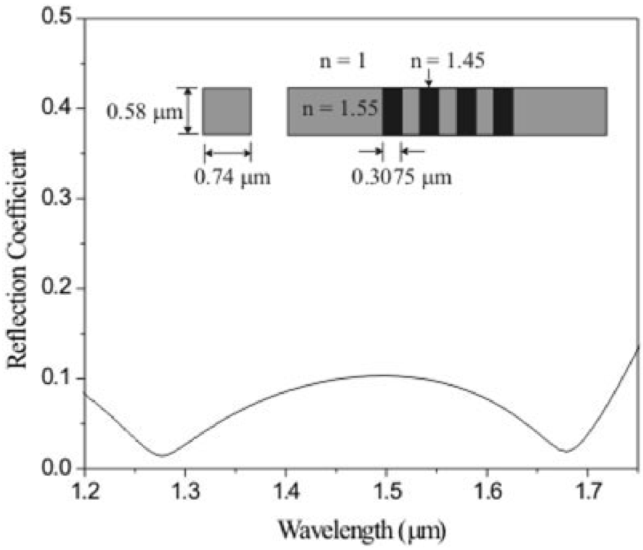

Figure 5 TE_{10}-mode reflection coefficient of a DFB reflector (inset) as a function of wavelength

the results. The waveguide presents a lowest transmission point around 14.3 GHz, suggesting a behavior similar to that of a band-reject filter.

To show the potential application of the 3D-method to integrated optics, the fifth example presents the TE10-mode reflection coefficient of a DFB structure (inset) designed to operate at 1.5 μm (see Fig. 5). A mesh with 44,437 tetrahedral elements was used, generating 588,168 unknown variables. The total time interval was 240 fs. The time step Δt was 0.02 fs. The simulation time was 9059 s. It is important to point out that the 2D analysis is unsuitable to model such a structure. This is due to the fact that the 3D method takes into account the whole wave scattering produced by the grating, which is partially neglected by the 2D calculations.

5. CONCLUSION

In this article, a new set of 3D-orthogonal vector basis functions was presented. These new basis functions originated from the Whitney's edge elements, preserving the same characteristics as those of conventional basis functions. The formulation allows for diagonal matrices to naturally appear without the use of the lumping method. Thus, the resulting system of equations can be solved directly from simple diagonal matrix inversion. To validate the formulation proposed here, eigenvalue problems of cavities composed of homogeneous and nonhomogenous media were analyzed and propagation characteristics of some structures using the time-domain full-band method in 3D for complex slow varying field amplitudes were investigated. All numerical results are in good agreement with those published previously in the literature. At the moment, other applications in time domain are under analysis, mainly related to photonic devices.

ACKNOWLEDGMENT

The authors thank the partial support of CePOF/FAPESP, CAPES, FAEPEX/UNICAMP, and CNPq, Brazil.

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TAILORING OF THE SUBWAVELENGTH FOCUS

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Received 1 February 2007

ABSTRACT: *Subwavelength focusing by an ensemble of magnetically coupled metamaterial elements is studied theoretically for the case when all the dimensions are small relative to the electromagnetic wavelength. It is shown that in contrast to expectation a focus can be placed in any desired position for any transmitter-lens configuration by the appropriate tuning of the receiver. A mathematical proof is provided and two examples are shown demonstrating a focal width of λ/60.* © 2007 Wiley Periodicals, Inc. Microwave Opt Technol Lett 49: 2228–2231, 2007; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.22689

Key words: *metamaterials; subwavelength lens; subwavelength focusing*

1. INTRODUCTION

Ever since the publication of Veselago's flat lens [1] and Pendry's perfect lens [2], the subject of subwavelength imaging and focusing has been widely investigated. Maslovski et al. [3] suggested that a negative index lens is not necessary for imaging and it is sufficient to rely on the excitation of two specified surfaces. At optical frequencies focusing and imaging by classical lenses are

