of the models is shown. The role of collisions and diffusion is discussed and the direction of future work aimed at devices is indicated. The requirement for velocity matching implies that eventually periodic structures and space harmonics are needed so that these devices will form a new type of amplifying metamaterials.

1. Introduction

Plasmas are a big subject: there have been innumerable papers devoted to their multifarious properties. It is mostly the hunting ground of physicists with occasional intrusion by engineers. When it comes to drifting plasmas it is engineers who are more interested. Drifting plasmas gave rise to a number of devices in the 1950s and they hold the promise of new devices in the still unexplored THz region.

A plasma is a set of charged particles. In the simplest one-dimensional case their dispersion equation is $\omega^2 = \omega_p^2$ where ω and ω_p are the frequency and plasma frequency respectively. How will the dispersion equation modify when this plasma is bodily moved with a velocity, v_0? We can find that easily by introducing the concept of Doppler shift, i.e. ω is to be replaced by $\omega - kv_0$ where k is the propagation constant. The corresponding dispersion equation is $\omega = kv_0 \pm \omega_p$. The two solutions are known as the fast and slow space charge waves. The first device application was in Travelling Wave Tubes where amplification was achieved by the interaction of the slow space charge wave with a slow electromagnetic wave (see e.g. [1]). In most devices the slow electromagnetic wave was produced by a helix [2] which slowed down the electromagnetic wave simply because it had to follow the longer path along the helix. A slow electromagnetic wave of velocity v can be represented by the dispersion equation $\omega = kv$. The waves interact via their longitudinal electric fields. The frequency range in which the interaction occurs is in the vicinity of the point where the two dispersion characteristics intersect. The result is a gap both in frequency and in propagation constant leading to growing waves. The mathematical solution was always obtained by deriving a coupling term between the two waves in a heuristic manner [1]. The first aim of our research is to find the growing waves in self-consistent models without the need of any heuristic coupling terms. We have though a number of other aims as well. Using our models we wish to investigate the interaction in regimes in which both collisions and the diffusion of the carriers are important. There was a time when such possibility seemed very unlikely that is until the advent of a new device in the 1960s, the acoustic wave amplifier [3]. Acoustic waves were amplified in a piezoelectric crystal (it was CdS although the effect was found in a few other crystals as well) by interaction with drifting carriers (the drift was achieved by applying a pulsed d.c. voltage). Amplification occurred when the drift velocity of the charge carriers exceeded the sound velocity in the solid. This device offered a clear indication that growing wave interactions are still possible in a collision dominated regime. The next logical step, taken in 1966, was [4] to design a Solid State Travelling Wave Amplifier to work by the interaction of drifting carriers with electromagnetic fields provided by a meander line. However due to the nearly-simultaneous appearance of the Gunn diode [5] there was little motivation to continue the research in that direction. The idea did not die: there have been one or two papers per year on the subject ever since. For a recent one see e.g. Ref. 6 concerned with another type of travelling wave interaction (two-stream instability). We believe there are strong arguments in favour of resurrecting and intensifying the research on travelling wave interactions in solids with a view to their applications in THz devices. There are many ways at the moment to generate THz waves but none of them satisfactory. A simple, inexpensive device would make possible a series of applications. The chances of realising such a device are much better than it was half a century ago due to the advances in manufacturing small features and to the ease with which complex mathematical solutions can nowadays be obtained. We shall briefly discuss various models in the present paper, and show the presence of growing waves.

2. Models and mathematical formulation

In the models considered we shall have two types of media and assume that wave propagation is in the *z* direction. Medium 1 is a lossless dielectric and medium 2 is capable of providing drifting charge carriers. The simplest model is shown in Fig. 1a. Both media are infinite in the *z* and *y* directions and semi-infinite in the *x* direction. In the second model (Fig. 1b) medium 2 is surrounded on both sides by medium 1. In the third model (Fig. 1c) medium 1 is bounded by a perfect electric conductor. In all three cases TM waves are assumed which have only E_x, E_z and H_y components. Our three models turn actually into six models considering the presence or absence of an applied spatially constant and infinitely large longitudinal magnetic field. Although this assumption is not realistic it may be approximated in practice, and besides it was often used in the theory of microwave tubes in order to simplify the mathematics. The simplification comes from the fact that in the presence of such magnetic field the transverse motion of the charge carriers is prohibited.

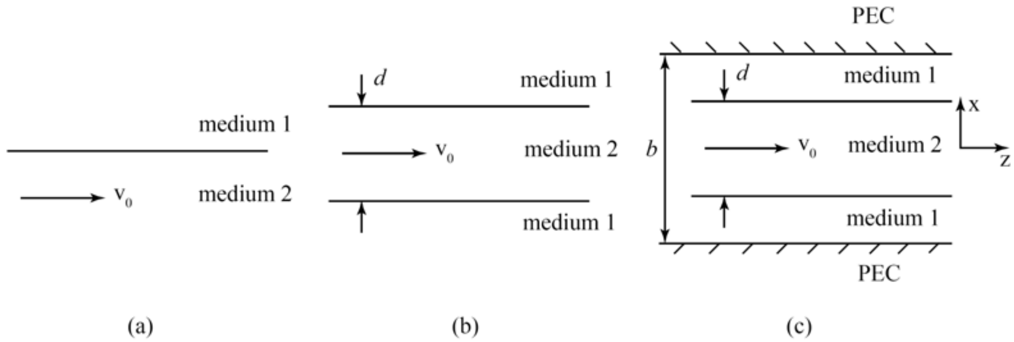

Fig. 1: Models.

