Mechanical Engineering Data and Formulae

Department of Mechanical Engineering, Imperial College London

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VERSION 2.00

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Version History

Version	Date	Description
1.0	May 2019	Previous major update
2.0	04/09/2024	Major update to all sections. End of annual reissue policy. New versions will only be issued when major changes are made.

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A General information

Table A.1: SI Units and abbreviations

Quantity	**Unit**	**Unit symbol**
Basic units		
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Derived units		
Acceleration, linear	metre/second^2	$\mathrm{m\,s^{-2}}$
Acceleration, angular	radian/second^2	$\mathrm{rad\,s^{-2}}$
Area	metre^2	$\mathrm{m^2}$
Density	kilogram/metre^3	$\mathrm{kg\,m^{-3}}$
Force	newton	$\mathrm{N\ (= kg\,m\,s^{-2})}$
Frequency	hertz	$\mathrm{(Hz = s^{-1})}$
Impulse, linear	newton-second	$\mathrm{N\,s}$
Impulse, angular	newton-metre-second	$\mathrm{N\,m\,s}$
Moment of force	newton-metre	$\mathrm{N\,m}$
Second moment of area	metre^4	$\mathrm{m^4}$
Moment of inertia	kilogram-metre^2	$\mathrm{kg\,m^2}$
Momentum, linear	kilogram-metre/second	$\mathrm{kg\,m\,s^{-1}}$
Momentum, angular	kilogram-metre^2/second	$\mathrm{kg\,m^2\,s^{-1}}$
Power	watt	$\mathrm{W\ (= J\,s^{-1} = N\,m\,s^{-1})}$
Pressure, stress	pascal	$\mathrm{Pa\ (= N\,m^{-2})}$
Stiffness (linear), spring constant	newton/metre	$\mathrm{N\,m^{-1}}$
Velocity, linear	metre/second	$\mathrm{m\,s^{-1}}$
Velocity, angular	radian/second	$\mathrm{rad\,s^{-1}}$
Volume	metre^3	$\mathrm{m^3}$
Work, energy	joule	$\mathrm{J\ (= N\,m)}$
Electrical units		
Potential	volt	$\mathrm{V\ (= W\,A^{-1})}$
Resistance	ohm	$\Omega\ \mathrm{(= V\,A^{-1})}$
Charge	coulomb	$\mathrm{C\ (= A\,s)}$
Capacitance	farad	$\mathrm{F\ (= A\,s\,V^{-1})}$
Electric field strength	volt/metre	$\mathrm{V\,m^{-1}}$
Electric flux density	coulomb/metre^2	$\mathrm{C\,m^{-2}}$
Magnetic units		
Magnetic flux	weber	$\mathrm{Wb\ (= V\,s)}$
Inductance	henry	$\mathrm{H\ (= V\,s\,A^{-1})}$
Magnetic field strength	—	$\mathrm{A\,m^{-1}}$
Magnetic flux density	—	$\mathrm{Wb\,m^{-2}}$

Table A.2: Conversion factors from Imperial to SI units

To convert	from	to	multiply by
Acceleration	foot/second^2 (ft/sec^2)	metre/second^2 (m s^{-2})	0.3048
	inch/second^2 (in/sec^2)	metre/second^2 (m s^{-2})	0.0254
Area	foot^2 (ft^2)	metre^2 (m^2)	0.092903
	inch^2 (in.^{2})	metre^2 (m^2)	6.4516×10^{-4}
Density	pound mass/inch^3 (lbm/in^3)	kilogram/metre^3 (kg m^{-3})	2.7680×10^4
	pound mass/foot^3 (lbm/ft^3)	kilogram/metre^3 (kg m^{-3})	16.018
Force	kip (1000 lb)	newton (N)	4.4482×10^3
	pound force (lb)	newton (N)	4.4482
Length	foot (ft)	metre (m)	0.3048
	inch (in)	metre (m)	0.0254
	mile (mi), U.S. statute	metre (m)	1.6093×10^3
	mile (mi), international nautical	metre (m)	1.852×10^3
Mass	pound mass (lbm)	kilogram (kg)	0.45359
	slug (lb-sec^2 /ft)	kilogram (kg)	14.594
	ton (2000 lbm)	kilogram (kg)	907.18
Moment of force	pound-foot (lb-ft)	newton-metre (N m)	1.3558
	pound-inch (lb-in.)	newton-metre (N m)	0.11298
Moment of inertia	pound-foot-second^2(lb-ft-sec^2)	kilogram-metre^2 (kg m^2)	1.3558
Momentum, linear	pound-second (lb-sec)	kilogram-metre/second (kg m s^{-1})	4.4482
Momentum, angular	pound-foot-second (lb-ft-sec)	newton-metre-second (N m s)	1.3558
Power	foot-pound/minute (ft-lb/min)	watt (W)	0.022597
	horsepower (550 ft-lb/sec)	watt (W)	745.70
Pressure, stress	atmosphere (std) (14.7 lb/in^2)	newton/metre^2 (N m^{-2} or Pa)	1.0133×10^5
	pound/foot^2(lb/ft^2)	newton/metre^2 (N m^{-2} or Pa)	47.880
	pound/inch^2(lb/in.^{2}or psi)	newton/metre^2 (N m^{-2} or Pa)	6.8948×10^3
Second moment of area	inch^4	metre^4 (m^4)	41.623×10^{-8}
Stiffness (linear)	pound/inch (lb/in.)	newton/metre (N m^{-1})	175.13
Velocity	foot/second (ft/sec)	metre/second (m s^{-1})	0.3048
	knot (nautical mi/hr)	metre/second (m s^{-1})	0.51444
	mile/hour (mi/hr)	metre/second (m s^{-1})	0.44704
	mile/hour (mi/hr)	kilometre/hour (km h^{-1})	1.6093
Volume	foot^3(ft^3)	metre^3 (m^3)	0.028317
	inch^3(in.^{3})	metre^3 (m^3)	1.6387×10^{-5}
	UK gallon	metre^3 (m^3)	4.546×10^{-3}
Work, Energy	British thermal unit (BTU)	joule (J)	1.0551×10^3
	foot-pound force (ft-lb)	joule (J)	1.3558
	kilowatt-hour (kw-h)	joule (J)	3.60×10^6

Table A.3: Decimal prefixes

Multiplication factor[a]			Prefix	Symbol
1 000 000 000 000	=	10^{12}	tera	T
1 000 000 000	=	10^{9}	giga	G
1 000 000	=	10^{6}	mega	M
1 000	=	10^{3}	kilo	k
100	=	10^{2}	hecto [a]	h
10	=	10	deka [a]	da
0.1	=	10^{-1}	deci [b]	d
0.01	=	10^{-2}	centi	c
0.001	=	10^{-3}	milli	m
0.000 001	=	10^{-6}	micro	μ
0.000 000 001	=	10^{-9}	nano	n
0.000 000 000 001	=	10^{-12}	pico	p

[a] Use prefixes to keep numerical values generally between 0.1 and 1000

[b] The use of prefixes hecto, deka, deci and centi should be avoided except for certain areas or volumes where the numbers would otherwise become awkward.

Table A.4: Physical constants

Avogadro's number[a]	N	$6.022 \times 10^{23}\,\mathrm{mol^{-1}}$
Absolute zero of temperature	—	$0\,\mathrm{K} = -273.2$ řC
Boltzmann's constant	k	$1.380 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Characteristic impedance of vacuum	Z_0	$= \left(\dfrac{\mu_0}{\varepsilon_0}\right)^{1/2} = 120\pi\ \Omega$
Electron volt	eV	$1.602 \times 10^{-19}\,\mathrm{J}$
Electronic charge	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Electronic rest mass	m_e	$9.109 \times 10^{-31}\,\mathrm{kg}$
Electronic charge to mass ratio	$\left(\dfrac{e}{m_e}\right)$	$1.759 \times 10^{11}\,\mathrm{C\,kg^{-1}}$
Faraday's constant[a]	F	$9.65 \times 10^{4}\,\mathrm{C\,mol^{-1}}$
Gas constant[a]	$\overline{R}$	$8.314\,\mathrm{J\,mol^{-1}\,K^{-1}}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Permittivity of free space	ε_0	$\dfrac{1}{36\pi} \times 10^{-9}\ \mathrm{F\,m^{-1}}$
Planck's constant	h	$6.626 \times 10^{-34}\,\mathrm{J\,s}$
Standard gravitational acceleration	g	$9.807\,\mathrm{m\,s^{-2}}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8}\,\mathrm{J\,m^{-2}\,s^{-1}\,K^{-4}}$
Velocity of light in vacuum	c	$2.9979 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Volume of perfect gas at S.T.P.[b]	—	$22.42 \times 10^{-3}\,\mathrm{m^3}$

[a] These are conventional definitions in gram mol units. For SI calculations in kg mol units multiply the values given by 10^3

[b] At Standard Temperature (0 řC) and Pressure (one atmosphere pressure or $1.013 \times 10^{5}\,\mathrm{N\,m^{-2}}$)

B Mathematics and Computing

Contents

B.1 Algebra

B.1.1 Logarithms

If $b^y = x$, $y = \log_b(x)$ and:

$$\log(x_1 x_2) = \log x_1 + \log x_2$$
$$\log\left(\frac{x_1}{x_2}\right) = \log x_1 - \log x_2$$
$$\log\left(\frac{1}{x}\right) = -\log x$$
$$\log x^n = n \log x$$
$$\log 1 = 0$$

For natural logarithms $b = e = 2.718282$ and if $e^y = x$,

$$y = \log_e(x) = \ln(y)$$

Hence

$$\log_{10} x = 0.4343 \ln x.$$

B.1.2 Quadratic equations

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

and ($b^2 > 4ac$) for real roots.

B.1.3 Determinants

2nd order:

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

3rd order:

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = +a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 - a_3 b_2 c_1 - a_2 b_1 c_3 - a_1 b_3 c_2$$

B.1.4 Vector algebra

$$\mathbf{a} = (a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}) = (a_1, a_2, a_3) \ etc.$$

Scalar (dot) product:

$$\mathbf{a}.\mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Vector (cross) product:

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Scalar triple product:

$$[\mathbf{a}, \mathbf{b}, \mathbf{c}] = \mathbf{a}.\mathbf{b} \times \mathbf{c} = \mathbf{b}.\mathbf{c} \times \mathbf{a} = \mathbf{c}.\mathbf{a} \times \mathbf{b} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Vector triple product:

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}\,(\mathbf{a}.\mathbf{c}) - \mathbf{c}\,(\mathbf{a}.\mathbf{b})$$

B.1.5 Series

Binomial series:

$$(1+x)^{\alpha} = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \ldots \qquad (\alpha \text{ arbitrary, } |x| < 1).$$

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \ldots \qquad (|x| < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \ldots \qquad (|x| < \infty)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots \qquad (|x| < \infty)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \ldots \qquad (-\frac{\pi}{2} < x < \frac{\pi}{2})$$

$$\sinh x = \frac{e^x - e^{-x}}{2} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \ldots \qquad (|x| < \infty)$$

$$\cosh x = \frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots \qquad (|x| < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^n \frac{x^{n+1}}{(n+1)} + \ldots \qquad (-1 < x < 1)$$

Stirling's formula for $n!$ when n is large:

$$n! \cong \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$$

or

$$\ln(n!) \cong \left(n + \frac{1}{2}\right) \ln n - n + \frac{1}{2} \ln(2\pi)$$

or

$$\log_{10}(n!) \cong 0.39909 + \left(n + \frac{1}{2}\right) \log_{10} n - 0.43429n$$

Taylor Series Expansions:

Expansion of $f(x)$ about $x = a$:

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!} f''(a) + \ldots + \frac{h^n}{n!} f^{(n)}(a) + R_n(h)$$

Where:

$$R_n(h) = \frac{h^{n+1}}{(n+1)!} f^{(n+1)}(a + \theta h), \ (0 < \theta < 1)$$

Expansion of $f(x, \ y)$ about the point (a, b):

$$f(a+h, b+k) = f(a,b) + \left[hf_x + kf_y\right]_{a,b} + \frac{1}{2!}\left[h^2 f_{xx} + 2hkf_{xy} + k^2 f_{yy}\right]_{a,b} + \ldots$$

Fourier series

For a periodic function $f(x)$ of period L

$$f(x) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2n\pi x}{L} + b_n \sin \frac{2n\pi x}{L}\right)$$

$$a_n = \frac{2}{L} \int_d^{d+L} f(x) \cos \frac{2n\pi x}{L} dx$$

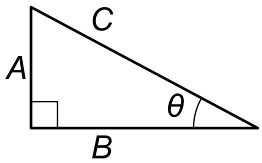
$$b_n = \frac{2}{L} \int_d^{d+L} f(x) \sin \frac{2n\pi x}{L} dx$$

If $f(x)$ is an *even* function of x, i.e. $f(-x) = f(x)$, then $b_n = 0, n = 1, 2, 3, \ldots$.

If $f(x)$ is an *odd* function of x, i.e. $f(-x) = -f(x)$, then $a_n = 0, n = 0, 1, 2, 3, \ldots$.

B.1.6 Trigonometry

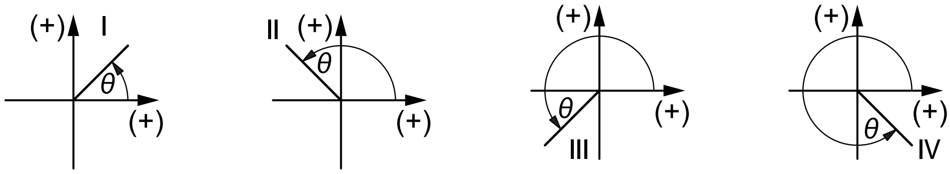
Definitions:



$$\sin\theta = \frac{A}{C} \quad \cos\theta = \frac{B}{C} \quad \tan\theta = \frac{A}{B}$$

$$\csc\theta = \frac{C}{A} \quad \sec\theta = \frac{C}{B} \quad \cot\theta = \frac{B}{A}$$

Signs of trigonometric functions in the four quadrants:



Quadrant:	I	II	III	IV
$\sin\theta$	+	+	−	−
$\cos\theta$	+	−	−	+
$\tan\theta$	+	−	+	−
$\csc\theta$	+	+	−	−
$\sec\theta$	+	−	−	+
$\cot\theta$	+	−	+	−

Trigonometric identities

$$\cos^2\theta + \sin^2\theta = 1$$

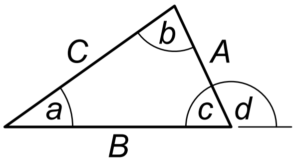
$$1 + \tan^2\theta = \sec^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\sin\frac{\theta}{2} = \sqrt{\frac{1}{2}(1-\cos\theta)}$$

$$\cos\frac{\theta}{2} = \sqrt{\frac{1}{2}(1+\cos\theta)}$$



Sine rule: $$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Cosine rule: $$C^2 = A^2 + B^2 - 2AB\cos c$$
$$C^2 = A^2 + B^2 + 2AB\cos d$$

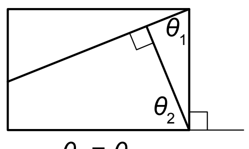
$$\sin(a+b) = \sin a \cos b + \cos a \sin b \qquad \sin(a-b) = \sin a \cos b - \cos a \sin b$$
$$\cos(a+b) = \cos a \cos b - \sin a \sin b \qquad \cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\sin a + \sin b = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right) \qquad \sin a - \sin b = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\cos a + \cos b = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right) \qquad \cos a - \cos b = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

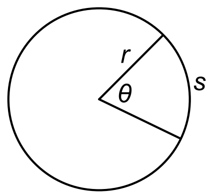
$$\sin \mathrm{i}z = \mathrm{i}\sinh z \qquad \sinh \mathrm{i}z = \mathrm{i}\sin z$$
$$\cos \mathrm{i}z = \cosh z \qquad \cosh \mathrm{i}z = \cos z$$

B.1.7 Geometry



$\theta_1 = \theta_2$

When the two intersecting lines are, respectively, perpendicular to two other lines, the angles formed by each pair are equal.

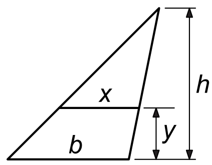


Circle:

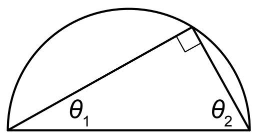
circumference $= 2\pi r$

Arc length $s = r\theta$

Sector area $= \frac{1}{2}r^2\theta$

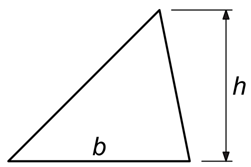


Similar triangles:
$$\frac{x}{b} = \frac{h-y}{h}$$



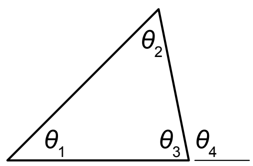
$\theta_1 + \theta_2 = \frac{1}{2}\pi$

Every triangle inscribed within a semicircle is a right triangle.



Any triangle:

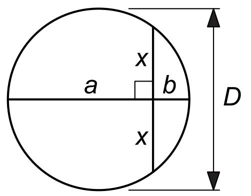
Area $= \frac{1}{2}bh$



Angles of a triangle:

$\theta_1 + \theta_2 + \theta_3 = \pi$

$\theta_4 = \theta_1 + \theta_2$

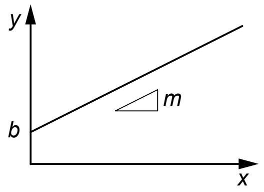


Intersecting chords:

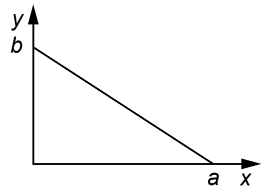
$x^2 = ab$

$x^2 \approx Db$ when $b \ll D$

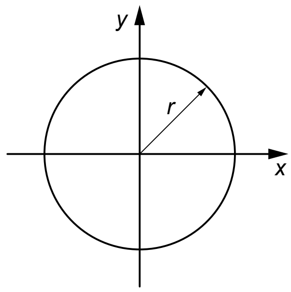
B.1.8 Analytic geometry



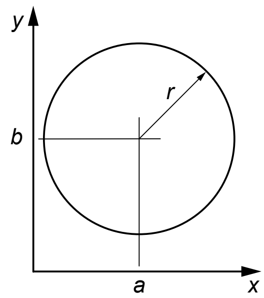
Straight line:
$y = b + mx$



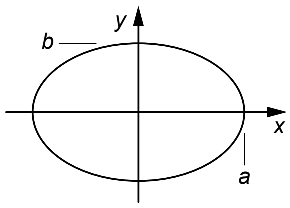
$$\frac{x}{a} + \frac{y}{b} = 1$$



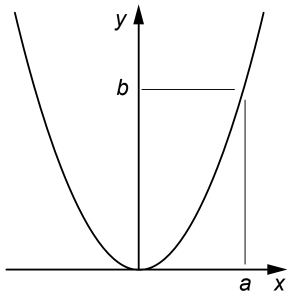
Circle:
$x^2 + y^2 = r^2$



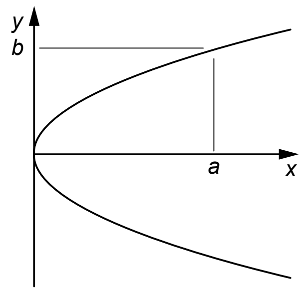
$$(x - a)^2 + (y - b)^2 = r^2$$



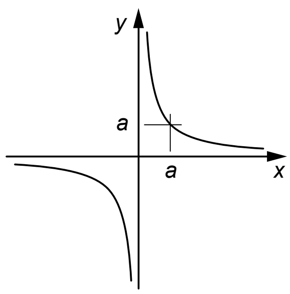
Ellipse:
$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$



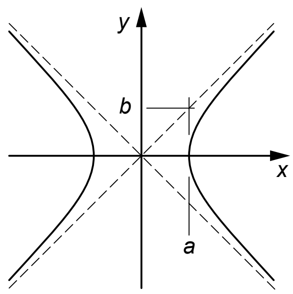
Parabola:
$$y = b\left(\frac{x}{a}\right)^2$$



$$x = a\left(\frac{y}{b}\right)^2$$

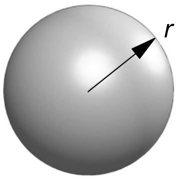


Hyperbola:
$xy = a^2$

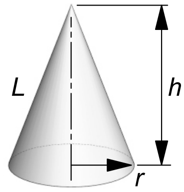


$$\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1$$

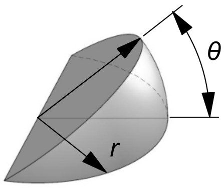
B.1.9 Solid geometry



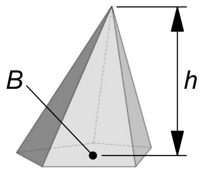
Sphere:
volume $= \frac{4}{3}\pi r^3$
surface area $= 4\pi r^2$



Right-circular cone:
volume $= \frac{1}{3}\pi r^2 h$
lateral area $= \pi r L$
$L = \sqrt{r^2 + h^2}$



Spherical wedge:
volume $= \frac{2}{3} r^3 \theta$



Any pyramid or cone:
volume $= \frac{1}{3} B h$
where $B =$ area of base.