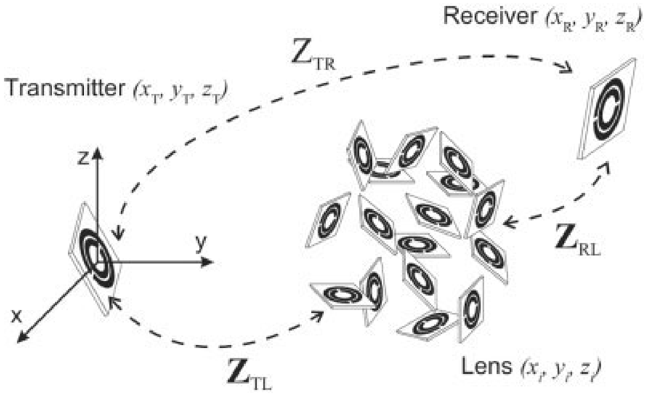

Figure 1 Schematic presentation of the configuration in the general case. The transmitter, receiver, and lens are at arbitrary positions and have arbitrary orientations. The coupling between the transmitter and the lens is Z_{TL} and that between the receiver and lens is Z_{RL}. The transmitter and receiver are coupled to each other via the mutual impedance Z_{TR}

strongly related to each other. However, there is a fine distinction between them when the dimensions are all small relative to the wavelength as pointed out by Freire and Marques [4, 5] whose lens consisted of two planes of magnetically coupled resonant elements. They found, scanning the space behind the lens by a small detector, that the power received was maximum at a particular point. This point is not a focus in the optical sense because the field is not a maximum there, but it could be regarded as a focus in another sense due to the fact that the power extracted by the receiver is higher there than that at any other point. The mechanism is matching the receiver to the transmitter [5]. In the experiments of Freire and Marques the transmitter-lens-receiver positions were those postulated by Pendry but that relationship is by no means unique.

The aim of the present article is to show that, wherever the positions of the transmitter and lens are, a focus can always be placed in an arbitrary position by adjusting the impedance of the receiver. We provide a general treatment of the problem in Section 2, proceed with examples of focusing in Section 3, and draw conclusions in Section 4. The mathematical proofs are relegated to Appendices A and B.

2. GENERAL FORMULATION

The general model is shown in Figure 1. We have $N + 2$ metamaterial elements, the transmitter, the receiver, and the N elements of the lens, which can have arbitrary positions and orientations. The self-impedances of the elements are: Z_T, Z_R, and Z_i ($i = 1 \ldots N$) where the subscripts T, R, and i refer to the transmitter, the receiver, and the elements of the lens. Their respective positions in space are at (x_T, y_T, z_T), (x_R, y_R, z_R), and (x_i, y_i, z_i). The mutual impedances are characterized by Z_{ij} ($i, j = 1 \ldots N$; $i \neq j$), the elements of the lens matrix, Z_{TLi} and Z_{RLi} ($i = 1 \ldots N$), elements of row vectors related to the transmitter and receiver respectively, and Z_{TR}, a scalar. Note that all the mutual impedances are assumed to be purely imaginary which implies that retardation is neglected, i.e. all dimensions are small relative to the electromagnetic wavelength. The elements of the lens are assumed to be lossless. The transmitter and the receiver have impedances of $Z_T = R_T + jX_T$ and $Z_R = R_R + jX_R$, the latter to be determined from the condition that for a voltage V_0 applied to the transmitter the power absorbed in the receiver should be maximum. The main steps of the analytical optimization are indicated in Appendix A. The optimum value of the receiver impedance is given by the expression

$$R_R^{(opt)} + jX_R^{(opt)} = Z_{RL}Z^{-1}Z_{RL}^T - \frac{(Z_{TR} - Z_{TL}Z^{-1}Z_{RL}^T)^2}{R_T - jX_T + Z_{TL}Z^{-1}Z_{TL}^T} \qquad (1)$$

where Z^{-1} is the inverse of the impedance matrix of the lens and the superscript T means the transposed of a vector.

