THz Letters

Derivation of Rectangular Metallic Waveguide Attenuation Constant for IEC 60153-2 and IEEE 1785.1 International Standards

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***Abstract*—The IEC 60153-2 and IEEE 1785.1 international standards for rectangular metallic waveguides are currently undergoing revision. Textbook derivations of the associated equations for the metal-pipe rectangular waveguide attenuation constant to be given in the revised standards are presented. These derivations provide important mathematical traceability for these equations, which has previously not been proven in the open literature for these standards. Furthermore, the historical approximations used in the original equations are identified and now corrected here. Results using the newly derived equations are demonstrated, with comparison to experimental results in the WM-380 (500 GHz to 750 GHz) band.**

***Index Terms*—Attenuation constant, derivation, international standards, loss, rectangular waveguide.**

I. Introduction

INTERNATIONAL standards provide technical user communities with a common set of specifications relevant to their field. This facilitates compatibility and interoperability between systems for the benefit of local and transnational cooperation and global trade.

For air-filled metal-pipe rectangular waveguides (hereby referred to as waveguides), both the International Electrotechnical Commission (IEC) 60153-2 [1] and IEEE 1785.1 [2] represent the current international standards, giving specifications for aperture sizes and other technical parameters related to waveguide dimensions and electromagnetic performance. One such parameter is the attenuation constant. This describes the dissipative loss per unit length of the waveguide and is a key performance metric for high-frequency transmission media.

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Recent years have seen an increasing number of manufacturers developing waveguides for operation above 110 GHz; for example, at sub-THz frequencies (e.g., upper-millimeter wave frequencies, between *ca.* 100 GHz and 300 GHz) [3], [4] and THz frequencies (i.e., at sub-millimeter-wave frequencies, between 300 GHz and 3 THz) [5], [6]. The two international standards specify aperture dimensions, for operation up to at least 3.3 THz, to reflect these developments. Above 110 GHz, it has been shown that dissipative losses are significant [7]. Having access to verified and standardized methods for calculating loss is, therefore, crucial for manufacturers to qualify waveguides for commercial use.

The 2nd edition of the IEC 60153-2 standard, published in 1974 [8], provided an equation for attenuation constant that has since been widely adopted by the waveguide user community. Equivalent forms of this equation have propagated to both the current 3rd edition of the IEC [1] and 1st edition of the IEEE [2] standards, and is used within these standards to provide reference values for loss across the standardized waveguide bands. However, with the former, in both 2nd and 3rd editions, no reference is cited for the derivation of this equation (which is not in the original 1st edition [9]). The exact mathematical integrity of the equation is therefore unproven, which is evidently problematic.

Currently, both the IEC 60153-2 and IEEE 1785.1 standards are being revised by working groups that include the authors of this paper. To this end, it is timely to update the standards with equations that can be precisely derived from an authoritative textbook source. This will achieve mathematical traceability for the equations provided in the revised standards, and thereby provide assurance to end-users employing these equations for assessing the performance of rectangular metallic waveguides.

The equations to be given in the revised standards are derived mathematically from Marcuvitz [10]. Through this process, the approximations made to achieve the original form of the equation [8] are identified for the first time. The updated equations either provide exact solutions or clearly state approximations with suitable justifications.

II. Derivation for the IEC 60153-2 Standard

In his seminal Waveguide Handbook [10], Marcuvitz gives the attenuation constant α for metal-pipe rectangular waveguides as follows:

$$\alpha = \frac{R_s}{\eta_0 b} \cdot \left(\frac{1 + \frac{2b}{a}\left(\frac{\lambda_0}{2a}\right)^2}{\sqrt{1 - \left(\frac{\lambda_0}{2a}\right)^2}} \right) \text{Np/m} \tag{1}$$

where a is the aperture broad-wall dimension (m), b is the aperture narrow-wall dimension (m), R_s is the classical surface resistance (Ω), η_0 is the intrinsic impedance of free space (Ω), and λ_0 is the wavelength in free space (m). This equation is derived from the well-known "power-loss" method, which is also the basis of those given by Collin [11] and Pozar [12]. It will now be shown how (1) can be reformulated, whereby R_s, η_0, and λ_0 do not need to be calculated, thus, being better suited for inclusion in an international standard.