B.2 Calculus

B.2.1 Differential calculus

Leibnitz's rule:

$$\mathrm{D}^n\left(fg\right) = f\left(\mathrm{D}^n g\right) + n\left(\mathrm{D}f\right)\left(\mathrm{D}^{n-1} g\right) + \frac{n(n-1)}{2!}\left(\mathrm{D}^2 f\right)\left(\mathrm{D}^{n-2} g\right) + \cdots + \left(\mathrm{D}^n f\right) g$$

where $\mathrm{D} = \dfrac{\mathrm{d}}{\mathrm{d}x}$, $f = f(x)$ and $g = g(x)$

Taylor's expansion of f(x) about x = a:

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \cdots + \frac{(x-a)^n}{n!}f^{(n)}(a) + \frac{(x-a)^{n+1}}{(n+1)!}f^{(n+1)}(\overline{x})$$

where $a < \overline{x} < x$. Substituting $h = x - a$ gives the following form:

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!}f''(a) + \cdots + \frac{h^n}{n!}f^{(n)}(a) + R_n(h)$$

where $R_n(h) = \dfrac{h^{n+1}}{(n+1)!}f^{(n+1)}\left(a + \theta h\right)$, $(0 < \theta < 1)$.

Taylor's expansion of f(x, y) about the point (a, b):

$$\begin{aligned} f(x,y) = {} & f(a,b) + \left[(x-a)f_x + (y-b)f_y\right]_{a,b} \\ & + \frac{1}{2!}\left[(x-a)^2 f_{xx} + 2\left(x-a\right)\left(y-b\right)f_{xy} + (y-b)^2 f_{yy}\right]_{a,b} + \ldots \end{aligned}$$

Substituting $h = x - a$ and $k = y - b$ gives the following form:

$$f(a+h, b+k) = f(a,b) + \left[hf_x + kf_y\right]_{a,b} + \frac{1}{2!}\left[h^2 f_{xx} + 2hkf_{xy} + k^2 f_{yy}\right]_{a,b} + \ldots$$

Partial differentiation:

If $y = Y(x)$, then $f(x, y) = f[x, Y(x)] \equiv F(x)$ and

$$\frac{\mathrm{d}F}{\mathrm{d}x} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{\mathrm{d}Y}{\mathrm{d}x}$$

If $x = X(t)$ and $y = Y(t)$, then $f(x, y) = F(t)$ and

$$\frac{\mathrm{d}F}{\mathrm{d}t} = \frac{\partial f}{\partial x}\frac{\mathrm{d}X}{\mathrm{d}t} + \frac{\partial f}{\partial y}\frac{\mathrm{d}Y}{\mathrm{d}t}$$

If $x = X(u, v)$ and $y = Y(u, v)$ then $f(x, y) = F(u, v)$ and

$$\frac{\partial F}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u}$$

$$\frac{\partial F}{\partial v} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial v}$$

Stationary points of $\mathbf{f}(\mathbf{x}, \mathbf{y})$:

These occur where $f_x = 0$, $f_y = 0$ simultaneously.
Let (a, b) be a stationary point: examine

$$K = \left[f_{xx}f_{yy} - (f_{xy})^2\right]_{a,b}$$

If:

- $K < 0$, then (a, b) is a *saddle point*;
- $K > 0$ and $f_{xx}(a, b) < 0$, then (a, b) is a *maximum*;
- $K > 0$ and $f_{xx}(a, b) > 0$, then (a, b) is a *minimum*.

Radius of curvature in Cartesian coordinates:

$$\rho_{xy} = \frac{\left[1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right]^{3/2}}{\frac{\mathrm{d}^2y}{\mathrm{d}x^2}}$$

Jacobian matrix

$$\mathbf{J} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

B.2.2 Standard differentials

$f(x)$	$\dfrac{\mathrm{d}f(x)}{\mathrm{d}x}$
x^n	nx^{n-1}
uv	$u\dfrac{\mathrm{d}v}{\mathrm{d}x}+v\dfrac{\mathrm{d}u}{\mathrm{d}x}$
$\dfrac{u}{v}$	$\dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x}-u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\mathrm{sech}^2 x$
$\log_e x=\ln x$	$\dfrac{1}{x}$
e^{ax}	ae^{ax}

B.2.3 Differential equations

The first-order linear equation

$$\frac{\mathrm{d}y}{\mathrm{d}x}+R(x)\,y=S(x)$$

has an integrating factor

$$\lambda(x)=\exp\left[\int R(x)\,\mathrm{d}x\right],$$

so that

$$\frac{\mathrm{d}}{\mathrm{d}x}(y\lambda)=S\lambda.$$

$$P(x,y)\,\mathrm{d}x+Q(x,y)\,\mathrm{d}y=0$$

is an exact equation if

$$\frac{\mathrm{d}P}{\mathrm{d}y}=\frac{\mathrm{d}Q}{\mathrm{d}x}.$$

B.2.4 Integral calculus

An important substitution:

$$\tan\frac{\theta}{2} = t.$$

Then

$$\sin\theta = \frac{2t}{(1+t^2)}$$

$$\cos\theta = \frac{(1-t^2)}{(1+t^2)}$$

and

$$d\theta = \frac{2}{(1+t^2)}\,dt.$$

Table B.1: Some indefinite integrals

$f(x)$	$\int f(x)\,dx$
$\sec x$	$\ln(\sec x + \tan x) = \ln\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)$
$\operatorname{cosec} x$	$\ln(\operatorname{cosec} x - \cot x) = \ln\tan\left(\frac{x}{2}\right)$
$\left(a^2-x^2\right)^{-1/2}$	$\sin^{-1}\left(\frac{x}{a}\right)$, $(\lvert x\rvert < a)$
$\left(a^2+x^2\right)^{-1/2}$	$\sinh^{-1}\left(\frac{x}{a}\right) = \ln\left[x+\left(a^2+x^2\right)^{1/2}\right] - \ln a = \ln\left[\frac{x}{a}+\left(1+\left(\frac{x}{a}\right)^2\right)^{1/2}\right]$
$\left(x^2-a^2\right)^{-1/2}$	$\cosh^{-1}\left(\frac{x}{a}\right) = \ln\left[x+\left(x^2-a^2\right)^{1/2}\right] - \ln a = \ln\left[\frac{x}{a}+\left(\left(\frac{x}{a}\right)^2-1\right)^{1/2}\right]$, $(x \ge a)$
$\left(a^2+x^2\right)^{-1}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
$\left(a^2-x^2\right)^{-1}$	$\frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) = \frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right)$, $(\lvert x\rvert < a)$
$\left(x^2-a^2\right)^{-1}$	$\frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right)$, $(\lvert x\rvert > a)$

Table B.2: Some definite integrals

$$I_n \equiv \int_0^{\pi/2} \sin^n x \, dx = \int_0^{\pi/2} \cos^n x \, dx = \frac{n-1}{n} I_{n-2}, \text{ where } I_0 = \frac{\pi}{2} \text{ and } I_1 = 1$$

$$I_{m,n} \equiv \int_0^{\pi/2} \sin^m x \cos^n x \, dx = \frac{m-1}{m+n} I_{m-2,n} = \frac{n-1}{m+n} I_{m,n-2}, (m > 1, n > 1)$$

$$\int_0^{\infty} e^{-ax} \sin bx \, dx = \frac{b}{\left(a^2+b^2\right)}, (a > 0)$$

$$\int_0^{\infty} e^{-ax} \cos bx \, dx = \frac{a}{\left(a^2+b^2\right)}, (a > 0)$$

$$\int_0^{\infty} e^{-x^2} \, dx = \frac{\sqrt{\pi}}{2}$$

Orthogonality of the trigonometric functions (where m and n are integers):

$$\int_0^L \sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{n\pi x}{L}\right) dx = \begin{cases} 0 & \text{if } m = n = 0 \\ L/2 & \text{if } m = n \neq 0 \\ 0 & \text{if } m \neq n \end{cases}$$

$$\int_0^L \cos\left(\frac{m\pi x}{L}\right) \cos\left(\frac{n\pi x}{L}\right) dx = \begin{cases} L & \text{if } m = n = 0 \\ L/2 & \text{if } m = n \neq 0 \\ 0 & \text{if } m \neq n \end{cases}$$

$$\int_0^L sin\left[(2m-1)\frac{\pi x}{2L}\right] sin\left[(2n-1)\frac{\pi x}{2L}\right] dx = \begin{cases} L/2 & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases}$$

$$\int_0^L cos\left[(2m-1)\frac{\pi x}{2L}\right] cos\left[(2n-1)\frac{\pi x}{2L}\right] dx = \begin{cases} L/2 & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases}$$

B.2.5 Vector calculus

Fundamental theorem of calculus for line integrals

$$\int_C \boldsymbol{\nabla}\phi \cdot \mathrm{d}\mathbf{r} = \phi(\mathbf{b}) - \phi(\boldsymbol{\alpha})$$

Divergence theorem

$$\int_V \boldsymbol{\nabla} \cdot \mathbf{A}\,\mathrm{d}V = \oint_S \mathbf{A} \cdot \hat{\mathbf{n}}\,\mathrm{d}S$$

Stokes' theorem

$$\int_S \boldsymbol{\nabla} \times \mathbf{A} \cdot \hat{\mathbf{n}}\,\mathrm{d}S = \oint_C \mathbf{A} \cdot \mathrm{d}\mathbf{r}$$

Green's theorem

$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy = \oint_C P\,dx + Q\,dy$$

Other related theorems, where f, $\mathbf{f}$ and F are scalar, vector and matrix fields respectively.

$$\int_V \boldsymbol{\nabla} f\,\mathrm{d}V = \oint_S f\,\hat{\mathbf{n}}\,\mathrm{d}S$$

$$\int_V \boldsymbol{\nabla} \times \mathbf{f}\,\mathrm{d}V = \oint_S \hat{\mathbf{n}} \times \mathbf{f}\,\mathrm{d}S$$

$$\int_V div\,F\,\mathrm{d}V = \oint_S F^T\,\hat{\mathbf{n}}\,\mathrm{d}S$$

Vector calculus in polar coordinates:

$$\boldsymbol{\nabla}\varphi \equiv \frac{\partial \varphi}{\partial r}\mathbf{u}_r + \frac{1}{r}\frac{\partial \varphi}{\partial \theta}\mathbf{u}_\theta$$

$$\boldsymbol{\nabla} \cdot \mathbf{v} = \frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}$$

$$\boldsymbol{\nabla}^2 \varphi \equiv \boldsymbol{\nabla} \cdot (\boldsymbol{\nabla}\varphi) = \frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \theta^2}$$

B.3 Statistics

B.3.1 Probabilities for events

For *events* A, B and C:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

The *odds* in favour of A are:

$$\frac{P(A)}{P(\overline{A})}$$

Conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad (\text{if } P(B) > 0)$$

The *chain rule*:

$$P(A \cap B \cap C) = P(A)P(B|A)P(C|A \cap B)$$

Bayes' rule:

$$P(A|B) = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(\overline{A})P(B|\overline{A})}$$

A and B are *independent* if

$$P(B|A) = P(B)$$

A, B and C are independent if

$$P(A \cap B \cap C) = P(A)P(B)P(C),$$

and $P(A \cap B) = P(A)P(B)$, $P(B \cap C) = P(B)P(C)$ and $P(C \cap A) = P(C)P(A)$.

B.3.2 Distribution, expectation and variance

The *probability distribution* for a *discrete* random variable X is the set $\{p_x\}$, where

$$p_x = P(X = x).$$

The *expectation* is

$$E(X) = \mu = \sum_x x p_x$$

From independent observations $x_1, x_2, \dots x_n$, the *sample mean*

$$\overline{x} = \frac{1}{n}\sum_k x_k$$

estimates μ.

The *variance* is

$$\mathrm{var}(X) = \sigma^2 = E\left\{(X-\mu)^2\right\} = E(X^2) - \mu^2,$$

where

$$E(X^2) = \sum_x x^2 p_x.$$

The *sample variance*:

$$s^2 = \frac{1}{n-1}\left\{\sum_k x_k^2 - \frac{1}{n}\left(\sum_j x_j\right)^2\right\}$$

estimates σ^2.

The *standard deviation* is:

$$\mathrm{sd}(X) = \sigma.$$

If the value y is observed with frequency n_y, then

$$n = \sum_y n_y, \qquad \sum_k x_k = \sum_y y n_y, \qquad \sum_k x_k^2 = \sum_y y^2 n_y.$$

For a function $g(x)$ of x,

$$E\{g(x)\} = \sum_x g(x) p_x.$$

B.3.3 Probability distributions for a continuous random variable

The *cumulative distribution function* (cdf) is

$$F(x) = P(X \leq x) = \int_{x_0=-\infty}^{x} f(x_0)\,\mathrm{d}x_0$$

The *probability density function* (pdf) is

$$f(x) = \frac{\mathrm{d}F(x)}{\mathrm{d}x}$$

$$\mu = \int_{-\infty}^{\infty} x f(x)\,\mathrm{d}x, \qquad \sigma^2 = E(X^2) - \mu^2$$

where

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,\mathrm{d}x$$

B.3.4 Discrete probability distributions

Binomial distribution *Binomial* (n, θ)

$$p_x = \binom{n}{x} \theta^x (1-\theta)^{n-x} \qquad (x = 0, 1, 2, \ldots, n)$$

$\mu = n\theta$, $\sigma^2 = n\theta(1-\theta)$.

Poisson distribution *Poisson* (λ)

$$p_x = \frac{\lambda^x e^{-\lambda}}{x!} \qquad (x = 0, 1, 2, \ldots) \qquad (\text{with } \lambda > 0)$$

$\mu = \lambda$, $\sigma^2 = \lambda$.

B.3.5 Continuous probability distributions

Uniform distribution *Uniform* (α, β)

$$f(x) = \begin{cases} \dfrac{1}{\beta - \alpha} & (\alpha < x < \beta) \\ 0 & (\text{otherwise}). \end{cases} \qquad \mu = \frac{\alpha + \beta}{2}, \sigma^2 = \frac{(\beta - \alpha)^2}{12}$$

Exponential distribution *Exponential* (λ)

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & (0 < x < \infty), \\ 0 & (-\infty < x \leq 0). \end{cases} \qquad \mu = \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2}.$$

Normal distribution $N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{1}{2} \left(\frac{x - \mu}{\sigma} \right)^2 \right\} \quad (-\infty < x < \infty), \qquad E(X) = \mu, \qquad \mathrm{var}(X) = \sigma^2$$

Standard normal distribution is $N(0, 1)$

If X is $N(\mu, \sigma^2)$, then $Y = \dfrac{X - \mu}{\sigma}$ is $N(0, 1)$.

B.3.6 System reliability

For a system of k devices, which operate independently, let

$$R_i = P(D_i) = P(\text{“device } i \text{ operates”}).$$

The *system reliability*, R, is the probability of a path of operating devices.

A system of devices *in series* fails if any device fails:

$$R = P(D_1 \cap D_2 \cap \cdots \cap D_k) = R_1 R_2 \ldots R_k$$

A system of devices *in parallel* operates if any device operates:

$$R = P(D_1 \cup D_2 \cup \cdots \cup D_k) = 1-(1-R_1)(1-R_2)\ldots(1-R_k)$$

B.4 Bias, standard error and mean square error

If t estimates θ and comes from a distribution having random variable T:

- *Bias* of t: $\quad \text{bias}(t) = E(T)-\theta$
- *Standard error* of t: $\quad \text{se}(t) = \text{sd}(t)$
- *Mean square error* of t: $\quad \text{MSE}(t) = E\left\{(t-\theta^2\right\} = \{\text{se}(t)\}^2 + \{\text{bias}(t)\}^2$

if $\overline{x}$ estimates μ, then $\text{bias}(\overline{x}) = 0$, $\text{se}(\overline{x}) = \dfrac{\sigma}{\sqrt{n}}$, $\text{MSE}(\overline{x}) = \dfrac{\sigma^2}{n}$, $\widehat{\text{se}}(\overline{x}) = \dfrac{s}{\sqrt{n}}$

B.4.1 Central limit property

If n is fairly large, $\overline{x}$ is approximately from $N\left(\mu, \dfrac{\sigma^2}{n}\right)$.

B.4.2 Confidence intervals

If $x_1, x_2, \ldots, x_n$ are independent observations from $N(\mu, \sigma^2)$ and σ^2 is known, then the *95% confidence interval* for μ is $\left(\overline{x} - 1.96\dfrac{\sigma}{\sqrt{n}}, \overline{x} + 1.96\dfrac{\sigma}{\sqrt{n}}\right)$.

If σ^2 is estimated then, from the table of $t_{(n-1)}$, we find $t_0 = t_{(n-1),0.05}$. Then the 95% CI for μ is $\left(\overline{x} - t_0\dfrac{s}{\sqrt{n}}, \overline{x} + t_0\dfrac{s}{\sqrt{n}}\right)$.

Table B.3: Standard normal table: values of pdf $\phi(y) = f(y)$ and cdf $\Phi(y) = F(y)$.

y	$\phi(y)$	$\Phi(y)$	y	$\phi(y)$	$\Phi(y)$	y	$\phi(y)$	$\Phi(y)$	y	$\Phi(y)$
0	0.399	0.5	0.9	0.266	0.816	1.8	0.079	0.964	2.8	0.997
0.1	0.397	0.540	1.0	0.242	0.841	1.9	0.066	0.971	3.0	0.998
0.2	0.391	0.579	1.1	0.218	0.864	2.0	0.054	0.977	0.841	0.8
0.3	0.381	0.618	1.2	0.194	0.885	2.1	0.044	0.982	1.282	0.9
0.4	0.368	0.655	1.3	0.171	0.903	2.2	0.035	0.986	1.645	0.95
0.5	0.352	0.691	1.4	0.150	0.919	2.3	0.028	0.989	1.96	0.975
0.6	0.333	0.726	1.5	0.130	0.933	2.4	0.022	0.992	2.326	0.99
0.7	0.312	0.758	1.6	0.111	0.945	2.5	0.018	0.994	2.576	0.995
0.8	0.290	0.788	1.7	0.094	0.955	2.6	0.014	0.995	3.09	0.999

Table B.4: Student t table: values $t_{m,p}$ of x for which $P(|X| > x) = p$, when X is t_m.

	p	0.10	0.05	0.02	0.01		p	0.10	0.05	0.02	0.01
m	1	6.31	12.71	31.82	63.66	m	9	1.83	2.26	2.82	3.25
	2	2.92	4.30	6.96	9.92		10	1.81	2.23	2.76	3.17
	3	2.35	3.18	4.54	5.84		12	1.78	2.18	2.68	3.05
	4	2.13	2.78	3.75	4.60		15	1.75	2.13	2.60	2.95
	5	2.02	2.57	3.36	4.03		20	1.72	2.09	2.53	2.85
	6	1.94	2.45	3.14	3.71		25	1.71	2.06	2.48	2.78
	7	1.89	2.36	3.00	3.50		40	1.68	2.02	2.42	2.70
	8	1.86	2.31	2.90	3.36		∞	1.645	1.96	2.326	2.576

B.5 Numerical methods

B.5.1 Interpolation

Methods to find an nth degree interpolation polynomial $p_n(x)$ of a function $f(x)$:

Lagrange:	$p_n(x) = \sum_{j=0}^{n} y_j \prod_{\substack{k=0 \\ k\neq j}}^{n} \frac{(x-x_k)}{(x_j-x_k)}$
Equal Spacing Newton Forward Divided Difference:	$p_n(x) = \sum_{k=0}^{n} \binom{r}{k} \Delta^k y_0$
Equal Spacing Newton Backward Divided Difference:	$p_n(x) = \sum_{k=0}^{n} (-1)^k \binom{-r}{k} \nabla^k y_0$

where $r = (x - x_0)/h$

Interpolation error:

$$\epsilon_n(x) = \frac{f^{n+1}(\xi)}{(n+1)!} \prod_{j=0}^{n} (x - x_j)$$

Cubic spline interpolation:

$$q_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3$$

where:

$$a_j = y_j$$

$$b_j = v_j$$

$$c_j = \frac{3\nabla y_{j+1}}{(x_{j+1} - x_j)^2} - \frac{v_{j+1} + 2v_j}{x_{j+1} - x_j}$$

$$d_j = \frac{-2\nabla y_{j+1}}{(x_{j+1} - x_j)^3} + \frac{v_{j+1} + v_j}{(x_{j+1} - x_j)^2}$$

$$v_j = q_j'(x_j)$$

$$\frac{v_{j-1}}{x_j - x_{j-1}} + 2\left(\frac{1}{x_j - x_{j-1}} + \frac{1}{x_{j+1} - x_j}\right) v_j + \frac{v_{j+1}}{x_{j+1} - x_j} = 3\left(\frac{\nabla y_j}{(x_j - x_{j-1})^2} + \frac{\nabla y_{j+1}}{(x_{j+1} - x_j)^2}\right)$$