Once the optimum load impedance pertaining to the (x_R, y_R, z_R) point has been found it follows that moving the receiver out of that point can only lead to lower extracted power (see Appendix B for a rigorous proof). Hence (x_R, y_R, z_R) must be the focal point.

We wish to emphasize here that this focal point can be put anywhere in three-dimensional space subject to the condition that all dimensions are small relative to the electromagnetic wavelength. It follows then clearly that the focal width must be subwavelength. The potential application of this new phenomenon is the ability to recognize a predetermined position by the fact that the received power is maximum in that point.

3. TWO EXAMPLES OF FOCUSING

It is beyond the scope of the present paper to pose additional conditions e.g. to find the optimum transmitter-lens-receiver configuration for realizing a desired power distribution in the focal region. We shall, however, demonstrate the possibility of specifying the focal point in two examples. For simplicity, we drop the third dimension and assume that all the elements are in the (x, y)-plane.

In our first example, the lens consists of two parallel layers of capacitively loaded loops. Each layer is build up by five elements in a planar arrangement as can be seen in Figure 2(a). The radius of the elements is $r_0 = 5$ mm and the distance between them in a layer is $d = 2.1r_0 = 10.5$ mm. The separation between the layers is $h = 2r_0 = 10$ mm. The transmitter and receiver are two metallic loops identical to the elements of the lens. The transmitter is placed at the (0, 0) point at a distance l from the lens. We chose the operating frequency to be 500 MHz. The self-inductance of the lens elements and that of the transmitter of 12.2 nH is compensated by loading them by 8.3 pF capacitors. The transmitter has an internal resistance of 50 Ω. The mutual inductances between the transmitter, receiver, and lens can be calculated using expressions available for the loops [6].

We place then the receiver at three different positions and for each position find the corresponding optimum impedance. Initially, the receiver is placed symmetrically to the transmitter at the distance $f = l$ in front of the lens [see Fig. 2(a)]. In this configuration the separation between the lens layers is twice larger than the distance between the transmitter/receiver and the lens, i.e. obeys the relation used in Refs. 4 and 5. Then, the receiver is moved along the y-axis on the distances $f = 1.5r_0$, $2r_0$ away from the lens. The values of optimum impedance corresponding to each position of the lens are given in Table 1.

The results for the extracted power, when the receiver with optimum loading is translated parallel to itself from its initial position, are shown in Figures 2(b)–2(d) for all three positions of the receiver. The power is plotted using contour plots relative to the maximum received power of $|V_0|^2/(4R_T)$. It can be clearly seen that focusing of the received power is possible not only for the symmetrical "$h/2 - h - h/2$" configuration but also for the other two. The position of the focus shifts together with the receiver when the later is moved away from the lens. Moreover, the maximum of the received power is independent of the position of the receiver and has a value of $|V_0|^2/(4R_T)$ for all three cases.

For the second example (Fig. 3) we chose quite a different lens configuration. It consists now of two parallel axially arranged arrays with the same element size but of seven elements. The