Given that $\lambda_0 = c/f$, where c is the speed of electromagnetic waves in vacuum (defined as 299 792 458 m/s), f is the frequency of operation (Hz), cut-off frequency of the waveguide $f_c = c/(2a \cdot \sqrt{\varepsilon_r})$ (Hz) [2] and ε_r is the relative permittivity of the dielectric (taken here to be unity). Thus, $\lambda_0 = 2a\,(f_c/f)$. Equation (1) can now be represented as follows:

$$\alpha = \frac{R_s}{\eta_0 b} \cdot \left(\frac{1 + \frac{2b}{a}\left(\frac{f_c}{f}\right)^2}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \right) \text{Np/m.} \tag{2}$$

Multiplying numerator and denominator by $(f/f_c)^2$ gives the following:

$$\alpha = \frac{R_s}{\eta_0 b} \cdot \left(\frac{\left(\frac{f}{f_c}\right)^2 + \frac{2b}{a}}{\left(\frac{f}{f_c}\right)\sqrt{\left(\frac{f}{f_c}\right)^2 - 1}} \right) \text{Np/m}\ . \tag{3}$$

Then, separating out $\sqrt{f/f_c}$ from the denominator and substituting in $R_s = \sqrt{\omega\mu_0/2\sigma}$, where $\omega = 2\pi f$ is the angular frequency of operation (rad/s), μ_0 is the vacuum magnetic permeability (H/m), and σ is the conductivity of the metallic waveguide walls (S/m) gives

$$\alpha = \frac{\sqrt{\pi f \mu_0/\sigma}}{\eta_0 b} \cdot \frac{1}{\sqrt{\frac{f}{f_c}}} \cdot \left(\frac{\left(\frac{f}{f_c}\right)^2 + \frac{2b}{a}}{\sqrt{\frac{f}{f_c}}\sqrt{\left(\frac{f}{f_c}\right)^2 - 1}} \right) \text{Np/m.} \tag{4}$$

Substituting the above expression for f_c into only the middle term and simplifying yields the following:

$$\alpha = \frac{\sqrt{\pi\mu_0 c/2\sigma}}{\eta_0 b\sqrt{a}} \cdot \left(\frac{\left(\frac{f}{f_c}\right)^2 + \frac{2b}{a}}{\sqrt{\frac{f}{f_c}}\sqrt{\left(\frac{f}{f_c}\right)^2 - 1}} \right) \text{Np/m.} \tag{5}$$

Typically, the generic unit for attenuation constant of Np/m is, for convenience, converted to dB/m when dealing with larger waveguides or dB/cm with smaller waveguides. In the case of the IEC standard, waveguides of all standardized sizes are included and so dB/m is adopted. Furthermore, the IEC standard equation incorporates a reference conductivity σ_0 by substituting $\sqrt{1/\sigma} \Rightarrow \sqrt{\sigma_0}\sqrt{1/(\sigma \cdot \sigma_0)}$. Updating (5) accordingly gives the following:

$$\alpha = \frac{20}{\ln(10)} \cdot \frac{\sqrt{\pi\mu_0 c/2\sigma_0}}{\eta_0} \cdot \sqrt{\frac{\sigma_0}{\sigma}} \cdot \frac{1}{b\sqrt{a}} \cdot \left(\frac{\left(\frac{f}{f_c}\right)^2 + \frac{2b}{a}}{\sqrt{\frac{f}{f_c}}\sqrt{\left(\frac{f}{f_c}\right)^2 - 1}} \right) \text{dB/m.} \tag{6}$$

This is the full analytical form for attenuation constant. From here, a simplified version suited for end user convenience can be calculated.

With international standards, it is common practice to represent a variable in more convenient units. With f and f_c both in Hz, they can both be given to any order of magnitude (GHz in the case of the two standards). In addition, with σ_0 input in GS/m, a and b can then be given in mm to provide equivalent results without any further manipulation. The final step is to apply all known input terms to calculate a leading constant. Using the current value [13], [14] for $\mu_0 = 1.25663706127(20) \times 10^{-6}$ H/m $\cong 4\pi \times 10^{-7}$ H/m, $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ (Ω), where $\varepsilon_0 = 1/\mu_0 c^2$ is the vacuum electric permittivity (F/m) and $\sigma_0 = 0.058$ GS/m for standard annealed copper at a room temperature of 20 °C [15] gives

$$\alpha \approx 2.3289 \cdot \sqrt{\frac{\sigma_0}{\sigma}} \cdot \frac{1}{b\sqrt{a}} \cdot \left(\frac{\left(\frac{f}{f_c}\right)^2 + \frac{2b}{a}}{\sqrt{\frac{f}{f_c}}\sqrt{\left(\frac{f}{f_c}\right)^2 - 1}} \right) \text{dB/m.} \tag{7}$$