Bilinear interpolation:

$$p_2(x, y) = a_0 + a_1 x + a_2 y + a_3 xy$$

where:

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \frac{1}{(y_1 - y_2)(x_1 - x_2)} \begin{bmatrix} x_2 y_2 & -x_1 y_2 & -x_2 y_1 & x_1 y_1 \\ -y_2 & y_2 & y_1 & -y_1 \\ -x_2 & x_1 & x_2 & -x_1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} f(x_1, y_1) \\ f(x_2, y_1) \\ f(x_1, y_2) \\ f(x_2, y_2) \end{bmatrix}$$

Barycentric interpolation:

$$f(r) = \lambda_1 f(r_1) + \lambda_2 f(r_2) + \lambda_3 f(r_3)$$

where:

$$\lambda_1 = \frac{(y_2 - y_3)(x - x_3) + (x_3 - x_2)(y - y_3)}{(y_2 - y_3)(x_1 - x_3) + (x_3 - x_2)(y_1 - y_3)}$$

$$\lambda_2 = \frac{(y_3 - y_1)(x - x_3) + (x_1 - x_3)(y - y_3)}{(y_2 - y_3)(x_1 - x_3) + (x_3 - x_2)(y_1 - y_3)}$$

$$\lambda_3 = 1 - \lambda_1 - \lambda_2$$

B.5.2 Numerical differentiation/Finite difference approximations

Forward	$y_n^k \approx \dfrac{\Delta^k y_n}{h^k}$	and	$\Delta^k y_n = \sum_{i=0}^{k} (-1)^i {k \atop i} y_{n+(k-i)}$
Backward	$y_n^k \approx \dfrac{\nabla^k y_n}{h^k}$	and	$\nabla^k y_n = \sum_{i=0}^{k} (-1)^i \binom{k}{i} y_{n-i}$
Central	$y_n^k \approx \dfrac{\delta^k y_n}{h^k}$	and	$\delta^k y_n = \sum_{i=0}^{k} (-1)^i \binom{k}{i} y_{n+(k/2-i)}$

Binomial coefficient:

$$\binom{a}{b} = \frac{a!}{b!\,(a-b)!}$$

B.5.3 Numerical integration

Estimating $I = \int_A^B f(x)\,dx$ with a subinterval width $h = (B - A)/N$:

Rectangular Rule:

$$I \approx h \sum_{n=1}^{N} f(x_n^*)$$

Compound Trapezium Rule:

$$I \approx h\left(\frac{f(A)}{2} + \sum_{n=1}^{N-1} f(x_n) + \frac{f(B)}{2}\right)$$

Compound Simpsons 1/3 Rule:

$$I \approx \frac{h}{3}\left(y_0 + y_N + 4\sum_{\substack{i=1 \\ i\ \text{odd}}}^{N-1} y_i + 2\sum_{\substack{i=2 \\ i\ \text{even}}}^{N-2} y_i\right)$$

Trapezium Rule Error:

$$-\frac{(B-A)\,h^2}{12} f^2(\xi)$$

Simpsons Rule Error:

$$-\frac{(B-A)\,h^4}{180}f^4\,(\xi)$$

Richardson's error estimation formula for use with Simpson's rule:

Let

$$I=\int_a^b f(x)\,\mathrm{d}x$$

and let I_1, I_2 be two estimates of I obtained using Simpson's rule with intervals h_1 and h_2, where $h_2 < h_1$ (i.e. $h_1 = \frac{b-a}{n_1}$, $h_2 = \frac{b-a}{n_2}$, where n_1, n_2 are even). Then a better estimate of I is given by:

$$I = I_2 + \frac{(I_2 - I_1)}{\left[\left(\frac{h_1}{h_2}\right)^4 - 1\right]}.$$

If $h_2 = \frac{1}{2}h_1$ then $I = I_2 + \frac{1}{15}(I_2 - I_1)$.

Gauss Quadrature:

$$I \approx \frac{B-A}{2}\sum_{j=1}^{n} w_j f\left(t_j\right) \quad \text{and} \quad x = \frac{1}{2}\left[A\,(1-t) + B\,(t+1)\right]$$

Number of Gauss Nodes n	**Nodes** t_j	**Weightings** w_j	**Degree of Precision**
1	0	2	1
2	$\pm\frac{1}{\sqrt{3}}$	1	3
3	$\pm\sqrt{\frac{3}{5}}$	$\frac{5}{9}$	5
	0	$\frac{8}{9}$	
4	$\pm\sqrt{\frac{3}{7}+\frac{2}{7}\sqrt{\frac{6}{5}}}$	$\frac{18-\sqrt{30}}{36}$	7
	$\pm\sqrt{\frac{3}{7}-\frac{2}{7}\sqrt{\frac{6}{5}}}$	$\frac{18+\sqrt{30}}{36}$	

B.5.4 Numerical methods for solving ODEs

General form a multistep method:

$$\sum_{i=0}^{s} \alpha_i y_{n+1-i} = h \sum_{i=0}^{s} \beta_i f_{n+1-i}$$

Forward Euler: $y_{n+1} \approx y_n + hf(t_n, y_n)$

Backward Euler: $y_{n+1} \approx y_n + hf(t_{n+1}, y_{n+1})$

Heun:
$$\begin{aligned} k_1 &= hf(t_n, y_n) \\ k_2 &= hf(t_n + h, y_n + k_1) \\ y_{n+1} &= y_n + \frac{1}{2}(k_1 + k_2) \end{aligned}$$

4th Order Runge-Kutta:
$$\begin{aligned} k_1 &= hf(t_n, y_n) \\ k_2 &= hf\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1\right) \\ k_3 &= hf\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2\right) \\ k_4 &= hf(t_n + h, y_n + k_3) \\ y_{n+1} &= y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \end{aligned}$$

4th Order Adams Bashford: $$y_{n+1} = y_n + \frac{h}{24}(55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3})$$

2nd Order Backward Differentiation Formula: $$y_{n+1} = \frac{4}{3}y_n - \frac{1}{3}y_{n-1} + \frac{2h}{3}f(t_{n+1}, y_{n+1})$$

where $t_{n+1} = t_n + h$ and $f_n = f(t_n, y_n)$

Convergence $\Leftrightarrow$ consistency and zero-stability.

Consistency:

$$\lim_{h \to 0}\left(\frac{\varepsilon_{local}}{h}\right) = 0$$

Zero-stability test equation:

$$y_n = z^n y_0$$

Eigenvalue decomposition of a matrix **M**:

$$\mathbf{M} = \mathbf{V}\boldsymbol{\Lambda}\mathbf{V}^{-1}$$

where **V** is a matrix of eigenvectors, and $\boldsymbol{\Lambda}$ a matrix of eigenvalues (λ_i).

The spectral condition number, S is defined as:

$$S = \frac{\max|\lambda_i|}{\min|\lambda_i|}$$

B.5.5 Numerical methods for solving PDEs

PDE classification:

$$Au_{xx} + 2Bu_{xy} + Cu_{yy} = f(x, y, u, u_x, u_y)$$

Type	Condition
Elliptic	$B^2 - 4AC < 0$
Parabolic	$B^2 - 4AC = 0$
Hyperbolic	$B^2 - 4AC > 0$

Non-dimensionalisation of the diffusion equation:

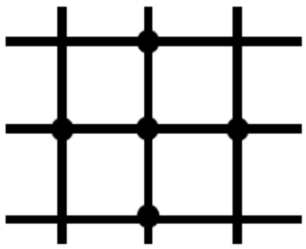
$$\tilde{x} = \frac{x}{L}, \quad \tilde{u} = \frac{u}{u_0}, \quad \tilde{t} = \frac{\alpha}{L^2} t$$

Non-dimensionalisation of the wave equation:

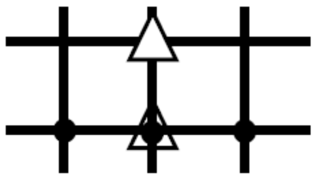
$$\tilde{x} = \frac{x}{L}, \quad \tilde{u} = \frac{u}{u_0}, \quad \tilde{t} = \frac{c}{L} t$$

Finite Difference Stencils

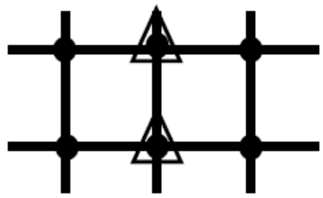
Laplace:



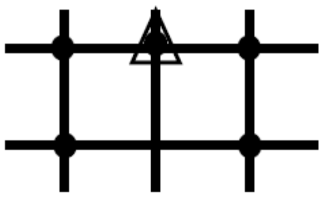
Explicit for parabolic:



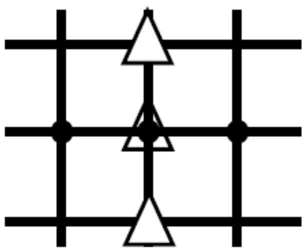
Crank-Nicolson for parabolic, general:



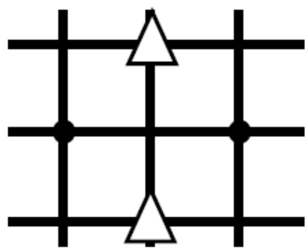
Crank-Nicolson for parabolic, $r = k/h^2 = 1$:



Explicit for hyperbolic General:



Explicit for hyperbolic, $r = k^2/h^2 = 1$:



where circles = space discretisation, and triangles = time discretisation.

Finite difference for Poisson Equation in 2D with irregular boundaries:

$$\frac{2}{h^2}\left(\frac{u_A}{a(a+c)} + \frac{u_C}{(a+c)c} - \frac{u_O}{ac}\right) + \frac{2}{k^2}\left(\frac{u_B}{b(b+d)} + \frac{u_D}{(b+d)d} - \frac{u_O}{bd}\right) = f(x, y)$$

$$u_O = u(x,\ y); u_A = u(x - ah,\ y); u_B = u(x,\ y - bk); u_C = u(x + ch,\ y); u_D = u(x,\ y + dk)$$

Finite volume flux equation for the 2D Laplace equation:

$$\left[k_e A_e \left(\frac{\partial u}{\partial x}\right)_e - k_w A_w \left(\frac{\partial u}{\partial x}\right)_w\right] + \left[k_n A_n \left(\frac{\partial u}{\partial y}\right)_n - k_s A_s \left(\frac{\partial u}{\partial y}\right)_s\right] + \bar{S}\Delta \mathrm{A} = 0$$

Linearised source term:

$$\bar{S}\Delta\ \mathrm{A} = S_u + S_p u_P$$

Nodal point equation:

$$a_P u_P = a_E u_E + a_W u_W + a_N u_N + a_S u_S + S_u$$

Where:

$$a_W = \frac{k_w A_w}{\delta x_{WP}};\ a_E = \frac{k_e A_e}{\delta x_{PE}};\ a_S = \frac{k_s A_s}{\delta y_{SP}};\ a_N = \frac{k_n A_n}{\delta y_{PN}};\ a_P = a_W + a_E + a_S + a_N - S_p$$

B.5.6 Approximate solution of an algebraic equation

- An iterative method for $x = \phi(x)$ converges when $\left|\phi'(x)\right| < 1$ near the root: if a root occurs near to $x = a$ take $x_0 = a$ and

$$x_{n+1} = \phi(x_n), n = 0, 1, 2, \ldots$$

- If a root of $f(x) = 0$ occurs near to $x = a$, take $x_0 = a$ and:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, n = 0, 1, 2, \ldots$$

  (the *Newton-Raphson method*).

B.6 Transforms

Dirac Delta:

$$\delta\,(x - x_0) = \begin{cases} \infty & \text{if } x = x_0 \\ 0 & \text{otherwise} \end{cases} \qquad \text{and} \qquad \int_{-\infty}^{\infty} \delta\,(x - x_0)\,dt = 1$$

Dirac Delta sifting property:

$$\int_{-\infty}^{\infty} \delta\,(x - x_0) f\,(x)\,dx = f\,(x_0)$$

and

$$\int_{0}^{\infty} \delta\,(x - a) f\,(x)\,dx = f\,(a)$$

for any $a > 0$.

B.6.1 Laplace transform

$$F(s) = \int_0^\infty f(t)\, e^{-st} dt$$

	Function/Signal	**Laplace Transform**
Linearity:	$af(t) + bg(t)$	$aF(s) + bG(s)$
Shift in time:	$f(t-a)\, u(t-a)$	$e^{-as}F(s)$
Shift in frequency:	$e^{at}f(t)$	$F(s-a)$
First Derivative:	$\dfrac{df(t)}{dt}$	$sF(s) - f(0)$
Second Derivative:	$\dfrac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0) - f'(0)$
Integral	$\int_0^t f(t)\,\mathrm{d}t$	$\dfrac{F(s)}{s}$
Scaling	$f(at)$	$\dfrac{1}{\lvert a\rvert}F\left(\dfrac{s}{a}\right)$
Constant (k)	$kf(t)$	$kF(s)$

Common transform pairs:	$\sin \omega t$	$\dfrac{\omega}{\left(s^2+\omega^2\right)}$ $(s > 0)$
	$\cos \omega t$	$\dfrac{s}{\left(s^2+\omega^2\right)}$ $(s > 0)$
	e^{at}	$\dfrac{1}{s-a}$ $(s > a)$
	e^{-at}	$\dfrac{1}{s+a}$
	$1 - e^{-at}$	$\dfrac{a}{s(s+a)}$
	$u(t)$	$\dfrac{1}{s}$
	$\delta(t)$	1
	$u(t)\, e^{-at}$	$\dfrac{1}{s+a}$
	$u(t) \cos at$	$\dfrac{s}{s^2+a^2}$
	$u(t) \sin at$	$\dfrac{a}{s^2+a^2}$

Laplace Transforms continue over page

Laplace Transforms continued

Function	Transform
$tf(t)$	$-\dfrac{\mathrm{d}F(s)}{\mathrm{d}s}$
$\dfrac{\partial f(t,a)}{\partial a}$	$\dfrac{\partial F(s,a)}{\partial a}$
$\int_0^t f(u)g(t-u)\,\mathrm{d}u$	$F(s)G(s)$
$t^n,\ n=1,2\ldots$	$\dfrac{n!}{s^{n+1}} \quad (s>0)$
$t^n\ (n>0)$	$\dfrac{\Gamma(1+n)}{s^{1+n}}^{\dagger}$
$\dfrac{1}{(n-1)!}t^{n-1}e^{-at}$	$\dfrac{1}{(s+a)^n}$
$\dfrac{1}{(b-a)}\left(e^{-at}-e^{-bt}\right)$	$\dfrac{1}{(s+a)(s+b)}$
$\dfrac{1}{(b-a)}\left[(c-a)e^{-at}-(c-b)e^{-bt}\right]$	$\dfrac{s+c}{(s+a)(s+b)}$
$1-\dfrac{b}{(b-a)}e^{-at}+\dfrac{a}{(b-a)}e^{-bt}$	$\dfrac{ab}{s(s+a)(s+b)}$
$\dfrac{e^{-at}}{(b-a)(c-a)}+\dfrac{e^{-bt}}{(c-b)(a-b)}+\dfrac{e^{-ct}}{(a-c)(b-c)}$	$\dfrac{1}{(s+a)(s+b)(s+c)}$
$c-\dfrac{b(c-a)}{(b-a)}e^{-at}+\dfrac{a(c-b)}{(b-a)}e^{-bt}$	$\dfrac{ab(s+c)}{s(s+a)(s+b)}$
$\dfrac{\sqrt{(a^2+\omega^2)}}{\omega}\sin(\omega t+\phi),\ \phi=\tan^{-1}\left(\dfrac{\omega}{a}\right)$	$\dfrac{s+a}{\left(s^2+\omega^2\right)} \quad (s>0)$
$e^{-at}\sin\omega t$	$\dfrac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos\omega t$	$\dfrac{s+a}{(s+a)^2+\omega^2}$
$\dfrac{1}{\omega}\sqrt{(c-a)^2+\omega^2}\,e^{-at}\sin(\omega t+\phi),\ \phi=\tan^{-1}\left(\dfrac{\omega}{c-a}\right)$	$\dfrac{(s+c)}{(s+a)^2+\omega^2}$
$\dfrac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{(1-\zeta^2)}\,t,\ (\zeta<1)$	$\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$
$\dfrac{1}{a^2+\omega^2}-\dfrac{1}{\omega\sqrt{a^2+\omega^2}}e^{-at}\sin(\omega t-\phi),\ \phi=\tan^{-1}\dfrac{-\omega}{a}$	$\dfrac{1}{s\left[(s+a)^2+\omega^2\right]}$
$1-\dfrac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t+\phi\right),\ \phi=\cos^{-1}\zeta,\ \zeta<1$	$\dfrac{\omega_n^2}{s\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}$
$H(t-T) \quad (=0,\ t<T;\ =1,\ t>T)$	$\dfrac{1}{s}e^{-sT} \quad (s,T>0)$
$f(t-a)H(t-a)$	$e^{-as}F(s)$

† Γ is the Gamma function such that $\Gamma(n+1)=n\Gamma(n)=n!$

B.6.2 Fourier Transform

The Fourier transform and its inverse:

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t} dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{i\omega t} d\omega$$

	Function/Signal	**Fourier Transform**
Linearity:	$af(t) + bg(t)$	$aF(\omega) + bG(\omega)$
Shift in time:	$f(t-a)$	$e^{-ia\omega}F(\omega)$
Shift in frequency:	$e^{iat}f(t)$	$F(\omega - a)$
Differentiation:	$\frac{df(t)}{dt}$	$i\omega F(\omega)$
Scaling:	$f(at)$	$\frac{1}{\lvert a\rvert}F\left(\frac{\omega}{a}\right)$
Common transform pairs:	1	$2\pi\delta(\omega)$
	$\delta(t)$	1
	e^{iat}	$2\pi\delta(\omega - a)$
	$\cos at$	$\pi[\delta(\omega - a) + \delta(\omega + a)]$
	$\sin at$	$\frac{\pi}{i}[\delta(\omega - a) - \delta(\omega + a)]$
	$u(t)$	$\pi\delta(\omega) + \frac{1}{i\omega}$

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C Mechatronics and Control

Contents

C.1 Logic

De Morgan's Law

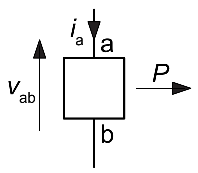
$$\mathrm{A} + \mathrm{B} = \overline{(\overline{\mathrm{A}} \cdot \overline{\mathrm{B}})}$$

C.2 Charge, current, voltage and power

Nomenclature

q	charge
i	current $= \dfrac{\mathrm{d}q}{\mathrm{d}t}$
v	electrical potential (*voltage*)
P	power leaving network
U	energy stored
R	resistance
C	capacitance
L	inductance

Subscript and arrow notations



Potential difference:

$$v_{\mathrm{ab}} = v_{\mathrm{a}} - v_{\mathrm{b}}$$

Power P leaving network between terminals a and b:

$$P = v_{\mathrm{ab}} i_{\mathrm{a}}$$

Passive components

Resistor:	$v = iR$ (Ohm's law)	Power dissipated: $P = i^2 R = \dfrac{v^2}{R}$
Inductor:	$v = L\dfrac{\mathrm{d}i}{\mathrm{d}t}$	Energy stored $U = \frac{1}{2}Li^2$
Capacitor:	$i = C\dfrac{\mathrm{d}v}{\mathrm{d}t}$, $q = Cv$	Energy stored $U = \frac{1}{2}Cv^2$

C.2.1 Colour codes for resistors etc.

Colour	Digit		
Black	0	Green	5
Brown	1	Blue	6
Red	2	Violet	7
Orange	3	Grey	8
Yellow	4	White	9

C.2.2 Standard values for components

E3 series	E6 series	E12 series	E24 series
10	10	10	10
			11
		12	12
			13
	15	15	15
			16
		18	18
			20
22	22	22	22
			24
		27	27
			30
	33	33	33
			36
		39	39
			43
47	47	47	47
			51
		56	56
			63
	68	68	68
			75
		82	82
			91

C.3 Networks

Kirchhoff's voltage law (KVL):

$$\sum(\text{p.d.s around loop}) = 0$$

Kirchhoff's current law (KCL):

$$\sum(\text{currents into node}) = 0$$

Resistors in series:

$$R_{\text{ser}} = R_1 + R_2 + \ldots$$

Resistors in parallel:

$$\frac{1}{R_{\text{par}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

i.e. for two resistors

$$R_{\mathrm{par}} = \frac{R_1 R_2}{R_1 + R_2}$$

Potential divider (with R_2 as output resistor):

$$v_{\mathrm{out}} = \frac{R_2}{R_1 + R_2} v_{\mathrm{in}}$$

C.4 Transients

$x(t)$	=	instantaneous voltage v or current i
X_0	=	initial value $x(0)$
X_{f}	=	final (steady state) value $x(\infty)$.
τ	=	time constant.