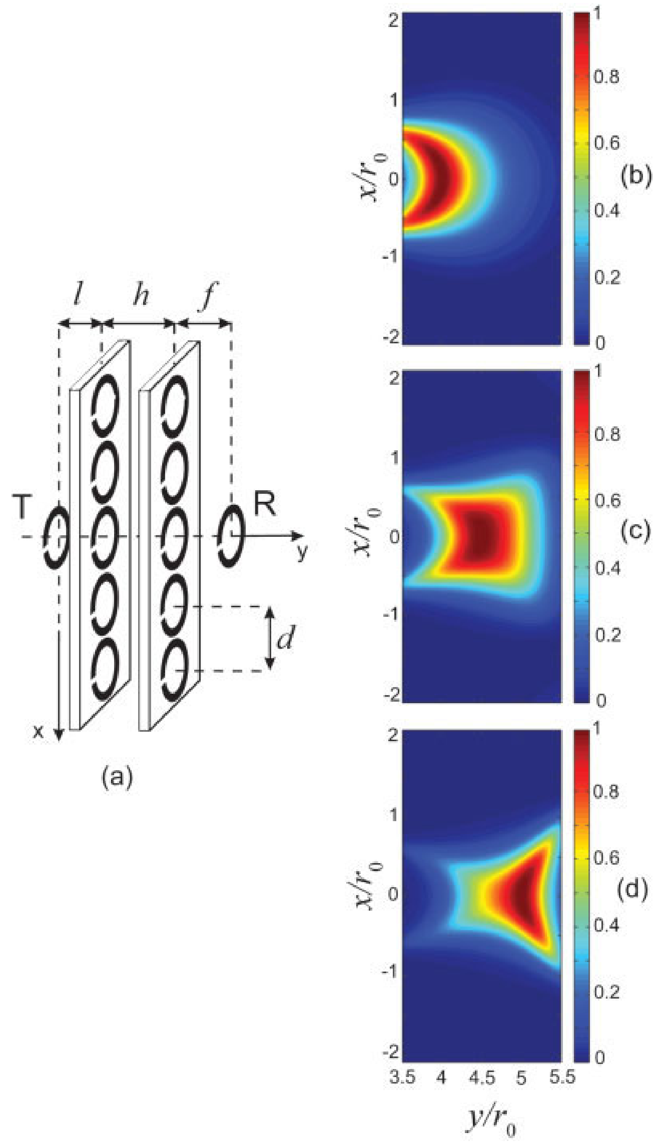

Figure 2 (a) Schematic presentation of the focusing with a two layered lens, (b–d) contour plots of the normalized extracted power corresponding to three different positions of the receiver, $f = r_0$, $1.5r_0$, $2r_0$. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

distance between the elements in an array is $d = 0.5r_0 = 2.5$ mm and the separation between the arrays is $h = 2.1r_0$. The arrays are placed symmetrically relative to the y-axis [see Fig. 3(a)]. Similarities to the first example are the choice of resonant frequency at 500 MHz, of the purely resistive impedance of the transmitter as 50 Ω and of the position of the transmitter at the point (0, 0). The distance from the transmitter to the lens is equal to $l = 0.5r_0$. Analogously to the previous example we place the receiver into

TABLE 1 Values of the Optimum Impedance for the Configuration of Figure 2 Corresponding to Three Different Positions of the Receiver, $f = r_0, 1.5r_0, 2r_0$

Distance, f/r_0	Optimum Impedance (Ω)
1	$0.96 + j0.40$
1.5	$0.22 - j0.34$
2	$0.06 - j0.29$