The leading constant is rounded to five significant figures, to be consistent with the historic IEC standard [8]. Note that the leading constant in (7) differs from the original value of 2.3273 [8], raising concerns regarding the accuracy of the original equation. Through investigation, it was found that a value of 2.3273 can indeed be obtained by using the widely adopted approximation for $\eta_0 = 376.730313412(59)\ \Omega \approx 120\pi\ \Omega$, a σ_0 value of $1/\rho_0$ where $\rho_0 \approx 17.241$ nΩ·m and a Neper to dB conversion factor $20/\ln(10) \approx 8.686$. These approximations were not fully documented and, therefore, the origin of this leading constant has not been traced in the open literature until now. Furthermore, the newly computed leading constant in (7) does not make these approximations and so provides a greater degree of accuracy. Equation (7) represents the corrected form and is being adopted in the on-going revision of IEC 60153-2.

III. Derivation for the IEEE 1785.1 Standard

The on-going revision of IEEE 1785.1 standard incorporates formulas in dB/cm, as it pertains to smaller waveguides, and adopts resistivity ρ in nΩ·m instead of conductivity. Furthermore, as an update to the first edition of this standard, a reference resistivity ρ_0 for copper is no longer incorporated. The full analytical form is achieved by taking (5), substituting $\sigma = 1/\rho$ and applying the appropriate conversion factor for

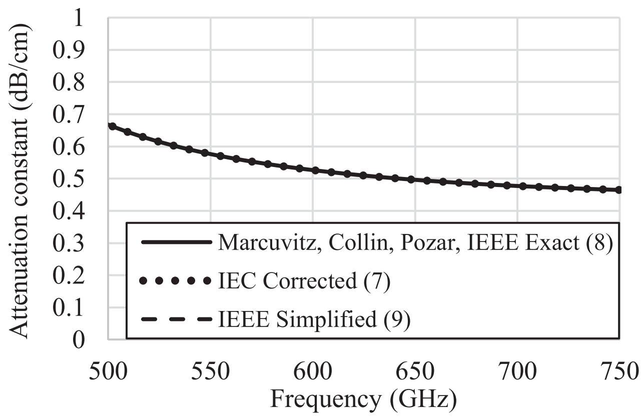

Fig. 1. Calculated attenuation constant in WM-380 band.

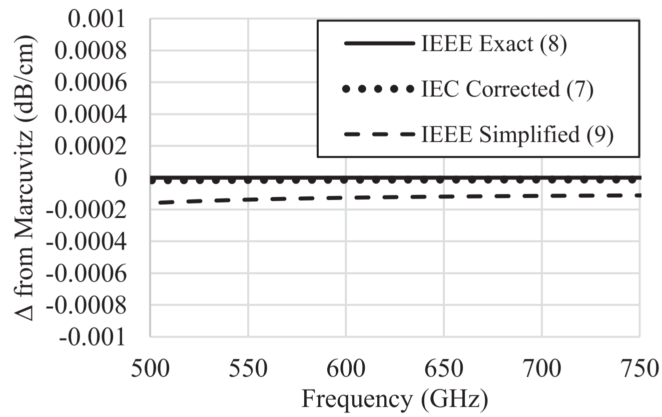

Fig. 2. Difference in the calculated attenuation constant results from the adopted Marcuvitz benchmark reference.

dB/cm to give the following:

$$\alpha = \frac{20}{100 \cdot \ln(10)} \cdot \frac{\sqrt{\frac{\pi \mu_0 \rho c}{2}}}{\eta_0 b \sqrt{a}} \cdot \left(\frac{\left(\frac{f}{f_c}\right)^2 + \frac{2b}{a}}{\sqrt{\frac{f}{f_c}} \sqrt{\left(\frac{f}{f_c}\right)^2 - 1}} \right) \text{dB/cm.} \tag{8}$$

Subsequently, the leading constant can be calculated as follows:

$$\alpha \approx 0.00561 \cdot \frac{\sqrt{\rho}}{b\sqrt{a}} \cdot \left(\frac{\left(\frac{f}{f_c}\right)^2 + \frac{2b}{a}}{\sqrt{\frac{f}{f_c}} \sqrt{\left(\frac{f}{f_c}\right)^2 - 1}} \right) \text{dB/cm.} \tag{9}$$

This simplified version provides a leading constant that is rounded to three significant figures. Equations (8) and (9) are both given in the revised IEEE 1785.1 standard [17].