At time $t = 0$ a switch operates so that the network of resistors and d.c. voltage sources connected to a capacitor or inductor changes, instantaneously. Then for $t \geq 0$:

$$x(t) = X_{\mathrm{f}} - (X_{\mathrm{f}} - X_0) \exp\left(-\frac{t}{\tau}\right)$$

For a capacitor:

- v remains unchanged through $t = 0$;
- $i \to 0$ as $t \to \infty$;
- $\tau = R_s C$

For an inductor:

- i remains unchanged through $t = 0$;
- $v \to 0$ as $t \to \infty$;
- $\tau = \frac{L}{R}$

C.5 Sensors and transducers

Nomenclature

G	gauge factor
e	mechanical strain
V	load cell excitation voltage (V)
T_0	reference temperature (°C for RTD; K for thermistors)
R_0	reference resistance (Ω)
β	RTD constant (Ω/K)
B	thermistor constant (K)

Strain gauge

$$\frac{\mathrm{dR}}{R} = Ge$$

Full bridge load cell (four complementary strain gauges)

$$v_{out} = VGe$$

Platinum resistance thermometer

$$R = R_0 + \beta(T - T_0)$$

Thermistor

$$\frac{R}{R_0} = \exp\left[\mathrm{B}\left(\frac{1}{\mathrm{T}} - \frac{1}{\mathrm{T_0}}\right)\right]$$

Thermocouples

$$E = E_x(T, T_{amb}) - E_y(T, T_{amb}) = E_{xy}(T, T_{amb})$$

C.6 AC networks

Nomenclature

X_m	*peak amplitude* (or semi-amplitude)
X_av	mean value $x(\infty)$.
X_pp	$2X_\mathrm{m}$ is *peak-to-peak* amplitude
f	frequency (Hz), and $\omega = 2\pi f$ (rad s^{-1})
T	$\frac{1}{f}$ is period

C.6.1 Average and root mean square values

General definitions for any periodic waveform:

$$X_\mathrm{av} = \frac{1}{T}\int_0^T x\,\mathrm{d}t$$

$$X_\mathrm{rms} = \sqrt{\frac{1}{T}\int_0^T x^2\,\mathrm{d}t}$$

For a waveform consisting of N samples of equal duration:

$$X_\mathrm{rms} = \sqrt{\frac{1}{N}\sum_{n=1}^{N} x_n{}^2}$$

For a sinusoidal waveform $x = X_\mathrm{m}\sin(\omega t + \phi)$:

$$X_\mathrm{rms} = \frac{1}{\sqrt{2}}X_\mathrm{m}$$

and for a sinusoidal positive half-cycle:

$$X_\mathrm{av} = \frac{2}{\pi}X_\mathrm{m}$$

C.6.2 Rectifiers and smoothing

Ripple Voltage

$$V_\mathrm{rpp} \approx \frac{\mathrm{dV_C}}{\mathrm{dt}} \cdot \Delta t = \frac{V_{DC}}{R_L C} \cdot \frac{1}{f}$$

$$V_{DC} = V_{max} - \frac{1}{2}V_{rpp}$$

C.6.3 Phasors and complex impedance

CIVIL: current leads voltage for a capacitor, voltage leads current for an inductor.

Current is common phasor for series circuits, voltage is common phasor for parallel circuits.

Inductive reactance: $X_L = \omega L$;

Capacitative reactance $X_C = \dfrac{1}{\omega C}$.

Complex impedance: $\overline{V} = \bar{I} \cdot \overline{Z}$ where $\overline{V}$, $\bar{I}$, and $\overline{Z}$ are complex quantities, and

$$\overline{\overline{Z}} = R \pm \mathrm{j}X$$

where $\overline{\overline{Z}}$ is impedance.

C.6.4 3-phase electrical power

For a three phase, star-connected voltage source with RMS voltage V_S:

Phase Voltage

$$V_\mathrm{P} = V_\mathrm{S}$$

Line voltage

$$V_\mathrm{L} = \sqrt{3} V_\mathrm{P}$$

Power factor

$$P_\mathrm{real} = V_\mathrm{S} I \cos\phi$$

Star-connected load

$$P_\mathrm{S\ Total} = 3\frac{{V_\mathrm{S}}^2}{R}$$

Delta-connected load

$$P_\mathrm{D\ Total} = 9\frac{{V_\mathrm{S}}^2}{R}$$

C.6.5 Electromagnetism

Nomenclature

N	number of coil turns
ℓ	magnetic flux path length (m)
ϕ	magnetic flux (Wb)
B	magnetic flux density (T)
μ_r	relative permeability
μ_0	permeability of free space (H/m)
A	cross-sectional area of magnetic flux path (m^2)
L	length of conductor in magnetic field (m)
S	reluctance (A/Wb)

Magnetomotive force

$$MMF = Ni$$

Flux density

$$B = \frac{\phi}{A}$$

Reluctance of magnetic path

$$S = \frac{\mathrm{MMF}}{\phi} = \frac{\ell}{\mu_r \mu_0 A}$$

energy in a magnetic field

$$energy = \frac{1}{2}\phi^2 S$$

Magnetic force of attraction

$$F = \frac{B^2 A}{2\mu}$$

Force acting on a conductor

$$F = BiL$$

Induced e.m.f.

$$E = N\frac{\mathrm{d}\phi}{\mathrm{d}t}$$

Inductance of a coil

$$L = \frac{\mu_r \mu_0 A N^2}{\ell}$$

C.6.6 Transformers

N_1 = primary turns

N_2 = secondary turns

Ideal transformer:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1},$$

C.7 DC machines

Nomenclature

T_e	electromagnetic torque (Nm)
T_s	shaft torque (Nm)
T_f	frictional torque (Nm)
K_e	electric machine induced emf constant (Vs/rad)
K_t	electric machine torque constant (Nm/A)
R_a	armature resistance (Ω)
v_a	armature voltage (V)
i_a	armature current (A)
ω	angular velocity (rad/s)
K_{cf}	constant friction coefficient (Nm)
K_f	speed dependent friction coefficient (Nm s/rad)

Induced EMF speed relationship for DC machine

$$E = K_e\omega$$

Electromagnetic torque

$$T_e = K_t i_a$$

where

$$K_t = K_e$$

Constant frictional torque

$$T_f = -\text{sign}(\omega)K_{cf}$$

Speed dependent frictional torque

$$T_f = -K_f\omega$$

Operating relationship

$$v_a = E + i_a R_a$$

Torque

$$J\dot{\omega} = T_e + T_f + T_s$$

where J is the moment of inertia of the electric machine.

The system can be simplified by ignoring the moment of inertia of the electric machine ($J = 0$) such that

$$T_e + T_f + T_s = 0$$

C.8 AC machines

Nomenclature

ω_{rotor}	rotational speed of the rotor (rpm)
ω_{stator}	rotational speed of the magnetic fields in the stator (rpm)
f	frequency of the emf (Hz)
p	number of poles
$Slip$	difference in speed between rotor and stator
T	torque of the motor

Synchronous speed of 3-phase AC synchronous machine

$$\omega_{\text{rotor}} = \frac{120 * f}{p}$$

Slip relationship

$$Slip = \frac{\omega_{\text{stator}} - \omega_{\text{rotor}}}{\omega_{\text{stator}}}$$

Kloss equation for torque

$$T = \frac{2T_{max}}{\dfrac{slip}{slip_{max}} + \dfrac{slip_{max}}{slip}} = \frac{P}{\omega_{\text{rotor}}}$$

C.9 Communications

Information (in bits) communicated by each of N equally probable messages:

$$I = \frac{1}{\log_{10} 2} \log N$$

C.10 Step function response and frequency response

Nomenclature

$\theta_{\text{in}}, \theta_{\text{out}}$	input and output variables
τ	time constant
ω_{n}	natural frequency
ζ	damping factor
H	gain
ϕ	phase shift

The *transfer function* for any linear system is generally expressed as a linear function of the Laplace variable s.

C.10.1 First-order systems

Transfer function of first order low pass (lag):

$$\frac{\theta_{\text{out}}}{\theta_{\text{in}}} = \frac{1}{1 + \tau s}$$

Figure C.1 shows the time plot for response to a unit step input:

$$\theta_{\text{in}} = \begin{cases} 0 & (t < 0) \\ 1 & (t > 0) \end{cases}$$

Gain (power ratio) in decibels (dB): $|H| = 20 \log_{10} \left(\dfrac{V_{\text{out}}}{V_{\text{in}}} \right)$.

The gain and phase relationships for first-order *passive* and *active* low-pass and high-pass

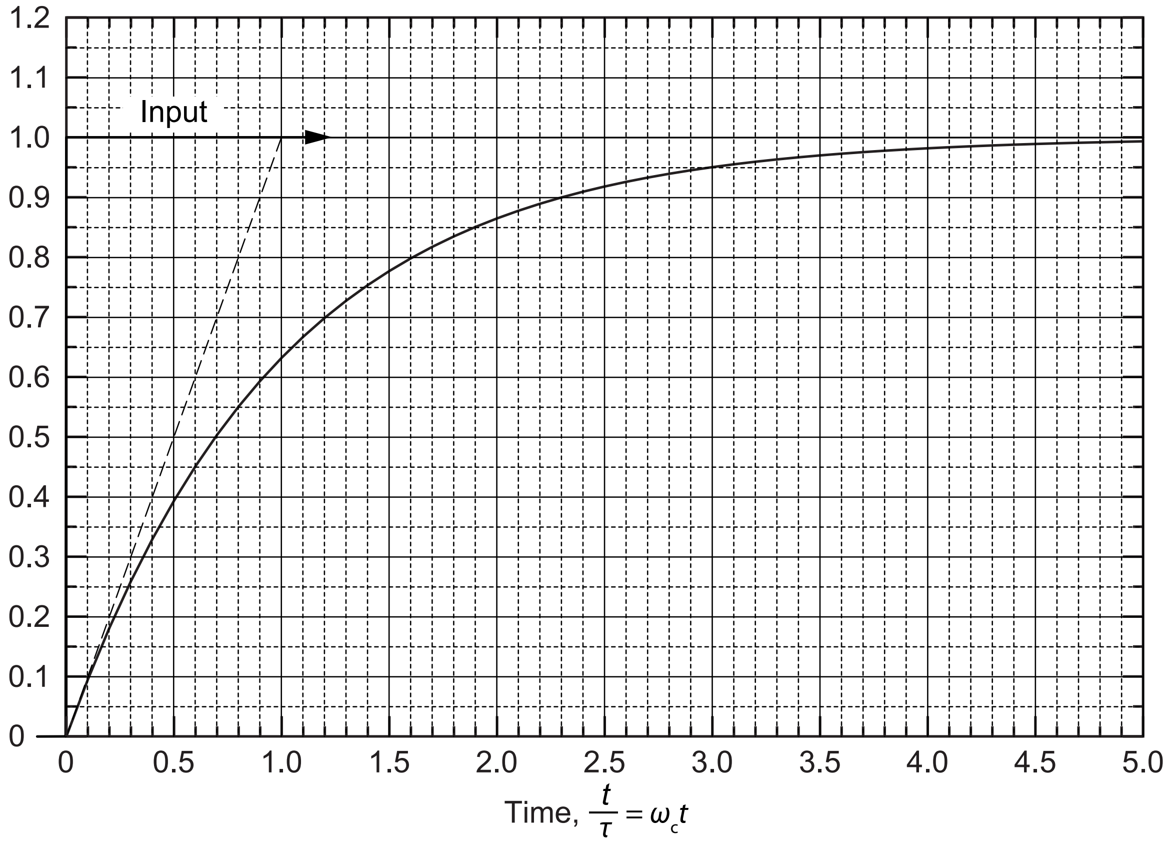


Figure C.1: Step response of a first-order low pass filter

filters are shown in Table C.1):

	Low-pass filter	High-pass filter
Passive	$\lvert H\rvert = \dfrac{1}{\sqrt{1+(\omega RC)^2}}$ $\phi = -\tan^{-1}(\omega RC)$	$\lvert H\rvert = \dfrac{\omega RC}{\sqrt{1+(\omega RC)^2}}$ $\phi = 90° - \tan^{-1}(\omega RC)$
Active	$\lvert H\rvert = \dfrac{R_2}{R_1}\dfrac{1}{\sqrt{1+(\omega R_2 C)^2}}$ $\phi = 180° - \tan^{-1}(\omega R_2 C)$	$\lvert H\rvert = \dfrac{C_1}{C_2}\dfrac{\omega RC_2}{\sqrt{1+(\omega RC_2)^2}}$ $\phi = -90° - \tan^{-1}(\omega RC)$

Figure C.2 shows the Bode plots for sinusoidal input $\theta_{in} = \hat{\theta_{in}} \sin(\omega t - \phi)$ to first-order low-pass and high-pass filters.

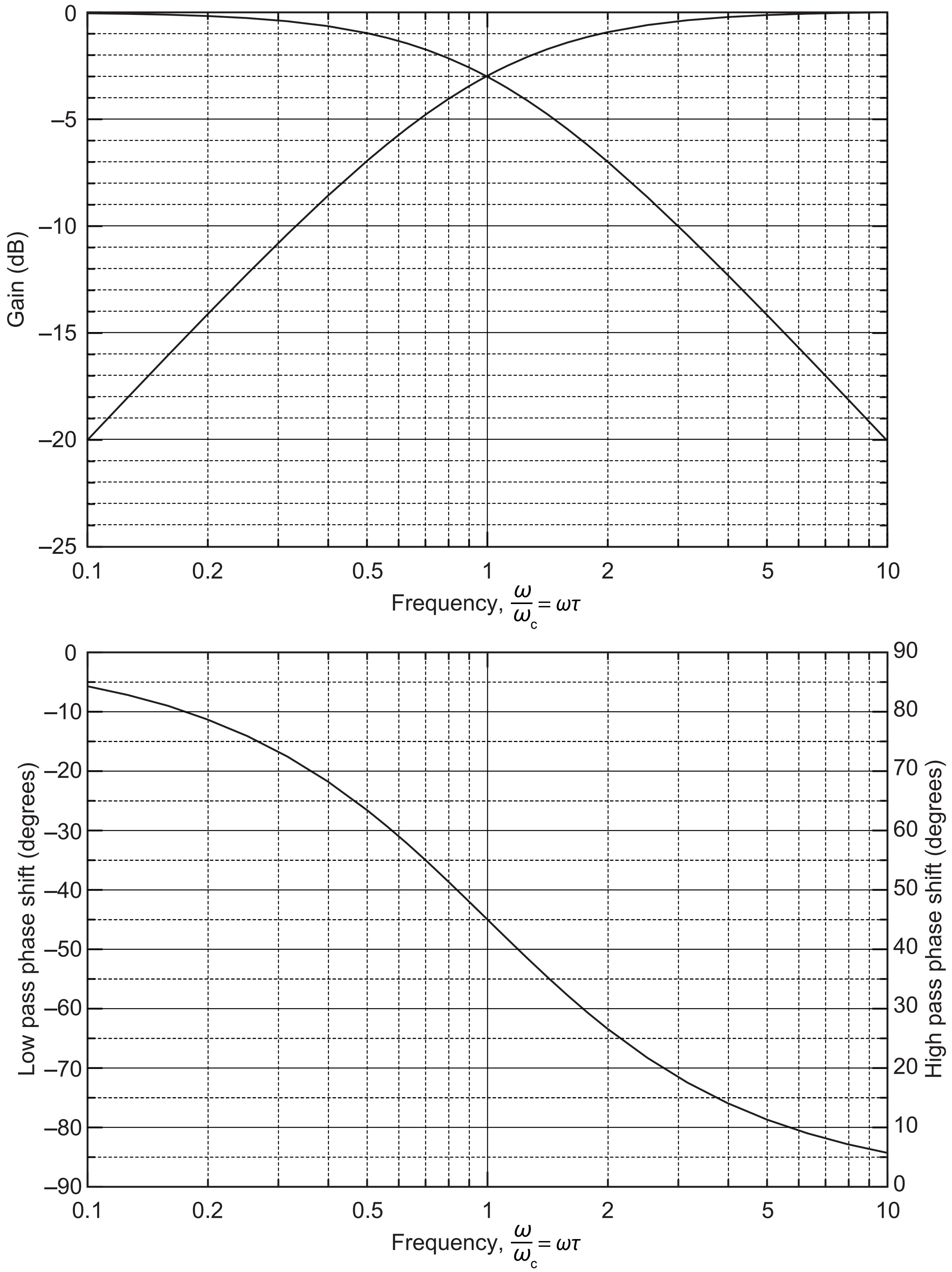


Figure C.2: Bode plot for first-order low and high pass filters

C.10.2 Second-order systems

Transfer function of a second-order low-pass system:

$$\frac{\theta_{\text{out}}}{\theta_{\text{in}}} = \frac{1}{\left(1 + 2\dfrac{\zeta}{\omega_n}s + \dfrac{1}{\omega_n^2}s^2\right)}$$

Unit step and frequency response are shown in Figs. C.3 and C.4.

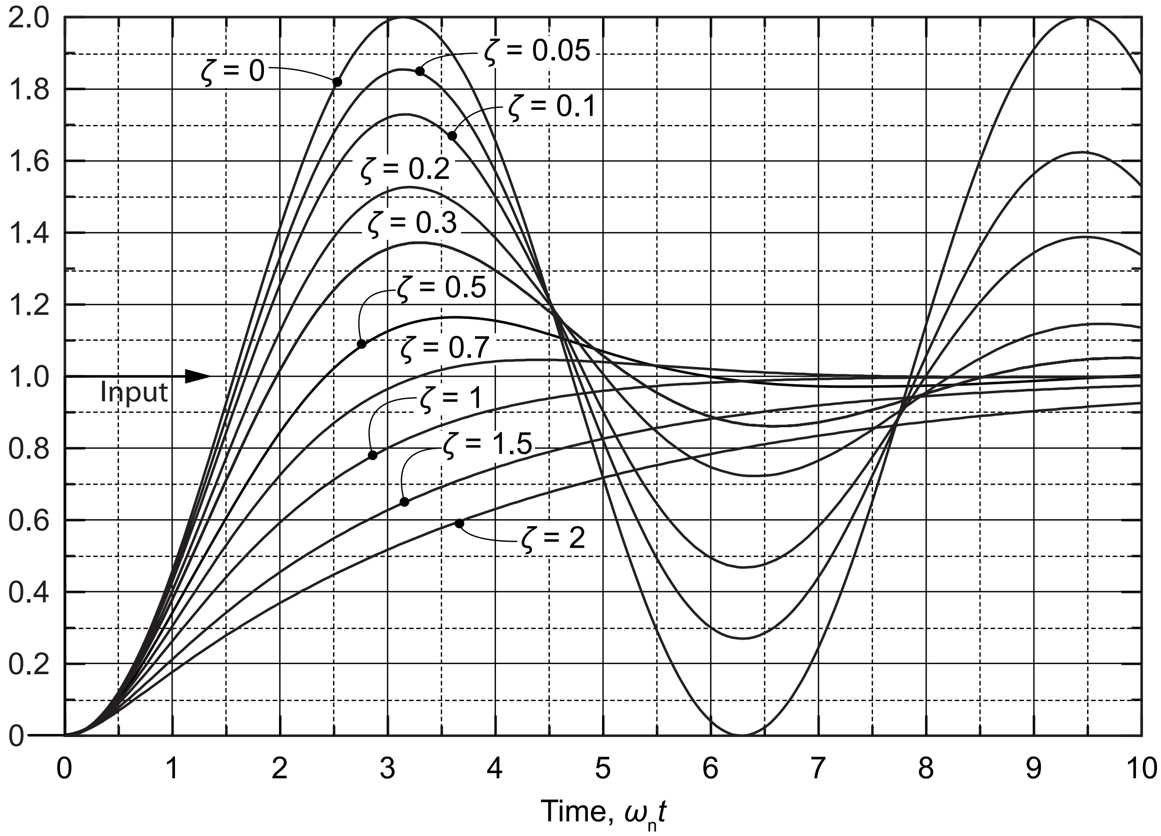


Figure C.3: Step response of a second-order low pass filter

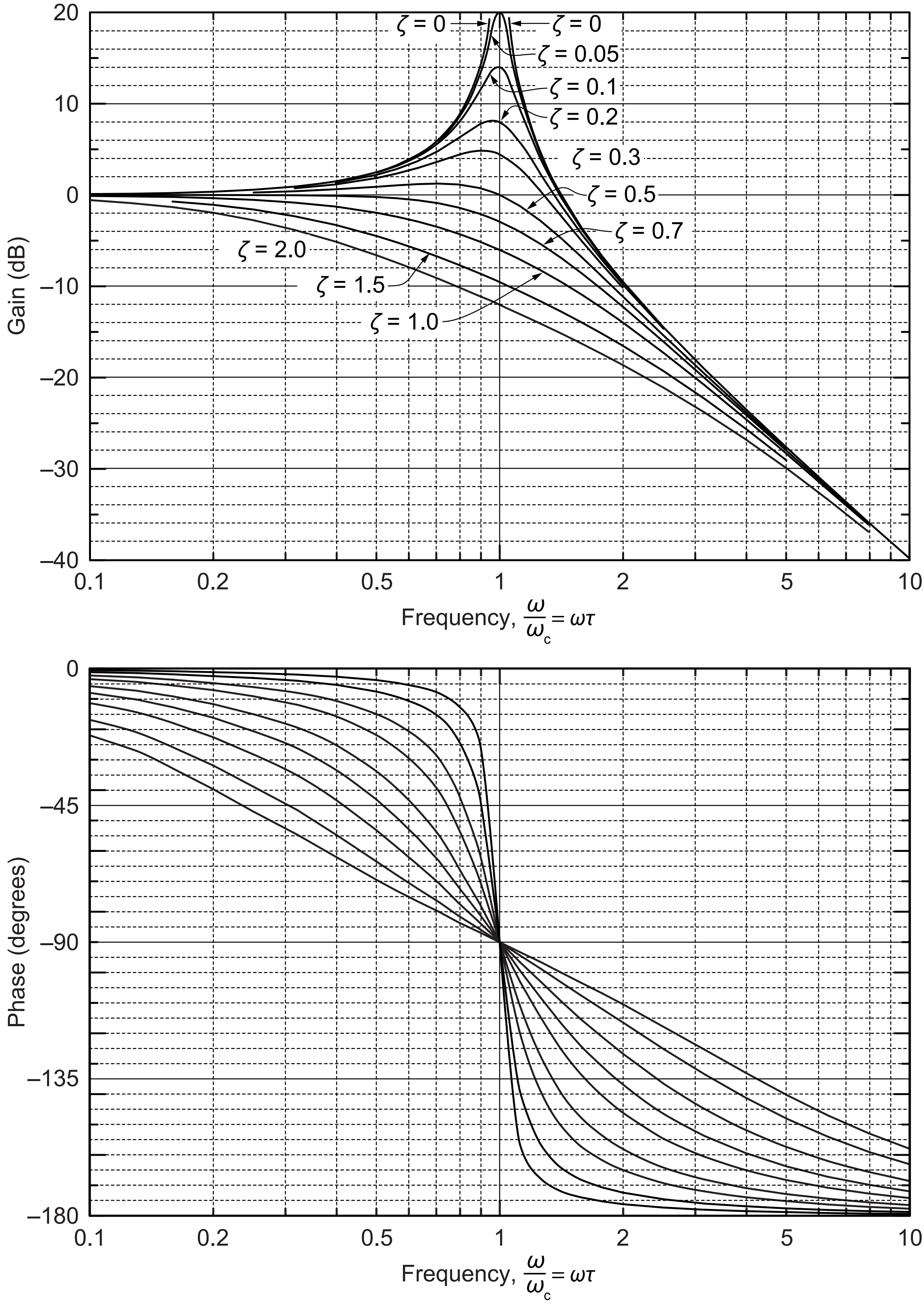
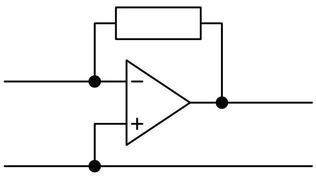