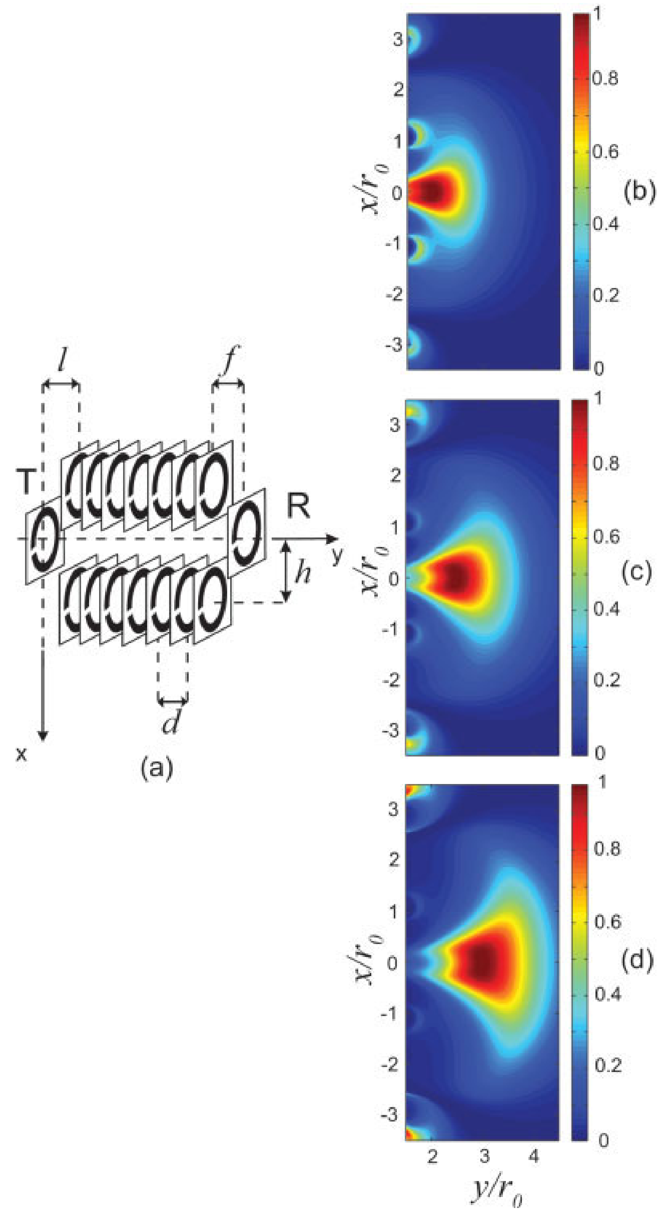

Figure 3 (a) Schematic presentation of the focusing with two parallel axial arrays, (b–d) contour plots of the normalized extracted power corresponding to three different positions of the receiver, $f = 0.5r_0$, $1.0r_0$, $1.5r_0$. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

three different positions on the y-axis, at the distances $f = 0.5r_0$, $1.0r_0$, $1.5r_0$. The corresponding values of the optimum impedance are given in Table 2. The power extracted from the receiving loop as it is moved parallel to itself from its initial optimum position is shown in Figures 3(b)–3(d) in the same manner as in Figures

TABLE 2 Values of the Optimum Impedance for the Configuration of Figure 3 Corresponding to Three Different Positions of the Receiver, $f = 0.5r_0, 1.0r_0, 1.5r_0$

Distance, f/r_0	Optimum Impedance (Ω)
0.5	$0.58 - j0.34$
1.0	$0.21 - j0.05$
1.5	$0.08 + j0.02$

2(b)–2(d) in the first example. There are clear maximums for each case corresponding to the different positions of the receiver.

The examples we have given relate to a small number of elements. The aim was merely to show the potential of the method proposed. Our numerical techniques, developed for magnetoinductive waves [7], could cope with several thousand elements.

4. CONCLUSIONS

We have shown that the power extracted from the receiver may (*i*) reach its maximum and (*ii*) be focused in an arbitrary pre-selected point independently of the actual arrangement of the metamaterial elements provided that the impedance of the receiver is tuned to the optimum value. Applications can be expected in RF problems where there is need to identify a point in space.

APPENDIX A

Denoting the voltage applied to the transmitter by V_0 we can write equations for the currents in the elements in the matrix form

$$\begin{pmatrix} Z_T & Z_{TL} & Z_{TR} \\ Z_{TL}^T & Z & Z_{RL}^T \\ Z_{TR} & Z_{RL} & Z_R \end{pmatrix} \begin{pmatrix} I_T \\ I \\ I_R \end{pmatrix} = \begin{pmatrix} V_0 \\ 0 \\ 0 \end{pmatrix}, \quad \text{(A1)}$$

where Z_T and Z_R are the impedances of the transmitter and receiver, respectively; I_T and I_R are the currents flowing into them; I is the N-element column-vector of the currents in the elements of the lens and 0 is a column-vector of N zeros.