IV. Verification of Equations

The equivalence of these equations will now be established by example. Results calculated using the equations derived here and those from textbook sources for a nominal WM-380 waveguide plated with gold; with $a = 0.38$ mm, $b = 0.19$ mm, and $\rho = 22.0$ nΩ·m [2] (with $\sigma = 0.045$ GS/m). The recommended operating band for this waveguide size is 500 GHz to 750 GHz [2]. All results are represented in dB/cm. The exact results given by Marcuvitz [10], Collin [11], Pozar [12] and (8) are identical as shown in Fig. 1.

The results shown in Fig. 1 appear to demonstrate good agreement between all the calculated results. However, Fig. 2 shows the difference, Δ, between results using (7)–(9) and that from Marcuvitz (adopted here as the benchmark reference). Given the precision of the attenuation constant values to be reported in the revised standards are 0.001 dB/cm at the highest, all calculated differences are negligibly small.

IV. Discussion

The equations provided here assume homogeneous conductivity, classical skin effect modelling, perfectly smooth waveguide walls and apertures of uniform rectangular dimensions. In practice, however, losses incurred by real waveguides suffering from imperfections in all these aspects require in-depth scientific study. Differences between calculated and measured waveguide attenuation have been widely reported (e.g., [7]). To account for these differences, rigorous approaches can be followed to consider additional aspects affecting wave propagation such as inhomogeneous conductivity (e.g., [18]), nonclassical skin effect (e.g., [19]) and surface roughness of the metal plating (e.g., [20]).

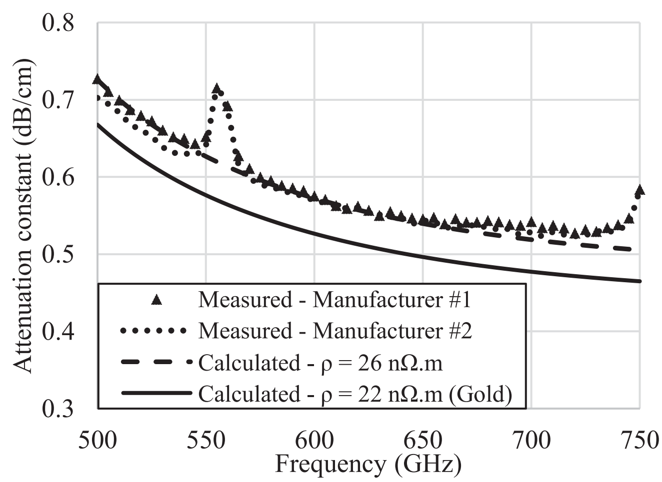

Fig. 3. Calculated and measured attenuation constant for WM-380 waveguides. Results were calculated using (8).

As a practical compromise, ρ can be considered (rather than adopting any default dc value found in textbooks), as the *effective* resistivity (or conductivity) of the waveguide wall conductors. This approach recognizes that the attenuation of a practical waveguide will usually be greater than that calculated using the textbook dc value for the resistivity of its conductors. Experimentally determined values for ρ have been reported for waveguides operating at THz frequencies in a number of studies (e.g., [21]). Users of the aforementioned international standards are encouraged to adopt this approach. As an example, attenuation constant results extracted from scattering (*S*-)parameter measurements of two gold plated WM-380 waveguides (from different manufacturers) are compared with results calculated using (8); with $\rho = 22.0$ nΩ·m (Gold) [2] and $\rho = 26$ nΩ·m determined experimentally. The 18% increase in the value of ρ evidently provides much closer agreement with the measured results, as shown in Fig. 3. The peaks seen at around 560 GHz and 750 GHz are due to atmospheric absorption by water vapour molecules, which is known to affect transmission in this band [21].

V. Conclusion

The equations for rectangular waveguide attenuation constant to be given in the revised IEC 60153-2 and IEEE 1785.1 standards have all been derived mathematically from Marcuvitz [10]—which is widely cited as an authoritative source for waveguide theory. Our derivation process provides valuable assurance to end users for obtaining calculated values of attenuation for their rectangular waveguides that are mathematically traceable to [10]. Furthermore, an error (due to approximations) present in an equation in an historical and widely used IEC standard has been identified and corrected. The new equations in these international standards are now fully mathematically traceable with all derivational steps documented in this letter.

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