Figure C.4: Bode plot for a second-order low pass filter

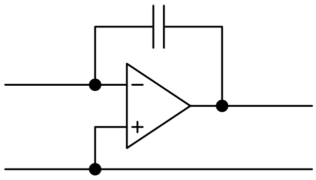
C.11 Operational amplifier stages

Table C.1 shows op-amp networks which implement various signal processing operations.

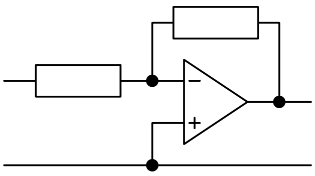
Table C.1: Operational amplifier signal processing stages



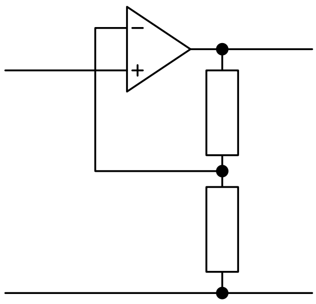
Current to voltage



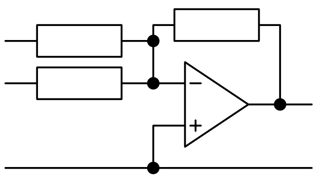
Charge to voltage



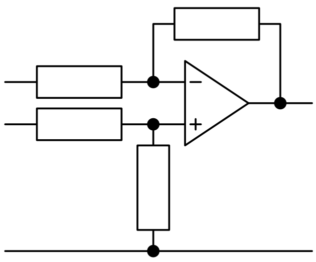
Inverting amp



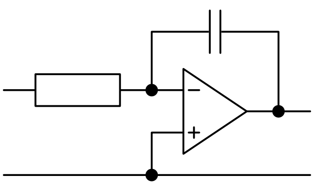
Non-inverting amp



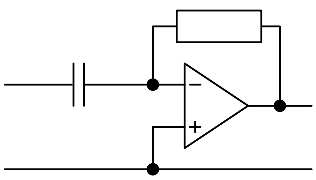
Summing amp



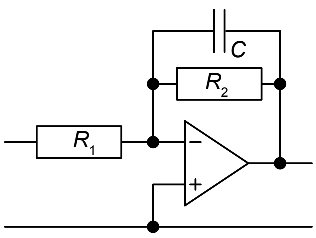
Difference amp



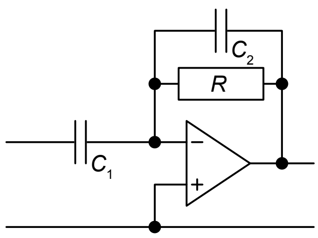
Integrator



Differentiator



First order low pass



First order high pass

C.12 Laplace and z-transforms for advanced control

Time domain	Laplace domain	Discrete domain	Z-domain
$x(t),\ t>0$	$X(s)=\int_0^\infty x(t)e^{-st}dt$	$x(nT)$	$X(z)=\sum_{k=0}^{k=\infty} x(kT)z^{-k}$
$\delta(t)$	1	$1,\ n=0;0,\ n\neq 0$	1
$\delta(t-kT)$	e^{-kTs}	$1,\ n=k;0,\ n\neq k$	z^{-k}
$1,\ t>0$	$\frac{1}{s}$	$1(nT)$	$\frac{z}{z-1}$
t	$\frac{1}{s^2}$	nT	$\frac{Tz}{(z-1)^2}$
$\frac{t^2}{2!}$	$\frac{1}{s^3}$	$\frac{1}{2!}(nT)^2$	$\frac{T^2}{2}\frac{z(z+1)}{(z-1)^3}$
$\frac{t^3}{3!}$	$\frac{1}{s^4}$	$\frac{1}{3!}(nT)^3$	$\frac{T^3}{6}\frac{z(z^2+4z+1)}{(z-1)^4}$
t^{m-1}	$\frac{(m-1)!}{s^m}$	$(kT)^{m-1}$	$\lim_{a\to 0}(-1)^{m-1}\frac{\partial^{m-1}}{\partial a^{m-1}}\left[\frac{z}{z-e^{-aT}}\right]$
e^{-at}	$\frac{1}{s+a}$	e^{-anT}	$\frac{z}{z-e^{-aT}}$
te^{-at}	$\frac{1}{(s+a)^2}$	nTe^{-anT}	$\frac{Tze^{-aT}}{\left(z-e^{-aT}\right)^2}$
$t^m e^{-at}$	$\frac{(m-1)!}{(s+a)^m}$	$(nT)^m e^{-amT}$	$(-1)^m\frac{\partial^m}{\partial a^m}\left[\frac{z}{z-e^{-aT}}\right]$
$1-e^{-at}$	$\frac{a}{s(s+a)}$	$1-e^{-anT}$	$\frac{z\left(1-e^{-aT}\right)}{(z-1)\left(z-e^{-aT}\right)}$
$\frac{1}{a}\left(at-1+e^{-at}\right)$	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}\left(anT-1+e^{-anT}\right)$	$\frac{z\left[\left(aT-1+e^{-aT}\right)z+\left(1-e^{-aT}-aTe^{-aT}\right)\right]}{a(z-1)^2\left(z-e^{-aT}\right)}$
$\left(e^{-at}-e^{-bt}\right)$	$\frac{b-a}{(s+a)(s+b)}$	$\left(e^{-anT}-e^{-bnT}\right)$	$\frac{\left(e^{-aT}-e^{-bT}\right)z}{\left(z-e^{-aT}\right)\left(z-e^{-bT}\right)}$

Time domain	Laplace domain	Discrete domain	Z-domain
$(1-at)\,e^{-at}$	$\dfrac{s}{s(s+a)^2}$	$(1-anT)\,e^{-anT}$	$\dfrac{z\left[z-e^{-aT}\,(1+aT)\right]}{\left(z-e^{-aT}\right)^2}$
$1-e^{-at}\,(1-at)$	$\dfrac{a^2}{s(s+a)^2}$	$1-e^{-anT}\,(1-anT)$	$\dfrac{z\left[z\left(1-e^{-aT}-aTe^{-aT}\right)+e^{-2aT}-e^{-aT}+aTe^{-aT}\right]}{(z-1)\left(z-e^{-aT}\right)^2}$
$be^{-bt}-ae^{-at}$	$\dfrac{(b-a)\,s}{(s+a)\,(s+b)}$	$be^{-bnT}-ae^{-anT}$	$\dfrac{z\left[z\,(b-a)-\left(be^{-aT}-ae^{-bT}\right)\right]}{\left(z-e^{-aT}\right)\left(z-e^{-bT}\right)}$
$\sin at$	$\dfrac{a}{s^2+a^2}$	$\sin anT$	$\dfrac{z\sin aT}{z^2-(2\cos aT)\,z+1}$
$\cos at$	$\dfrac{s}{s^2+a^2}$	$\cos anT$	$\dfrac{z\,(z-\cos aT)}{z^2-(2\cos aT)\,z+1}$
$e^{-at}\ \cos bt$	$\dfrac{s+a}{(s+a)^2+b^2}$	$e^{-anT}\ \cos bnT$	$\dfrac{z\left(z-e^{-aT}\cos bT\right)}{z^2-2e^{-aT}\,(\cos bT)\,z+e^{-2aT}}$
$e^{-at}\ \sin bt$	$\dfrac{b}{(s+a)^2+b^2}$	$e^{-anT}\ \sin bnT$	$\dfrac{ze^{-aT}\sin bT}{z^2-2e^{-aT}\,(\cos bT)\,z+e^{-2aT}}$
$1-e^{-at}\left(\cos bt+\dfrac{a}{b}\ \sin bt\right)$	$\dfrac{a^2+b^2}{s\left[(s+a)^2+b^2\right]}$	$1-e^{-anT}\left(\cos bnT+\dfrac{a}{b}\ \sin bnT\right)$	$\dfrac{z\,(Az+B)}{(z-1)\left(z^2-2e^{-aT}\,(\cos bT)\,z+e^{-2aT}\right)}$, $A=1-e^{-aT}\cos bT-\dfrac{a}{b}e^{-aT}\ \sin bT$, $B=e^{-2aT}+\dfrac{a}{b}e^{-aT}\ \sin bT-e^{-aT}\cos bT$

D Solid Mechanics

Contents

D.1 Mechanics and dynamics

D.1.1 Square screw threads

M	=	moment required to *raise* an axially loaded nut
W	=	axial load on nut
ϕ	=	helix angle of thread
μ_s	=	static coefficient of friction
d_m	=	mean thread diameter

$$M = \left(\frac{\tan\phi + \mu_s}{1 - \mu_s \tan\phi}\right) W \frac{d_m}{2}$$

D.1.2 Belt friction

T_1, T_2	=	Belt tensions with $T_1 > T_2$
μ_s	=	static coefficient of friction
α	=	angle of lap

$$\frac{T_1}{T_2} = e^{\mu_s \alpha}$$

D.1.3 Flat clutches

T	=	maximum torque transmitted
F	=	thrust
R_1, R_2	=	outer and inner radii for annular clutch
μ_s	=	static coefficient of friction

For uniform pressure conditions:

$$T = \frac{2}{3}\mu_s F \left(\frac{R_1^3 - R_2^3}{R_1^2 - R_2^2}\right)$$

For uniform wear conditions:

$$T = \mu_s F \left(\frac{R_1 + R_2}{2}\right)$$

D.1.4 Kinematics of particles

$\mathbf{a}$	=	acceleration vector
s	=	distance travelled
v	=	tangential velocity
t	=	time
$\mathbf{e}_\mathrm{n}, \mathbf{e}_\mathrm{t}$	=	unit vectors in n-t coordinates
$\mathbf{e}_\mathrm{r}, \mathbf{e}_\theta$	=	unit vectors in r-θ coordinates
ρ	=	instantaneous radius of path curvature

For normal and tangential components:

$$\mathbf{a} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}\mathbf{e}_t + \frac{v^2}{\rho}\mathbf{e}_n$$

For polar components:

$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_\theta$$

D.1.5 Kinematics of rigid bodies with sliding contacts

$\mathbf{v}$	=	velocity vector
$\mathbf{a}$	=	acceleration vector
$\mathbf{v}_\mathrm{rel}$	=	velocity vector relative to rotating body (sliding velocity)
$\mathbf{a}_\mathrm{rel}$	=	acceleration vector relative to rotating body (sliding acceleration)
ω	=	angular velocity vector
α	=	angular acceleration vector
$\mathbf{r}$	=	position vector

$$\mathbf{v} = \mathbf{v}_\mathrm{rel} + \omega \times \mathbf{r}$$

$$\mathbf{a} = \mathbf{a}_\mathrm{rel} + 2\omega \times \mathbf{v}_\mathrm{rel} + \alpha \times \mathbf{r} + \omega \times (\omega \times \mathbf{r})$$

D.1.6 Mass moments of inertia

m = total mass of body

G = centre of mass (centre of gravity)

I_G = Mass moment of inertia about G

Body	Mass moment of inertia
Rectangular lamina, $b \times h$	$I_G = \frac{1}{12}m(b^2 + h^2)$
Solid disc, radius r	$I_G = \frac{1}{2}mr^2$
Uniform slender rod, total length L	$I_G = \frac{1}{12}mL^2$
Solid sphere, radius r	$I_G = \frac{2}{5}mr^2$
Spherical shell, radius r	$I_G = \frac{2}{3}mr^2$
Cylindrical ring, radius r	$I_G = mr^2$

D.1.7 Rigid body dynamics

G	=	centre of mass
m	=	mass of rigid body
F	=	force
I_G	=	moment of inertia about G
M_G	=	moment about G
a_G	=	linear acceleration at G
$\alpha = \ddot{\theta}$	=	angular acceleration
$v = \dot{r} = \dot{x} = \dot{y}$	=	linear velocity
$\omega = \dot{\theta}$	=	angular velocity
p	=	linear momentum
H	=	angular momentum
$\frac{\mathrm{d}\mathbf{H}}{\mathrm{d}t}$	=	rate of change of angular momentum
$\mathbf{M}$	=	gyroscopic moment vector
$\boldsymbol{\omega_p}$	=	precession velocity vector
$\boldsymbol{\omega_s}$	=	spin velocity vector

The equations of motion for rigid planar motion are:

$$\sum F_x = m\,(a_G)_x$$

$$\sum F_y = m\,(a_G)_y$$

$$\sum M_G = I_G\ddot{\theta} = I_G\alpha$$

The work done-kinetic energy equations for rigid planar motion are:

$$\int_{r_1}^{r_2}\sum F\mathrm{d}r = \frac{1}{2}m\left(v_2^2 - v_1^2\right) = \frac{1}{2}m\left(\dot{r}_2^2 - \dot{r}_1^2\right)$$

$$\int_{\theta_1}^{\theta_2}\sum M_G\mathrm{d}\theta = \frac{1}{2}I_G\left(\omega_2^2 - \omega_1^2\right) = \frac{1}{2}I_G\left(\dot{\theta}_2^2 - \dot{\theta}_1^2\right)$$

The impulse-momentum equations for rigid planar motion are:

$$\int_{t_1}^{t_2}\sum F\mathrm{d}t = m\,(v_2 - v_1) = m\,(\dot{r}_2 - \dot{r}_1) = \Delta p$$

$$\int_{t_1}^{t_2}\sum M_G\mathrm{d}t = I_G\,(\omega_2 - \omega_1) = I_G\left(\dot{\theta}_2 - \dot{\theta}_1\right) = \Delta H_G$$

The angular momentum about a general point, O:

$$H_O = H_G + p_x\,(y_G - y_O) - p_y\,(x_G - x_O) = I_G\dot{\theta} + m\dot{x}_G\,(y_G - y_O) - m\dot{y}_G\,(x_G - x_O)$$

The gyroscopic moment equation is:

$$\frac{\mathrm{d}\mathbf{H}}{\mathrm{d}t} = \boldsymbol{\omega_p} \times\; I\boldsymbol{\omega_s} = \mathbf{M}$$

D.1.8 Mass flow problems

$\mathbf{F}$	=	internal force vector exerted from the emitted mass
$\boldsymbol{\alpha}$	=	acceleration vector
m	=	mass of object
m_{f}	=	emitted mass
$\mathbf{v}_{\mathrm{f}}$	=	velocity vector of emitted mass relative to object

$$m\boldsymbol{\alpha} = -\frac{\mathrm{d}m_{\mathrm{f}}}{\mathrm{d}t}\mathbf{v}_{\mathrm{f}} = \mathbf{F}$$

D.1.9 Vibrations

Undamped SDOF

$$\omega_n = \sqrt{\frac{k}{m}} \quad \text{or} \quad \omega_n = \sqrt{\frac{k_\theta}{I_G}}$$

$$f = \frac{1}{T} = \frac{\omega_n}{2\pi}$$

$$T_{\mathrm{max}} = V_{\mathrm{max}}$$

Damped SDOF

$$\zeta = \frac{c}{2\sqrt{km}}$$

$$x(t) = e^{-\zeta\omega_n t}(Acos\sqrt{1-\zeta^2}\ \omega_n t + Bsin\sqrt{1-\zeta^2}\ \omega_n t\ )$$

$$\sqrt{1-\zeta^2}\ \omega_n = \omega_d$$

$$x\,(t) = e^{-\zeta\omega_n t}\,(A\ cos\ \omega_d t + Bsin\ \omega_d t) + x_0 \sin(\omega t - \varphi)$$

Forced SDOF

$$\frac{|x_0|}{|F|} = \frac{1}{\sqrt{(k-m\omega^2)^2 + (c\omega)^2}} \quad \text{and} \quad \tan\varphi = \frac{c\omega}{k-m\omega^2}.$$

$$\frac{kx_0}{F} = \frac{1}{\sqrt{\left(1-r^2\right)^2 + (2\zeta r)^2}} \quad \text{and}\, \tan\varphi = \frac{2\zeta r}{\left(1-r^2\right)}.$$

$$F_{\mathrm{exi}} = m' r\omega^2$$

Vibration Isolation

$$T = \left|\frac{F_T}{F}\right| = \sqrt{\frac{k^2 + (c\omega)^2}{\left(k-\omega^2\right)^2 + (c\omega)^2}}$$

$$T = \left|\frac{F_T}{F}\right| = \sqrt{\frac{1 + (2\zeta r)^2}{\left(1 - r^2\right)^2 + (2\zeta r)^2}}$$

Base Excitation

$$\left|\frac{x_0}{y_0}\right| = \sqrt{\frac{k^2 + (c\omega)^2}{\left(k - \omega^2\right)^2 + (c\omega)^2}}$$

$$\left|\frac{x_0}{y_0}\right| = \sqrt{\frac{1 + (2\zeta r)^2}{\left(1 - r^2\right)^2 + (2\zeta r)^2}}$$

$$\varphi = \operatorname{atan}\frac{c\omega}{k - m\omega^2} - \operatorname{atan}\frac{c\omega}{k}$$

2DOF

$$\left(\mathbf{K} - \mathbf{M}\omega^2\right)\mathbf{x} = 0,$$

$$\beta_i = \left[\frac{x_2}{x_1}\right]_{\omega_{ni}}$$

$$x_1(t) = A_1 \sin(\omega_{n1} t - \varphi_1) + A_2 \sin(\omega_{n2} t - \varphi_2)$$

$$x_2(t) = \beta_1 A_1 \sin(\omega_{n1} t - \varphi_1) + \beta_2 A_2 \sin(\omega_{n2} t - \varphi_2)$$

D.2 Mechanical properties of materials

D.2.1 Definitions and Characterisation

Stress

Engineering stress, σ $$\sigma = \frac{P}{A_0}$$

True stress, σ_t $$\sigma_t = \frac{P}{A}$$

Their relationship $$\sigma_t = \sigma(1+\varepsilon)$$

where P is the force, A_0 is original and A the current cross-sectional area.

Strain

Engineering strain, ε $$\varepsilon = \frac{x}{l_0}$$

True strain, ε_t $$\varepsilon_t = \ln\left(\frac{l}{l_o}\right) = \ln\left(\frac{A_o}{A}\right)$$

Their relationship $$\varepsilon_t = \ln(1+\varepsilon)$$

where x is the extension of an original length l_o and l is the current length.