The current in the receiver is, from Eq. (A1),

$$I_R = -\frac{Z_{TR} - Z_{RL}Z^{-1}Z_{TL}^T}{(Z_R - Z_{RL}Z^{-1}Z_{RL}^T)(Z_T - Z_{TL}Z^{-1}Z_{TL}^T) - (Z_{TR} - Z_{RL}Z^{-1}Z_{TL}^T)^2} V_0 \quad \text{(A2)}$$

and the current in the transmitter

$$I_T = -\frac{Z_R - Z_{RL}Z^{-1}Z_{RL}^T}{(Z_R - Z_{RL}Z^{-1}Z_{RL}^T)(Z_T - Z_{TL}Z^{-1}Z_{TL}^T) - (Z_{TR} - Z_{RL}Z^{-1}Z_{TL}^T)^2} V_0. \quad \text{(A3)}$$

The power extracted from the receiver, $P_R = 1/2|I_R|^2 \mathrm{Re} Z_R$, depends on its impedance: $P_R = P_R(R_R, X_R)$. Since the elements of the lens are lossless, $Z_{RL}Z^{-1}Z_{RL}^T$, $Z_{TL}Z^{-1}Z_{TL}^T$, and $Z_{RL}Z^{-1}Z_{TL}^T$ are purely imaginary. Finding the derivatives of P_R, $\partial P_R/\partial R_R$, and $\partial P_R/\partial X_R$, and equating them to zero, we find the following relationship for the values of the optimum resistance, $R_R^{(opt)}$, and reactance, $X_R^{(opt)}$

$$Z_1 = \frac{R_3^2}{Z_2^*}, \quad \text{(A4)}$$

where

$$Z_1 = Z_R^{(opt)} - Z_{RL}Z^{-1}Z_{RL}^T$$

$$Z_2 = Z_T - Z_{TL}Z^{-1}Z_{TL}^T, \quad \text{(A5)}$$

$$R_3^2 = -(Z_{TR} - Z_{RL}Z^{-1}Z_{TL}^T)^2$$

which yields Eq. (1).

The power extracted from the receiver in the optimum case is equal to that absorbed by the resistive load of the transmitter and has the value

$$P_R^{(max)} = P_T^{(max)} = |V_0|^2/(4R_T). \quad \text{(A6)}$$

APPENDIX B

The optimum value of the impedance of the receiver gives a focus in the chosen point if $\partial P_R/\partial u = 0$ $(u = x, y, z)$. The power depends on the coordinates indirectly via couplings $Z_{RL} = Z_{RL}(x, y, z)$ and $Z_{TR} = Z_{TR}(x, y, z)$ which change as the receiver is moved off from its initial position. We can then write for the power extracted from the receiver using Eq. (A2)

$$P_R = \frac{1}{2}\frac{R_3^2 R_1}{2(R_1R_2 - X_1X_2 + R_3^2)^2 + (X_1R_2 + X_2R_1)}|V_0|^2. \quad \text{(B1)}$$

Finding the derivatives of Eq. (B2) by each of the coordinates, $u = x, y, z$, and equating them to zero gives

$$\frac{\partial R_3}{\partial u}\{(R_3^{(opt)})^4[(R_1^{(opt)})^2 + X_1^2] - R_3^4[(R_1^{(opt)})^2 + (X_1^{(opt)})^2]\} - \frac{\partial X_1}{\partial u}R_3(R_3^{(opt)})^2[(R_3^{(opt)})^2X_1 - R_3^2X_1^{(opt)}] = 0, \quad \text{(B2)}$$

where $R_3^{(opt)}$, $R_1^{(opt)}$, and $X_1^{(opt)}$ are the values of R_3, R_1, and X_1 as obtained from Eq. (A5) under optimum conditions.

It can be seen that when $R_3 = R_3^{(opt)}$, $R_1 = R_1^{(opt)}$, and $X_1 = X_1^{(opt)}$, i.e. the receiver is at the optimum position, then Eq. (B3) holds. This means that the derivatives of the power extracted from the receiver by the coordinates are zero, $\partial P_R/\partial u = 0$, and the optimum position is a focus.

ACKNOWLEDGMENT

The authors are grateful to Profs. Ricardo Marques, Manuel Freire, Richard Syms and Mikhail Shamonin for a number of stimulating discussions. Financial support of the German Research council is acknowledged by O.S. and E. S. (GK 695 and Emmy-Nother Programme).

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