We need to solve Maxwell's equations in conjunction with the equation of motion subject to the boundary conditions. There is no difficulty in writing up Maxwell's equations for the three field components but we need to dwell on the accompanying equation of motion for the electrons. The usual formulation is in Lagrangian terms, i.e. it is concerned with the position of a single charge carrier as it varies as a function of time. It needs to be transformed into Eulerian terms, i.e. when the velocity of an electron is given at point *z* at a time *t*. The resultant nonlinear equation can then be linearized by assuming that the harmonically varying velocity is much smaller than the drift velocity. The left-hand-side of the equation of motion contains, besides the acceleration term, a collision term, and a pressure term, the latter depending on the spatial rate of change of the charge density. The boundary conditions are trivial on the perfect electric conductor but less so on the boundary between Medium 1 and Medium 2 when transverse motion of the charge carriers is permitted. In that case both surface charges and surface currents must be taken into account. For all six models we end up with a set of linear partial differential equations which can be reduced to dispersion equations by the assumption of a wave solution in the form exp[j(ω*t*-*kz*)].

3. Results

We show below the solution for only one of our models (Model 3 with infinite magnetic field in the *z* direction) for the lossless case when the dielectric constant of the dielectric is 1000, the velocity of the charge carriers is *c*/10 (where *c* is the velocity of light), the plasma frequency is f_p = 1 THz, $k_p b$ = 0.2, $k_p d$ = 0.1, $k_p = 2\pi f_p / c$ where *b* and *d* are dimensions in Fig. 1b and c. The transverse field distribution is harmonic in the plasma; in the dielectric it may be harmonic or hyperbolic depending on the variables. The dispersion curve for the plasma with zero drift velocity is shown in Fig.2a in terms of normalized coordinates, f/f_p plotted against k/k_p. It differs from the usual surface plasma dispersion curves partly because there are waveguide modes present (due to the confinement of the fields by the perfect electric conductor) and partly because the harmonic solution allows a large number of plasma waves which cannot be resolved on the scale used. As the drift velocity increases the plasma curves approximately rotate (Fig. 2b). When the drift velocity becomes higher than the velocity of the waveguide mode then the individual dispersion curves intersect resulting in the tell-tale gap in frequency and propagation constant giving rise to growing waves.

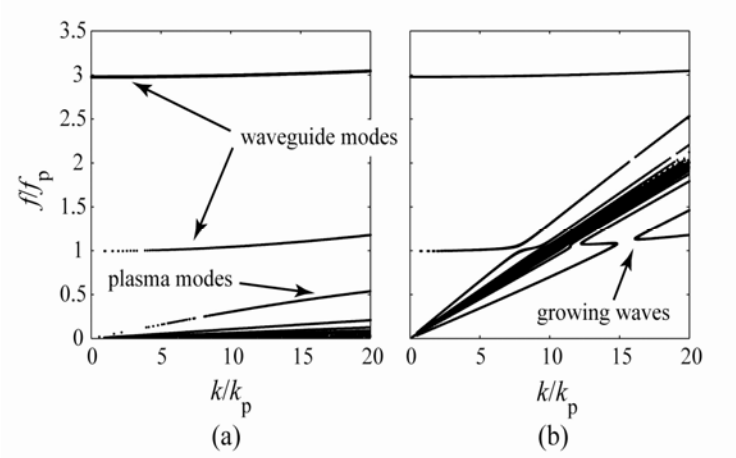

Fig. 2: Dispersion equations and growing waves. (a) v_0=0. (b) v_0=c/10.

4. Conclusions and future work

We have six models and an enormous range of possible parameters. In the present paper we have only scratched the surface of the problem. We have shown that growing waves appear in a self-consistent model. We intend to investigate in detail all six models for the lossless case and show the conditions under which growing waves appear. The higher is the rate of growth the more likely is that net gain can still be achieved in the presence of collisions and diffusion (as in the acoustic amplifier). Having found conditions for growth we shall need to investigate how those conditions can be satisfied with existing materials. The maximum carrier velocity that can be achieved in a solid is probably about *c*/200 in InSb. The high dielectric constant of 1000 assumed in our example cannot be achieved in practice. The practical solution must be a periodic structure in which the interaction can be achieved via higher spatial harmonics. And that combination of drifting plasmas with periodic structures places the set of problems discussed here firmly into the realm of metamaterials.

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