Ductility

% Elongation $$\frac{(l_f - l_0)}{l_0} \times 100\%$$

Percentage Reduction of Area (%RA or %ROA) $$\frac{(A_0 - A_f)}{A_0} \times 100\%$$

where l_f is the final length, l_0 the initial length and A_f is the final cross-sectional area.

Elastic properties and volume change

Young's modulus, E $$E = \frac{\sigma}{\varepsilon}$$

Poisson's ratio, ν $$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$

Fractional change in volume, $\delta V/V_0$ $$\frac{\delta V}{V_0} = \varepsilon_x(1-2\nu)$$

ν defined above for an applied tensile strain ε_x, and transverse strains $\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x$, V_0 is the original volume.

Hardness

Vickers hardness, H_V $$H_V = 1.854\frac{P}{d_1^2}$$

Brinell hardness, H_B $$H_B = \frac{2P}{\pi D\left(D-\sqrt{D^2-d^2}\right)}$$

where P is the applied force (in kg), d_1 is the indentation diagonal, D and d the diameter of the indenter and indent respectively (all in mm).

Interatomic bonding

Potential energy in a bond, U $$U = -Ar^{-m} + Br^{-n}$$

Force in a bond, F $$F = \frac{mA}{r^{m+1}} - \frac{nB}{r^{n+1}}$$

Their relationship $$U = \int F dr$$

where r is the interatomic spacing, and A, B, m, n are constants.

Crystal structure and yield stress

Theoretical density, ρ $$\rho = \frac{NM}{V \cdot N_A}$$

Hall-Petch relationship $$\sigma_y = \sigma_i + \frac{k}{\sqrt{d}}$$

where N is the number of atoms in a unit cell, M is the atomic mass, V is the volume of a unit cell, and N_A is Avogadro's number, σ_y is the yield strength, σ_i the lattice strength, d the grain size and k a constant.

Lever rule

In a two-phase solid-liquid system $$(X - x_L)M_L = (x_s - X)M_S$$

Proportion liquid $$M_L = \frac{x_S - X}{x_S - x_L}$$

Proportion solid $$M_S = \frac{X - x_L}{x_S - x_L}$$

where X is an arbitrary composition, x_s and x_L are the compositions of the solid and liquid in the two phase region.

Composites

Critical fibre length, l_c $$l_c = \frac{D}{2} \cdot \frac{\sigma_f}{\tau_B}$$

Modulus, E_c (iso-strain) parallel $$E_c = V_f E_f + \left(1 - V_f\right) E_m$$

Modulus, E_c (iso-stress) transverse $$\frac{1}{E_c} = \frac{V_f}{E_f} + \frac{\left(1 - V_f\right)}{E_m}$$

where D is the fibre diameter, σ_f is the fracture strength of the fibre, τ_B is the fibre-matrix interfacial shear strength, V_f is the fibre volume fraction, E_f and E_m are the tensile modulus of the fibre and matrix respectively.

D.2.2 Elastic constants of materials

ρ = Density
E = Young's modulus, modulus of elasticity
G = Shear modulus, modulus of rigidity
K = Bulk modulus
ν = Poisson's ratio
α = Coefficient of linear thermal expansion

Relationships between elastic constants:

$$G = \frac{E}{2\left(1+\nu\right)} \qquad K = \frac{E}{3\left(1-2\nu\right)}$$

Some typical values:

	ρ $\mathrm{kg\,m^{-3}}$	E GPa	G GPa	K GPa	ν	α $10^{-6}\mathrm{K}^{-1}$
Mild steel	7850	207	80	175	0.3	11
Aluminium alloy	2720	69	27	69	0.3	23
Brass	8410	103	38	117	0.35	19
Titanium alloy	5000	110	42	110	0.31	11
Hardwood (oak) along grain	920	23	2	3	0.37	7
Hardwood (oak) across grain	920	9	1	3	0.03	44
Concrete	2400	20	9	9	0.15	10
Water	1000			2		51 (ice @ 0° C)

D.2.3 Failure properties

Fracture

Stress Intensity Factor Solution, K

$$K = Y\sigma\sqrt{a}$$

Plastic zone size, r_y, ahead of a Mode I loaded crack

$$r_y = \frac{1}{\beta\pi}\left(\frac{K}{\sigma_y}\right)^2$$

where $\beta = 2$ for plane stress and $\beta = 6$ under plane strain conditions.

Plastic zone correction for K

$$K_{eff} = \sigma\sqrt{a_{eff}}\,Y\left(\frac{a_{eff}}{W}\right) \quad ; \quad a_{eff} = a + r_y$$

ASTM size requirement for K_{IC} testing

$$a,\ (W-a),\ B > 2.5\left(\frac{K_{IC}}{\sigma_y}\right)^2$$

The stress field directly ahead of a sharp crack for a linear-elastic material

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$$

Paris law for fatigue crack growth

$$\frac{da}{dN} = B\Delta K^m$$

Fatigue

Alternating stress σ_a	$\sigma_a = \dfrac{\sigma_{\max} - \sigma_{\min}}{2}$
Mean stress σ_m	$\sigma_m = \dfrac{\sigma_{\max} + \sigma_{\min}}{2}$
R Ratio	$R = \dfrac{\sigma_{\min}}{\sigma_{\max}}$

where σ_{max} and σ_{min} are the cyclic maximum and minimum stress respectively.

Integrated Paris law

$$\frac{2}{2-m}\left[\frac{1}{a_f^{\frac{m}{2}-1}} - \frac{1}{a_i^{\frac{m}{2}-1}}\right] = B\,(Y\Delta\sigma)^m\,N$$

Fatigue strength reduction factor

$$K_f = \frac{\sigma_{fs,\text{unnotched}}}{\sigma_{fs,\text{notched}}}$$

Goodman law

$$\frac{\sigma_a}{\sigma_{fs,0}} + \frac{\sigma_m}{UTS} = 1$$

Notch sensitivity index

$$q = \frac{K_f - 1}{K_t - 1}$$

Arrhenius rate law and creep

Arrhenius rate law	$D \propto e^{-Q/RT}$
Steady-state creep rate, $\dot{\varepsilon}_{ss}$	$\dot{\varepsilon}_{ss} = Ae^{-Q/RT} \cdot \sigma^n$
Creep modulus, $E_c(t)$	$E_c(t) = \dfrac{\sigma_0}{\varepsilon(t)}$
Larson-Miller parameter, P_{LM}	$P_{LM} = T(\log t_r + C)$

Corrosion

Change in mass during oxidation according to linear law

$$\Delta m = k_L t \quad ; \quad k_L = A_L e^{-Q_L/RT}$$

Change in mass during oxidation according to parabolic law

$$(\Delta m)^2 = k_p t \quad ; \quad k_p = A_p e^{-Q_p/RT}$$

Pilling-Bedworth Ratio

$$\frac{\text{Vol of Oxide Produced by Oxidation}}{\text{Vol of Oxide Consumed by Oxidation}} = \frac{M_{\text{oxide}}\ \rho_{\text{metal}}}{n\ M_{\text{metal}}\ \rho_{\text{oxide}}}$$

Weight change in corrosion

$$W = \frac{ItM}{nF} = \frac{iAtM}{nF}$$

Dislocation

The critical resolved shear stress

$$\tau_R = \frac{Fcos\lambda}{A/cos\phi} = \sigma(\cos\phi \cdot \cos\lambda)$$

Shear stress for solid solution strengthening

$$\tau_y = \frac{f_i}{\mathbf{b}} + \frac{f_{ss}}{\mathbf{b}}$$

Grain size evolution as a function of time at a given temperature

$$\left(d^n - d_0^n\right) \propto\ t$$

Line tension

$$\tau \mathbf{b} L = 2T$$

Friction caused by precipitates

$$f_0 = \frac{2T}{L}$$

Force on a dislocation

$$F = \tau\ b$$

Shear stress needed to curve a dislocation

$$\tau = \frac{\alpha G b}{R}$$

Shear stress due to the dislocation density

$$\tau_y = \tau_i + \alpha\ G b \sqrt{\rho}$$

Hall-Petch shear yield strength relationship

$$\tau_y = \tau_i + \frac{k}{\sqrt{d}}$$

von Mises equivalent stress in terms of direct and shear stresses

$$\sigma_{eq} = \frac{1}{\sqrt{2}} \left[\left(\sigma_{xx} - \sigma_{yy}\right)^2 + \left(\sigma_{yy} - \sigma_{zz}\right)^2 + (\sigma_{zz} - \sigma_{xx})^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2\right)\right]^{\frac{1}{2}}$$

D.3 Elementary stress analysis formulation

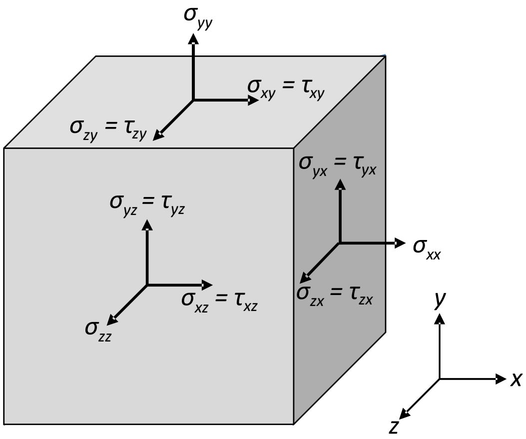


Figure D.1: Three dimensional stress systems

D.3.1 Equilibrium of stresses

3D problem, static case, no body forces, Cartesian coordinates:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = 0$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = 0$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = 0$$

3D, static case, no body forces, polar coordinates:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} \cdot \frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r} \cdot \frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \sigma_{\theta z}}{\partial z} + \frac{2 \cdot \sigma_{r\theta}}{r} = 0$$

$$\frac{\partial \sigma_{rz}}{\partial r} + \frac{1}{r} \cdot \frac{\partial \sigma_{\theta z}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} = 0$$

D.3.2 Strains and compatibility

Definition of infinitesimal strains (3D case):

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \qquad \varepsilon_{yy} = \frac{\partial v}{\partial y} \qquad \varepsilon_{zz} = \frac{\partial w}{\partial z}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \qquad \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \qquad \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

Compatibility of infinitesimal strains (2D case):

$$\frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2}$$

D.3.3 Constitutive equation for elasticity (Hooke's Law including thermal stresses)

$$\varepsilon_{xx} = \frac{1}{E} \cdot \left[\sigma_{xx} - \nu \cdot (\sigma_{yy} + \sigma_{zz}) \right] + \alpha \cdot \Delta T$$

$$\varepsilon_{yy} = \frac{1}{E} \cdot \left[\sigma_{yy} - \nu \cdot (\sigma_{zz} + \sigma_{xx}) \right] + \alpha \cdot \Delta T$$

$$\varepsilon_{zz} = \frac{1}{E} \cdot \left[\sigma_{zz} - \nu \cdot (\sigma_{xx} + \sigma_{yy}) \right] + \alpha \cdot \Delta T$$

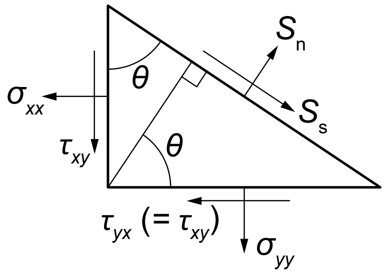
$$\gamma_{xy} = \frac{\sigma_{xy}}{G}$$

$$\gamma_{yz} = \frac{\sigma_{yz}}{G}$$

$$\gamma_{xz} = \frac{\sigma_{xz}}{G}$$

D.4 Transformation of stresses and strains

D.4.1 Normal and shear stress components



$$S_{\mathrm{n}} = \frac{\sigma_{xx}+\sigma_{yy}}{2} + \frac{\sigma_{xx}-\sigma_{yy}}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$S_{\mathrm{s}} = \frac{\sigma_{xx}-\sigma_{yy}}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

Principal stresses:

$$\sigma_1, \sigma_2 = \frac{\sigma_{xx}+\sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

The direction of the principal stresses (and of the normal to the principal planes) to the x axis is θ_{p} where:

$$\tan 2\theta_{\mathrm{p}} = \frac{2\tau_{xy}}{\sigma_{xx}-\sigma_{yy}}$$

Maximum shear stress: The maximum shear stress is half the difference of the principal stresses and acts on planes at 45° to the principal planes.

D.4.2 Normal and shear strain components

$$\epsilon_n = \frac{\epsilon_{xx}+\epsilon_{yy}}{2} + \frac{\epsilon_{xx}-\epsilon_{yy}}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$
$$\frac{\epsilon_s}{2} = \frac{\epsilon_{xx}-\epsilon_{yy}}{2}\sin 2\theta - \frac{\gamma_{xy}}{2}\cos 2\theta$$

where:

- ϵ_{xx}, ϵ_{yy} and ϵ_n are the direct strains acting in the same directions as, respectively, the stresses σ_{xx}, σ_{yy} and S_n above;
- γ_{xy} and ϵ_s are the shear strains associated with the stresses τ_{xy} and S_s

NOTE: the relevant strain relationships may be obtained from the stress relationships by substituting the appropriate direct stresses by the associated direct strain and shear stresses by *one half* of the associated shear strain.

D.4.3 Rotation of stresses in 2D problems ($Ox'y'$ rotated by θ from Oxy)

$$\sigma_{x'x'} = \sigma_{xx}\cdot\cos^2\theta + \sigma_{yy}\cdot\sin^2\theta + 2\cdot\sigma_{xy}\cdot\cos\theta\cdot\sin\theta$$
$$\sigma_{y'y'} = \sigma_{xx}\cdot\sin^2\theta + \sigma_{yy}\cdot\cos^2\theta - 2\cdot\sigma_{xy}\cdot\cos\theta\cdot\sin\theta$$
$$\sigma_{x'y'} = \left(\sigma_{yy}-\sigma_{xx}\right)\cdot\cos\theta\cdot\sin\theta + \sigma_{xy}\cdot\left(\cos^2\theta - \sin^2\theta\right)$$

D.5 Mechanics of structures

D.5.1 Elementary beam theory

σ	=	axial stress at axial position x and vertical distance y from neutral axis
τ	=	shear stress in vertical axial plane
d	=	total depth of beam (i.e. $d = y_{max} - y_{min}$)
M	=	bending moment about neutral axis at x
S	=	shear force at x
I	=	second moment of area about neutral axis
R	=	radius of curvature at x
v	=	vertical deflection at x
y	=	height above the neutral axis.

Bending about a principal axis :

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R} = -E\frac{\mathrm{d}^2 v}{\mathrm{d}x^2}$$

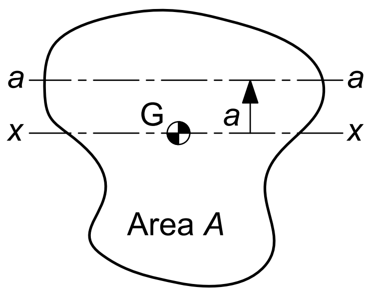
Moment curvature relationship

$$EI\frac{d^2 v}{dx^2} = M$$

$$EI\frac{dv}{dx} = \text{End slope}$$

$$EIv = \text{End deflection}$$

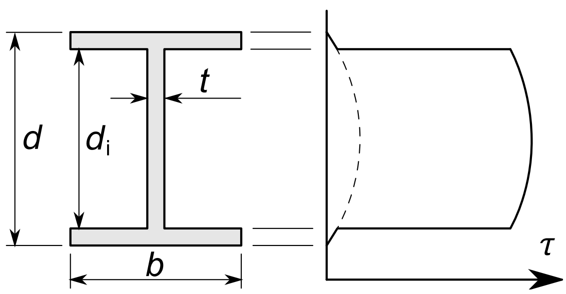
Parallel axis theorem:



$$I_{aa} = I_{xx} + Aa^2$$

Shear stress distribution for a simple cross-section is given by $\tau = \dfrac{S}{2I}\left(\dfrac{d^2}{4} - y^2\right)$.

For an I-beam:



$$\hat{\tau} = \frac{S}{8I}\left[d_i^2 + \frac{b}{t}\left(d^2 - d_i^2\right)\right]$$

where $\hat{\tau}$ is the maximum shear stress.

Table D.1: Second moments of area for simple cross-sections

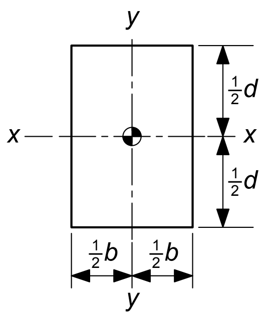
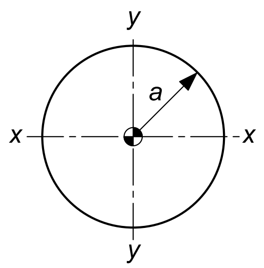
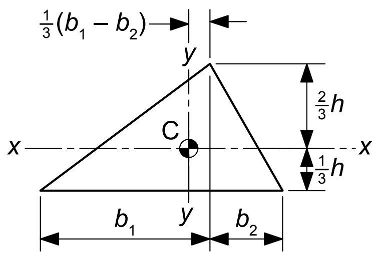
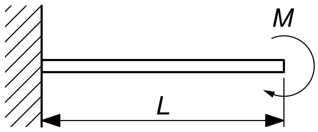
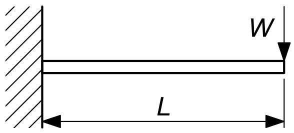
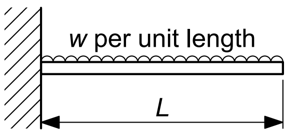
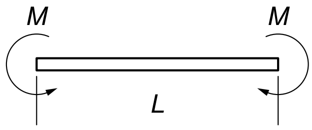
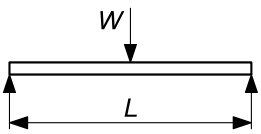
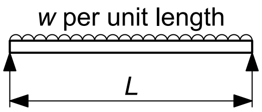
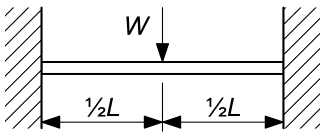
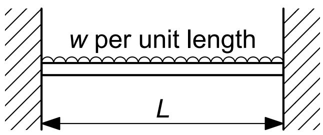
	$I_{xx} = \frac{1}{12} b d^3$ $I_{yy} = \frac{1}{12} b^3 d$
	$I_{xx} = I_{yy} = \frac{1}{4} \pi a^4$
	$I_{xx} = \frac{1}{36} (b_1 + b_2)\, h^3$ $I_{yy} = \frac{1}{36} (b_1 + b_2)\, h \left(b_1^2 + b_1 b_2 + b_2^2\right)$

Table D.2: Beams bent about principal axis

<table>
<tr><th></th><th>End slope</th><th>End deflection</th><th>Central deflection</th></tr>
<tr><td>
</td><td>$\frac{ML}{EI}$</td><td>$\frac{ML^2}{2EI}$</td><td></td></tr>
<tr><td>
</td><td>$\frac{WL^2}{2EI}$</td><td>$\frac{WL^3}{3EI}$</td><td></td></tr>
<tr><td>
</td><td>$\frac{wL^3}{6EI}$</td><td>$\frac{wL^4}{8EI}$</td><td></td></tr>
<tr><td>
</td><td>$\frac{ML}{2EI}$</td><td></td><td>$\frac{ML^2}{8EI}$</td></tr>
<tr><td>
</td><td>$\frac{WL^2}{16EI}$</td><td></td><td>$\frac{WL^3}{48EI}$</td></tr>
<tr><td>
</td><td>$\frac{wL^3}{24EI}$</td><td></td><td>$\frac{5wL^4}{384EI}$</td></tr>
<tr><th></th><th>End moment</th><th></th><th>Central deflection</th></tr>
<tr><td>
</td><td>$\frac{WL}{8}$</td><td></td><td>$\frac{WL^3}{192EI}$</td></tr>
<tr><td>
</td><td>$\frac{wL^2}{12}$</td><td></td><td>$\frac{wL^4}{384EI}$</td></tr>
</table>

D.5.2 Elastic torsion

Circular solid and hollow shafts

τ = shear stress at radius r
T = applied torque
J = polar second moment of area
d = diameter of circular section
θ = angle of twist over length L

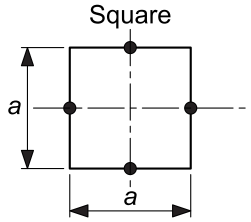
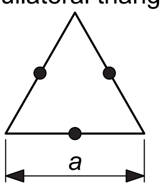
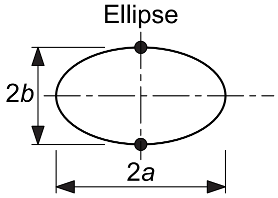
$$\frac{\tau}{r} = \frac{T}{J} = \frac{G\theta}{L}$$

For a solid circular section:

$$J = 2I_{xx} = \frac{\pi d^4}{32}$$

Non-circular compact sections

Table D.3: Torsion of solid non-circular sections (at the circular markers)

Shape of cross section	Maximum shear stress, τ_{max}	Angle of twist, θ
Square 	$4.81\dfrac{T}{a^3}$	$7.10\dfrac{TL}{a^4 G}$
Equilateral triangle 	$20\dfrac{T}{a^3}$	$46\dfrac{TL}{a^4 G}$
Ellipse 	$\dfrac{2}{\pi}\dfrac{T}{ab^2}$	$\left(a^2+b^2\right)\dfrac{TL}{\pi a^3 b^3 G}$

Torsional stiffness $$K_T = \frac{T}{\theta} = \frac{GJ}{L}$$

Torsional stiffness for a shaft with n different sections in series $$\frac{1}{K_T} = \frac{1}{K_{T1}} + \frac{1}{K_{T2}} + \ldots + \frac{1}{K_{Tn}}$$

Torsional stiffness for a compact elliptical cross section $$\frac{T}{\theta/L} = \frac{G \cdot A^4}{4 \cdot \pi^2 \cdot J}$$

Thin walled closed sections

A = enclosed area to mid-thickness
t = wall thickness
s = distance around perimeter
C = perimeter enclosing mid-thickness area

$$\tau = \frac{T}{2At}$$

Torsional stiffness:

$$\frac{T}{\theta/L} = \frac{4A^2G}{\oint \left(\frac{1}{t}\right) \mathrm{d}s}$$

Torsional stiffness for a thin-walled closed section with uniform thickness:

$$\frac{T}{\theta/L} = \frac{4 \cdot G \cdot A^2 \cdot t}{C}$$

D.5.3 Springs

d = wire diameter
D = helix diameter
δ = deflection
F = force

End deflection of a closed-helix, round wire spring:

$$\delta = \frac{8FD^3N}{Gd^4}$$

Maximum shear stress (torsion only):

$$\tau = \frac{8FD}{\pi d^3}$$

D.5.4 Thin walled pressure vessels

Hookes law (cylindrical pressure vessel)

$$\varepsilon_\theta = \frac{1}{E}\left[\sigma_\theta - \upsilon(\sigma_r + \sigma_z)\right] + \alpha\Delta T$$

$$\varepsilon_r = \frac{1}{E}\left[\sigma_r - \upsilon(\sigma_\theta + \sigma_z)\right] + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \upsilon(\sigma_\theta + \sigma_r)\right] + \alpha\Delta T$$

Hookes law (spherical pressure vessel)

$$\varepsilon_\theta = \frac{1}{E}\left[\sigma_\theta - \upsilon(\sigma_r + \sigma_\phi)\right] + \alpha\Delta T$$

$$\varepsilon_r = \frac{1}{E}\left[\sigma_r - \upsilon(\sigma_\theta + \sigma_\phi)\right] + \alpha\Delta T$$

$$\varepsilon_\phi = \frac{1}{E}\left[\sigma_\phi - \upsilon(\sigma_\theta + \sigma_r)\right] + \alpha\Delta T$$

Volumetric strain (cylindrical pressure vessel) $\varepsilon_v = 2\varepsilon_\theta + \varepsilon_z$

Volumetric strain (spherical pressure vessel) $\varepsilon_v = 3\varepsilon_\theta$

R = inner radius
t = wall thickness
p = internal pressure

Hoop stress in hollow, pressurised cylinder:

$$\sigma_\theta = p\left(\frac{R}{t}\right)$$

Axial stress (closed end cylinder):

$$\sigma_z = p\left(\frac{R}{2t}\right)$$

Stress in hollow, pressurised sphere :

$$\sigma = p\left(\frac{R}{2t}\right)$$

D.5.5 Thick-walled cylinders

For axi-symmetric systems, the circumferential and radial stresses at radius r are, respectively:

$$\sigma_{\theta\theta} = A + \frac{B}{r^2}$$

$$\sigma_{rr} = A - \frac{B}{r^2}$$

D.6 Stress functions formulation for elasticity

D.6.1 Cartesian coordinates

2D-stresses defined from the Airy stress function $\Phi(x, y)$:

$$\sigma_{xx}(x,y) = \frac{\partial^2 \Phi(x,y)}{\partial y^2} \qquad \sigma_{yy}(x,y) = \frac{\partial^2 \Phi(x,y)}{\partial x^2} \qquad \sigma_{xy}(x,y) = -\frac{\partial^2 \Phi(x,y)}{\partial x\, \partial y}$$

Bi-harmonic equation:

$$\nabla^4 \Phi(x,y) = 0 \Leftrightarrow \frac{\partial^4 \Phi(x,y)}{\partial x^4} + 2 \cdot \frac{\partial^4 \Phi(x,y)}{\partial x^2\, \partial y^2} + \frac{\partial^4 \Phi(x,y)}{\partial y^4} = 0$$

where $\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)$ is the Laplace operator.

D.6.2 Polar coordinates

2D-stresses defined from Airy stress function $\Phi(r, \theta)$:

$$\sigma_{rr}(r,\theta) = \frac{1}{r} \cdot \frac{\partial \Phi(r,\theta)}{\partial r} + \frac{1}{r^2} \cdot \frac{\partial^2 \Phi(r,\theta)}{\partial \theta^2}$$

$$\sigma_{\theta\theta}(r,\theta) = \frac{\partial^2 \Phi(r,\theta)}{\partial r^2}$$

$$\sigma_{r\theta}(r,\theta) = -\frac{\partial}{\partial r} \left(\frac{1}{r} \cdot \frac{\partial \Phi(r,\theta)}{\partial \theta} \right)$$

Laplace operator:

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial}{\partial r} + \frac{1}{r^2} \cdot \frac{\partial^2}{\partial \theta^2}$$

Michells table for stress fields (solution terms for the bi-harmonic equation, $\Phi_i(r,\theta)$ including calculated coefficients)

$\Phi_i(r,\theta)$	$\sigma_{rr}(r,\theta)$	$\sigma_{r\theta}(r,\theta)$	$\sigma_{\theta\theta}(r,\theta)$
r^2	2	0	2
$r^2 \cdot \ln(r)$	$2 \cdot \ln(r) + 1$	0	$2 \cdot \ln(r) + 3$
$\ln(r)$	r^{-2}	0	$-r^{-2}$
$1 \cdot \theta$	0	r^{-2}	0
$r^3 \cdot \cos(\theta)$	$2 \cdot r \cdot \cos(\theta)$	$2 \cdot r \cdot \sin(\theta)$	$6 \cdot r \cdot \cos(\theta)$
$r \cdot \theta \cdot \sin(\theta)$	$2 \cdot r^{-1} \cdot \cos(\theta)$	0	0
$r \cdot \ln(r) \cdot \cos(\theta)$	$r^{-1} \cdot \cos(\theta)$	$r^{-1} \cdot \sin(\theta)$	$r^{-1} \cdot \cos(\theta)$
$r^{-1} \cdot \cos(\theta)$	$-2 \cdot r^{-3} \cdot \cos(\theta)$	$-2 \cdot r^{-3} \cdot \sin(\theta)$	$2 \cdot r^{-3} \cdot \cos(\theta)$
$r^3 \cdot \sin(\theta)$	$2 \cdot r \cdot \sin(\theta)$	$-2 \cdot r \cdot \cos(\theta)$	$6 \cdot r \cdot \sin(\theta)$
$r \cdot \theta \cdot \cos(\theta)$	$-2 \cdot r^{-1} \cdot \sin(\theta)$	0	0
$r \cdot \ln(r) \cdot \sin(\theta)$	$r^{-1} \cdot \sin(\theta)$	$-r^{-1} \cdot \cos(\theta)$	$r^{-1} \cdot \sin(\theta)$
$r^{-1} \cdot \sin(\theta)$	$-2 \cdot r^{-3} \cdot \sin(\theta)$	$2 \cdot r^{-3} \cdot \cos(\theta)$	$2 \cdot r^{-3} \cdot \sin(\theta)$
$r^{n+2} \cdot \cos(n\theta),\ n > 1$	$-(n+1)(n-2) \cdot r^{n} \cdot \cos(n\theta)$	$n(n+1) \cdot r^{n} \cdot \sin(n\theta)$	$(n+1)(n+2) \cdot r^{n} \cdot \cos(n\theta)$
$r^{-(n-2)} \cdot \cos(n\theta),\ n > 1$	$-(n+2)(n-1) \cdot r^{-n} \cdot \cos(n\theta)$	$-n(n-1) \cdot r^{-n} \cdot \sin(n\theta)$	$(n-1)(n-2) \cdot r^{-n} \cdot \cos(n\theta)$
$r^{n} \cdot \cos(n\theta),\ n > 1$	$-n(n-1) \cdot r^{n-2} \cdot \cos(n\theta)$	$n(n-1) \cdot r^{n-2} \cdot \sin(n\theta)$	$n(n-1) \cdot r^{n-2} \cdot \cos(n\theta)$
$r^{-n} \cdot \cos(n\theta),\ n > 1$	$-n(n+1) \cdot r^{-(n+2)} \cdot \cos(n\theta)$	$-n(n+1) \cdot r^{-(n+2)} \cdot \sin(n\theta)$	$n(n+1) \cdot r^{-(n+2)} \cdot \cos(n\theta)$
$r^{n+2} \cdot \sin(n\theta),\ n > 1$	$-(n+1)(n-2) \cdot r^{n} \cdot \sin(n\theta)$	$-n(n+1) \cdot r^{n} \cdot \cos(n\theta)$	$(n+1)(n+2) \cdot r^{n} \cdot \sin(n\theta)$
$r^{-(n-2)} \cdot \sin(n\theta),\ n > 1$	$-(n+2)(n-1) \cdot r^{-n} \cdot \sin(n\theta)$	$n(n-1) \cdot r^{-n} \cdot \cos(n\theta)$	$(n-1)(n-2) \cdot r^{-n} \cdot \sin(n\theta)$
$r^{n} \cdot \sin(n\theta),\ n > 1$	$-n(n-1) \cdot r^{n-2} \cdot \sin(n\theta)$	$-n(n-1) \cdot r^{n-2} \cdot \cos(n\theta)$	$n(n-1) \cdot r^{n-2} \cdot \sin(n\theta)$
$r^{-n} \cdot \sin(n\theta),\ n > 1$	$-n(n+1) \cdot r^{-(n+2)} \cdot \sin(n\theta)$	$n(n+1) \cdot r^{-(n+2)} \cdot \cos(n\theta)$	$n(n+1) \cdot r^{-(n+2)} \cdot \sin(n\theta)$
$r^2 \cdot \theta$	$2 \cdot \theta$	-1	$2 \cdot \theta$
$r^2 \cdot \ln(r) \cdot \theta$	$\left[2 \cdot \ln(r) + 1\right] \cdot \theta$	$-\left[\ln(r) + 1\right]$	$\left[2 \cdot \ln(r) + 3\right] \cdot \theta$
$\ln(r) \cdot \theta$	$r^{-2} \cdot \theta$	$r^{-}2 \cdot \left[\ln(r) - 1\right]$	$-r^{-2} \cdot \theta$

D.6.3 Elastic torsion

Torsion stresses derived using Prandtl's stress function, $\Phi(x,y)$:

$$\sigma_{xz}(x,y)=\frac{\partial\Phi(x,y)}{\partial y} \qquad \sigma_{yz}(x,y)=-\frac{\partial\Phi(x,y)}{\partial x}$$

Compatibility of strains and displacements in torsion:

$$\nabla^2\Phi(x,y)=\frac{\partial^2\Phi(x,y)}{\partial x^2}+\frac{\partial^2\Phi(x,y)}{\partial y^2}=-2\cdot G\cdot\overline{\theta} \qquad or \qquad \oint_{\Gamma_i}\nabla\Phi\cdot\mathbf{n}\,\mathrm{d}s=-2\cdot G\cdot\overline{\theta}\cdot A_i$$

Boundary condition for Prandtl's stress function on the connected boundaries Γ_i :

$$\frac{\partial\Phi(x_\Gamma,y_\Gamma)}{\partial s}=0 \text{ for } (x_\Gamma,y_\Gamma)\in\Gamma_i \qquad and \qquad \Phi(x_{\Gamma_0},y_{\Gamma_0})=0 \text{ for } (x_\Gamma,y_\Gamma)\in\Gamma_0$$

Boundary condition for Prandtl's stress function on the cross-section domain Ω

$$T=2\cdot\iint_\Omega\Phi(x,y)\,\mathrm{d}\Omega, \quad \text{with } \Phi(x_{\Gamma_0},y_{\Gamma_0})=0 \text{ at the external boundary } \Gamma_0$$

Torsion of a thin rectangle (with thickness t and length b):

$$\Phi(x)=G\cdot\overline{\theta}\cdot\left[\left(\frac{t}{2}\right)^2-x^2\right] \qquad T=G\cdot\overline{\theta}\cdot b\cdot\frac{t^3}{3} \qquad \sigma_{yz}=2\cdot G\cdot\overline{\theta}\cdot x$$

Torsion of a section with $n\geq 1$ thin-walled closed compartments (wall thickness t):

$$T=2\cdot\sum_{i=1}^{n}A_{\mathrm{m},i}\cdot\Phi_i \qquad \tau_{ij}=\frac{\Phi_i-\Phi_j}{t_{ij}}$$

$$\overline{\theta}_i=\frac{1}{2\cdot G\cdot A_{\mathrm{m},i}}\cdot\oint_{\Gamma_{\mathrm{m},i}}\frac{\Phi_i-\Phi_j}{t_{ij}}\,\mathrm{d}s \qquad \text{where } i\in\{1,\ldots,n\} \text{ and } j \text{ is adjacent to } i$$

D.7 Plasticity

D.7.1 Yield criteria

Y = yield stress in uniaxial tension

In a three dimensional stress system having principal stresses σ_1, σ_2 and σ_3 where

$$\sigma_1\geq\sigma_2\geq\sigma_3.$$

Tresca yield criterion:

$$|\sigma_1-\sigma_3|=Y$$

von Mises yield criterion:

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2=2Y^2$$

D.7.2 Plasticity theory

Invariants

$$s^3 - J_1 s^2 - J_2 s - J_3 = 0$$

$$J_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$$

$$J_2 = \sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2 - \sigma_{xx}\sigma_{yy} - \sigma_{yy}\sigma_{zz} - \sigma_{zz}\sigma_{xx}$$

$$J_3 = \sigma_{xx}\sigma_{yy}\sigma_{zz} + 2\sigma_{xy}\sigma_{yz}\sigma_{zx} - \sigma_{xx}\sigma_{yz}^2 - \sigma_{yy}\sigma_{zx}^2 - \sigma_{zz}\sigma_{xy}^2$$

Equivalent stress

$$2\,\overline{\sigma}^2 = 6J_2' = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

$$= \left(\sigma_{xx} - \sigma_{yy}\right)^2 + \left(\sigma_{yy} - \sigma_{zz}\right)^2 + (\sigma_{zz} - \sigma_{xx})^2 + 6\left(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2\right)$$

Equivalent strain

$$\frac{9}{2}\,\overline{\varepsilon}^2 = (\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2$$

$$= \left(\varepsilon_{xx} - \varepsilon_{yy}\right)^2 + \left(\varepsilon_{yy} - \varepsilon_{zz}\right)^2 + (\varepsilon_{zz} - \varepsilon_{xx})^2 + 6\left(\varepsilon_{xy}^2 + \varepsilon_{yz}^2 + \varepsilon_{zx}^2\right)$$

$$\text{where } \varepsilon_{xy} = \frac{1}{2}\left(\frac{\mathrm{d}u}{\mathrm{d}y} + \frac{\mathrm{d}v}{\mathrm{d}x}\right) = \frac{1}{2}\gamma_{xy}, \quad \varepsilon_{yz} = \frac{1}{2}\gamma_{yz}, \quad \varepsilon_{zx} = \frac{1}{2}\gamma_{zx}$$

Levy-Mises equations

$$\frac{\mathrm{d}\varepsilon_{xx}}{\sigma'_{xx}} = \frac{\mathrm{d}\varepsilon_{yy}}{\sigma'_{yy}} = \frac{\mathrm{d}\varepsilon_{zz}}{\sigma'_{zz}} = \frac{\mathrm{d}\varepsilon_{xy}}{\sigma_{xy}} = \frac{\mathrm{d}\varepsilon_{yz}}{\sigma_{yz}} = \frac{\mathrm{d}\varepsilon_{zx}}{\sigma_{zx}} = \mathrm{d}\lambda$$

$$\mathrm{d}\varepsilon_{xx} = \frac{\mathrm{d}\overline{\varepsilon}}{\overline{\sigma}}\left(\sigma_{xx} - \frac{1}{2}\left(\sigma_{yy} + \sigma_{zz}\right)\right)$$

$$\mathrm{d}\varepsilon_{yy} = \frac{\mathrm{d}\overline{\varepsilon}}{\overline{\sigma}}\left(\sigma_{yy} - \frac{1}{2}\left(\sigma_{zz} + \sigma_{xx}\right)\right)$$

$$\mathrm{d}\varepsilon_{zz} = \frac{\mathrm{d}\overline{\varepsilon}}{\overline{\sigma}}\left(\sigma_{zz} - \frac{1}{2}\left(\sigma_{xx} + \sigma_{yy}\right)\right)$$

$$\mathrm{d}\varepsilon_{xy} = \frac{3}{2}\frac{\mathrm{d}\overline{\varepsilon}}{\overline{\sigma}}\sigma_{xy} \qquad \mathrm{d}\varepsilon_{yz} = \frac{3}{2}\frac{\mathrm{d}\overline{\varepsilon}}{\overline{\sigma}}\sigma_{yz} \qquad \mathrm{d}\varepsilon_{zx} = \frac{3}{2}\frac{\mathrm{d}\overline{\varepsilon}}{\overline{\sigma}}\sigma_{zx}$$

D.8 Viscoelasticity

$$e(t) = \int_0^t C(t-\tau)\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}\,\mathrm{d}\tau \qquad \sigma(t) = \int_0^t M(t-\tau)\frac{\mathrm{d}e}{\mathrm{d}\tau}\,\mathrm{d}\tau$$

$$\overline{e} = \overline{C}(s)\,\overline{\sigma}\,s \qquad \overline{\sigma} = \overline{M}(s)\,\overline{e}\,s$$

$$f(t) = t^n;\ \overline{f}(s) = \frac{\Gamma(1+n)}{s^{1+n}}, \text{ where } \Gamma(1+n) = n\,\Gamma(n)$$

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E Thermofluids

Contents

E.1 Dimensionless groups

Nomenclature

A	surface area
C_p	specific heat at constant pressure
D	pipe diameter
F_D, F_L	drag force, lift force
g	gravitational acceleration
h	convection heat transfer coefficient
k_f k_s	thermal conductivity of fluid, of solid
L	reference length
$\triangle P$	pressure drop
T_∞, T_s	temperature at infinity, at surface
U, U_∞	characteristic velocity, velocity at infinity
α	thermal diffusivity
β	coefficient of volumetric thermal expansion
ε	roughness height
μ	absolute viscosity
ρ	density
τ_w	surface shear stress

Table E.1: Dimensionless groups for Thermofluids

Parameter	Definition
Biot number (Bi)	$\dfrac{hL}{k_\mathrm{s}}$
Coefficient of lift (C_L)	$\dfrac{F_\mathrm{L}}{\frac{1}{2}\rho U_\infty^2 A}$
Coefficient of drag (C_D)	$\dfrac{F_\mathrm{D}}{\frac{1}{2}\rho U_\infty^2 A}$
Fourier number (Fo)	$\dfrac{\alpha t}{L^2}$
Friction factor (f)	$\dfrac{\triangle P}{\left(\frac{L}{D}\right)\frac{1}{2}\rho U_\infty^2}$
Friction coefficient (C_f)	$\dfrac{\tau_w}{\frac{1}{2}\rho U_\infty^2}$
Number of transfer units (NTU)	$\dfrac{\mathrm{U_{ht}A}}{\mathrm{C_{min}}}$
Effectiveness (ϵ)	$\dfrac{\mathrm{q}}{\mathrm{q_{max}}}$
Grashof number (Gr)	$\dfrac{\beta g\,(T_\mathrm{s}-T_\infty)\,L^3\rho^2}{\mu^2}$
Nusselt number (Nu)	$\dfrac{hL}{k_f}$
Prandtl number (Pr)	$\dfrac{\mu C_\mathrm{p}}{k_\mathrm{f}}$
Rayleigh number (Ra)	$\mathrm{Gr}\cdot\mathrm{Pr}$
Reynolds number (Re)	$\dfrac{U_\infty L\rho}{\mu}$
Roughness ratio	$\dfrac{\varepsilon}{L}$
Stanton number (St)	$\dfrac{h}{\rho U_\infty C_\mathrm{p}} = \dfrac{\mathrm{Nu}}{\mathrm{Re}\cdot\mathrm{Pr}}$

E.2 Heat transfer

Nomenclature

A	Area
Bi	Biot number
c	Specific heat capacity
C	Heat capacity rate
C_i	Coefficient to the series solutions in transient one-dimensional conduction solution
$C_{\min}$	Minimum heat capacity rate
$C_{\max}$	Maximum heat capacity rate
D	Diameter
Fo	Fourier number
h	Convective heat transfer coefficient
k	Thermal conductivity
L	Reference length
$\dot{m}$	Mass flow rate
NTU	Number of transfer units
Nu	Nusselt number
P	Perimeter
q	Heat transfer rate
$\dot{q}$	Rate of energy generation per unit volume
r	Radius
t	Time
T	Temperature
U	Overall heat transfer coefficient
V	Volume
x^*	Normalised distance of x
x, y, z	Cartesian coordinates
x_{fd}	Entry Length
ρ	Density
β	Coefficient of volumetric thermal expansion
ε	Heat exchanger effectiveness
θ^*	Normalised Temperature
ζ_i	Coefficient to the series solutions in transient one-dimensional conduction solution

Subscripts

b	Base
c	Cross-section or cold
f	Fluid
h	Hot
H	Hydraulic (diameter)
i	Inlet, or initial condition, or index
m	Mean
max	Maximum
o	Outlet
p	Constant pressure
s	Solid, or surface condition
∞	Free stream or at infinity

Heat Diffusion Equation

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho\ c\frac{\partial T}{\partial t}$$

Fin equation

$$\frac{d}{dx}\left(A_c\frac{dT}{dx}\right)-\frac{h}{k}\frac{dA_s}{dx}(T-\ T_\infty)=0$$

Transient conduction: Lumped Capacitance Method with convection

$$\theta^*=\frac{T-T_\infty}{T_i-T_\infty}=\exp\left[-\left(\frac{hA_s}{\rho Vc}\right)t\right]$$

Transient conduction: Solution for 1D plane wall with convection

$$\theta^*=\frac{T-T_\infty}{T_i-T_\infty}=\sum_{i=1}^{\infty}C_i\exp\left(-\zeta_i^2 Fo\right)\cdot\cos\left(\zeta_i x^*\right)$$

Governing equation for the heat transfer to the bulk flow (internal flow)

$$\frac{dT_m}{dx}=\frac{q_s''P}{\dot{m}c_p}=\frac{P}{\dot{m}c_p}h\left(T_s-T_m\right)$$

Non dimensional temperature inside a thermal boundary layer

$$\theta^*=\frac{T-T_s}{T_\infty-T_s}$$

Nusselt number in terms of non-dimensional temperature and distance from a surface

$$Nu=\left.\frac{\partial\theta^*}{\partial y^*}\right|_{y^*=0}\quad,\text{ where } y^*=\frac{y}{L}$$

Coefficient of volumetric thermal expansion

$$\beta=\left.\frac{-1}{\rho}\frac{\partial\rho}{\partial T}\right|_p\quad\text{for non ideal gasses}$$

$$\beta=\frac{1}{T}\quad\text{for ideal gasses}$$

Heat capacity rates ratio (heat exchangers)

$$\frac{C_h}{C_c} = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{h,o}}$$

Number of transfer units (NTU)

$$\mathrm{NTU} = \frac{UA}{C_{\min}}$$

Definition of max possible heat transfer rate in heat exchangers

$$q_{\max} = C_{\min}(T_{h,i} - T_{c,i})$$

Definition of heat exchanger effectiveness

$$\varepsilon = \frac{q}{q_{\max}}$$

Table E.2: (a) One-dimensional steady-state solutions to the heat equation with no generation

Name	Plane Wall	Cylindrical wall
Heat equation	$\frac{\partial^2 T}{\partial x^2} = 0$	$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0$
Temperature distribution	$T(\mathrm{x}=0) - \Delta T\frac{x}{\mathrm{L}}$	$T(r=r_2) + \Delta T\frac{ln\frac{r}{r_2}}{ln\frac{r_1}{r_2}}$
	$\Delta T = (T(\mathrm{x}=0) - T(\mathrm{x}=L))$	$\Delta T = (T(r=r_1) - T(r=r_2))$
Thermal resistance	$\frac{\mathrm{L}}{k_s A_c}$	$\frac{\ln\frac{r_2}{r_1}}{2\pi \mathrm{L} k_s}$

Table E.2: (b) Temperature distribution and heat loss for fins of uniform cross section

Case	Tip condition ($x = \mathrm{L}$)	Temperature distribution $\dfrac{\theta}{\theta_b}$	Fin heat transfer rate
A	Convection heat transfer	$\dfrac{\cosh m(\mathrm{L}-x) + \left(\dfrac{h}{mk}\right)\sinh m(\mathrm{L}-x)}{\cosh m\mathrm{L} + \left(\dfrac{h}{mk}\right)\sinh m\mathrm{L}}$	$M\dfrac{\sinh m\mathrm{L} + \left(\dfrac{h}{mk}\right)\cosh m\mathrm{L}}{\cosh m\mathrm{L} + \left(\dfrac{h}{mk}\right)\sinh m\mathrm{L}}$
B	Adiabatic	$\dfrac{\cosh m(\mathrm{L}-x)}{\cosh m\mathrm{L}}$	$M \tanh m\mathrm{L}$
C	Prescribed temperature	$\dfrac{\left(\dfrac{\theta_\mathrm{L}}{\theta_b}\right)\sinh mx + \sinh m(\mathrm{L}-x)}{\sinh m\mathrm{L}}$	$M\dfrac{\left(\cosh m\mathrm{L} - \dfrac{\theta_\mathrm{L}}{\theta_b}\right)}{\sinh m\mathrm{L}}$
D	Infinite fin	e^{-mx}	M
$\theta = T - T_\infty$		$m^2 = \dfrac{h\mathrm{P}}{k_s A_c}$	
$\theta_b = \theta(0) = T_b - T_\infty$		$M = \sqrt{h\mathrm{P}k_s A_c}\,\theta_b$	

Table E.2: (c) Empirical correlations for forced convection

Correlation	Conditions
Local laminar flow over a flat plate:	
$C_{\mathrm{f,x}} = \dfrac{0.664}{\mathrm{Re}_x^{1/2}}$	$\mathrm{Pr} \geq 0.6$
$\mathrm{Nu_x} = \dfrac{h(x)x}{k} = 0.332\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3}$	$\mathrm{Pr} \geq 0.6$
Average laminar flow over a flat plate:	
$\overline{C}_{\mathrm{f,L}} = \dfrac{1.328}{\mathrm{Re}_L^{1/2}}$	$\mathrm{Pr} \geq 0.6$
$\overline{\mathrm{Nu}}_\mathrm{L} = 0.664\mathrm{Re}_L^{1/2}\mathrm{Pr}^{1/3}$	$\mathrm{Pr} \geq 0.6$
Local turbulent flow over a flat plate (isothermal):	
$C_{\mathrm{f,x}} = \dfrac{0.0592}{\mathrm{Re}_x^{1/5}}$	$5 \cdot 10^5 \leq \mathrm{Re}_x \leq 10^8$
$\mathrm{Nu_x} = 0.0296\mathrm{Re}_x^{4/5}\mathrm{Pr}_x^{1/3}$	$\begin{cases} 0.6 \leq \mathrm{Pr} \leq 60 \\ 5 \cdot 10^5 \leq \mathrm{Re}_x \leq 10^7 \end{cases}$

Average turbulent flow over a flat plate (isothermal):

$\overline{C}_{\mathrm{f,L}} = \dfrac{0.074}{\mathrm{Re}_L^{1/5}}$ — $5 \cdot 10^5 \leq \mathrm{Re}_L \leq 10^8$

$\overline{\mathrm{Nu}}_{\mathrm{L}} = 0.037\mathrm{Re}^{4/5}\mathrm{Pr}^{1/3}$ — $\begin{cases} 0.6 \leq \mathrm{Pr} \leq 60 \\ 5 \cdot 10^5 \leq \mathrm{Re}_L \leq 10^8 \end{cases}$

Average mixed flow over a flat plate (isothermal):

$\overline{C}_{\mathrm{f,L}} = \dfrac{0.074}{\mathrm{Re}_L^{1/5}} - \dfrac{1742}{\mathrm{Re}_L}$ — $5 \cdot 10^5 \leq \mathrm{Re}_L \leq 10^8$

$\overline{\mathrm{Nu}}_{\mathrm{L}} = \left(0.037\mathrm{Re}_L^{4/5} - 871\right)\mathrm{Pr}^{1/3}$ — $\begin{cases} 0.6 \leq \mathrm{Pr} \leq 60 \\ 5 \cdot 10^5 \leq \mathrm{Re}_L \leq 10^8 \end{cases}$

Flat plate with uniform heat flux:

$\mathrm{Nu}_{\mathrm{x}} = 0.453\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3}$ — Laminar flow

$\mathrm{Nu}_{\mathrm{x}} = 0.0308\mathrm{Re}_x^{0.8}\mathrm{Pr}^{1/3}$ — Turbulent flow

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Thermal entry length for flow in a pipe:

$\dfrac{x_{\mathrm{fd},t}}{D} = 0.05\mathrm{Re}_D\mathrm{Pr}$ — Laminar flow ($\mathrm{Re}_D \leq 10^4$)

$\dfrac{x_{\mathrm{fd},t}}{D} = 10$ — Turbulent flow ($\mathrm{Re}_D > 10^4$)

Fully developed laminar flow in a pipe:

$\mathrm{D_H} = \dfrac{4\mathrm{A_c}}{\mathrm{P}}$

$\mathrm{C_{f,D}} = \dfrac{16}{\mathrm{Re}_D}$

$\mathrm{Nu_D} = 3.66$ — isothermal surface

$\mathrm{Nu_D} = 4.36$ — Constant heat flux

Fully developed turbulent flow in a pipe:

$\mathrm{Nu}_D = 0.023\mathrm{Re}_D^{4/5}\mathrm{Pr}^n$
($n = 0.4$ when flow is heated,
$n = 0.3$ when flow is cooled) — $\begin{cases} 0.7 \leq \mathrm{Pr} \leq 160 \\ \mathrm{Re_D} > 10^4 \end{cases}$

$C_{\mathrm{f,D}} = (0.1975\ln\mathrm{Re}_D - 0.41)^{-2}$ — (smooth pipe) $3 \cdot 10^3 \leq \mathrm{Re}_D \leq 5 \cdot 10^6$

Table E.3: Coefficients used in the one-term approximation to the series solutions for transient one-dimensional conduction

Data is based on TABLE 5.1, Bergman, T. L. et al. Incroperas Principles of Heat and Mass Transfer. Eighth edition, Theodore L. Bergman, Adrienne S. Lavine, Frank P. Incropera and David P. DeWitt. Hoboken, NJ: John Wiley & Sons, Inc., 2017. Print.

	Plane Wall		Infinite Cylinder		Sphere	
$Bi^{\dagger}$	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.03	0.1723	1.0049	0.2440	1.0075	0.2991	1.0090
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.05	0.2218	1.0082	0.3143	1.0124	0.3854	1.0149
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.07	0.2615	1.0114	0.3709	1.0173	0.4551	1.0209
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.09	0.2956	1.0145	0.4195	1.0222	0.5150	1.0268
0.10	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.15	0.3779	1.0237	0.5376	1.0365	0.6609	1.0445
0.20	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.25	0.4801	1.0382	0.6856	1.0598	0.8447	1.0737
0.30	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0932	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0919	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5994	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2402	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	1.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2881	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2733	2.4050	1.6018	3.1415	2.0000

$^{\dagger}Bi = hL/K$ for the plane wall and hr_o/k for the infinite cylinder and sphere

E.3 Fire Dynamics

General reaction equation for the combustion of a hydrocarbon fuel in air

$$C_\alpha H_\beta O_\gamma + \left(\alpha + \frac{\beta}{4} - \frac{\gamma}{2}\right)(O_2 + 3.76N_2) \rightarrow \alpha\, CO_2 + \frac{\beta}{2} H_2O + 3.76\left(\alpha + \frac{\beta}{4} - \frac{\gamma}{2}\right) N_2$$

Mass loss rate

$$\dot{m} = \frac{\mathrm{d}m}{\mathrm{d}t} = -k'm$$

Arrhenious reaction rate

$$k' = A \exp\left(-\frac{E_A}{RT}\right)$$

Heat release rate

$$\dot{Q}_c = \chi_c \dot{m}'' A_f \Delta H_C$$

Adiabatic flame temperature

$$T_{ad} = T_0 + \frac{\Delta H_c}{\sum \frac{N_i}{N_{fuel}} \bar{c}_{p,i}}$$

Solution of the 1D heat diffusion equation for a semi-infinite solid and convective boundary condition

$$\frac{\theta_s}{\theta_\infty} = 1 - \exp\left(\frac{\alpha t}{(k/h)^2}\right) \cdot \operatorname{erfc}\left(\frac{\sqrt{\alpha t}}{k/h}\right)$$

Effective emission coefficient of a flame

$$\varepsilon = 1 - \exp(-KL)$$

Equation for laminar flame speed

$$S_u = \left(\frac{2k}{\rho_0^2 c_p^2 (T_F - T_0)} \dot{Q}'''_{ave}\right)^{0.5}$$

Convective component of the heat release rate

$$\dot{Q}_{conv} = (1 - \chi_{rad})\, \dot{Q}_c$$

Non-dimensional heat release rate

$$\dot{Q}_c^* = \frac{\dot{Q}_c}{\rho_\infty c_p T_\infty \sqrt{gD} D^2}$$

Beyler equation for centreline plume temperature

$$\Delta T_0 = 26 \frac{\dot{Q}_{conv}^{2/3}}{z^{5/3}}$$

McCaffrey equation for diffusion flame height

$$\frac{l}{D} = 3.7 \left(\dot{Q}_c^*\right)^{2/5} - 1.02$$

Alpert equations for maximum gas temperature near the ceiling

$$T_{max} - T_0 = 5.38 \; \frac{\left(\dot{Q}_c/r\right)^{2/3}}{H} \quad \text{for } r > 0.18H$$

$$T_{max} - T_\infty = \frac{16.9\dot{Q}_c^{2/3}}{H^{5/3}} \quad \text{for } r \leq 0.18H$$

Zabedakis and Burgess equation for mass loss rate of a pool fire

$$\dot{m}'' = \dot{m}''_\infty \left(1 - \exp\left(-k\beta D\right)\right)$$

Mass loss rate of a fire

$$\dot{m}'' = \frac{\dot{Q}''_F + \dot{Q}''_E - \dot{Q}''_L}{L_v}$$

Heat release rate as a function of combustibility ratio

$$\frac{\dot{Q}_c}{A_F} = \dot{Q}''_{net} \chi_c \left(\frac{\Delta H_c}{L_v}\right)$$

Frank-Kamenetskii equation

$$\nabla^2\theta = -\delta \exp\left(\theta\right) \qquad \text{for } \delta = \frac{r_0^2 E_A \Delta H_c A C_i^n}{kRT_a^2} \exp\left(-E_A/RT_a\right)$$

Ignition time of a thermally thin fuel and symmetric convective heating

$$t_{ig} = \frac{\tau\rho c}{2h} \ln\left(\frac{T_\infty - T_0}{T_\infty - T_{ig}}\right)$$

Ignition time of a thermally thin fuel and symmetric external heat flux

$$t_{ig} = \rho c \tau \frac{\left(T_{ig} - T_0\right)}{\dot{Q}''_R}$$

Ignition time of a thermally thick fuel and external heat flux

$$t_{ig} = \frac{\pi}{4} k\rho c \frac{\left(T_{ig} - T_0\right)^2}{\dot{Q}''^2_R}$$

Flame spread over a thermally thin fuel

$$V \propto \frac{1}{T_P - T_0}$$

Flame spread over a thermally thick fuel

$$V \propto \frac{1}{(T_P - T_0)^2}$$

Quintieres equation for flame spread over a thermally thick fuel

$$V^{-1/2} = \left[\frac{(\pi k \rho c)^{1/2}}{2hl^{1/2}\dot{q}_F''}\right][h\left(T_{ig} - T_0\right) - \dot{q}_E'']$$

Spread rate of smouldering

$$V \approx \frac{k}{\rho c} \cdot \frac{1}{x} = \frac{\alpha}{x}$$

Growth rate of a compartment fire

$$\dot{Q}_c = \pi \Delta H_c \dot{m}'' V^2 t^2 = \alpha_f (t - t_0)^2$$

Heat release rate at flashover

$$\dot{Q}_{FO} = 750 A_w H^{1/2}$$

Time constraint for safe evacuation

$$t_p + t_a + t_{rs} < t_u$$

Lambert-Beer law for flame radiation

$$I = I_0 \exp(-\kappa C L)$$

Conditions for incapacitation by smoke

$$\sum FED_i > 1$$

Mass flowrate into smoke layer

$$\dot{M} = 0.071 \dot{Q}_c^{1/3} z^{5/3} [1 + 0.026 \dot{Q}_c^{2/3} z^{-5/3}]$$

Temperature of the smoke layer

$$\Delta T = \frac{\dot{Q}_c}{\dot{M} c_p}$$

Volumetric extraction rate of smoke

$$V_s = \frac{0.53 P_f}{\rho_s}$$

Time of smoke layer descent

$$t = \frac{20A}{P_f g^{1/2}} \left(\frac{1}{y^{1/2}} - \frac{1}{H^{1/2}}\right)$$

E.4 Continuity and equation of motion

Cartesian coordinates	(x, y, z)
Cylindrical coordinates	(r, θ, z)
Density	ρ
Dynamic viscosity	μ
Pressure	p
Velocity components, Cartesian coordinates	$\boldsymbol{u} = (u, v, w)$
Velocity components, cylindrical coordinates	$\boldsymbol{u} = (u_r, u_\theta, u_z)$
Viscous stress tensor, Cartesian coordinates	$\boldsymbol{\tau} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix}$
Viscous stress tensor, cylindrical coordinates	$\boldsymbol{\tau} = \begin{bmatrix} \tau_{rr} & \tau_{r\theta} & \tau_{rz} \\ \tau_{\theta r} & \tau_{\theta\theta} & \tau_{\theta z} \\ \tau_{zr} & \tau_{z\theta} & \tau_{zz} \end{bmatrix}$

Gradient in cylindrical coordinates

$$\mathrm{grad}(\boldsymbol{f}) = \begin{bmatrix} \partial f_r/\partial r & (1/r)(\partial f_r/\partial\theta - f_\theta) & \partial f_r/\partial z \\ \partial f_\theta/\partial r & (1/r)(\partial f_\theta/\partial\theta + f_r) & \partial f_\theta/\partial z \\ \partial f_z/\partial r & (1/r)\partial f_z/\partial\theta & \partial f_z/\partial z \end{bmatrix}$$

Continuity equation

Cartesian Coordinates
$$\frac{\partial\rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

Cylindrical Coordinates
$$\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial(r\rho u_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho u_\theta)}{\partial\theta} + \frac{\partial(\rho u_z)}{\partial z} = 0$$

E.4.1 The Cauchy equations

Cartesian Coordinates

- x-component of the momentum balance:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) + \frac{f_{\mathrm{body},x}}{\rho}$$

- y-component of the momentum balance:

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{1}{\rho}\left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right) + \frac{f_{\mathrm{body},y}}{\rho}$$

- z-component of the momentum balance:

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{1}{\rho}\left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right) + \frac{f_{\mathrm{body},z}}{\rho}$$

Cylindrical Coordinates

- radial component of the momentum balance:

$$\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\left(\frac{\partial u_r}{\partial \theta} - u_\theta\right) + u_z\frac{\partial u_r}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{1}{\rho}\left[\frac{1}{r}\frac{\partial (r\tau_{rr})}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta r}}{\partial \theta} + \frac{\partial \tau_{zr}}{\partial z} - \frac{\tau_{\theta\theta}}{r}\right] + \frac{f_{\mathrm{body},r}}{\rho}$$

- azimuthal component of the momentum balance:

$$\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right) + u_z\frac{\partial u_\theta}{\partial z} = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + \frac{1}{\rho}\left[\frac{1}{r}\frac{\partial (r\tau_{r\theta})}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{z\theta}}{\partial z} + \frac{\tau_{\theta r}}{r}\right] + \frac{f_{\mathrm{body},\theta}}{\rho}$$

- axial component of the momentum balance:

$$\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{1}{\rho}\left[\frac{1}{r}\frac{\partial (r\tau_{rz})}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}\right] + \frac{f_{\mathrm{body},z}}{\rho}$$

E.4.2 The Navier-Stokes equations for incompressible flow

Cartesian coordinates

- mass conservation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

- x-component of the momentum balance:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \frac{f_{\mathrm{body},x}}{\rho}$$

- y-component of the momentum balance:

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + \frac{f_{\mathrm{body},y}}{\rho}$$

- z-component of the momentum balance:

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) + \frac{f_{\mathrm{body},z}}{\rho}$$

Cylindrical coordinates

- mass conservation:

$$\frac{1}{r}\frac{\partial (r u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$

- radial component of the momentum balance:

$$\begin{aligned}\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\left(\frac{\partial u_r}{\partial \theta} - u_\theta\right) + u_z\frac{\partial u_r}{\partial z} &= \\ -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta} - \frac{u_r}{r^2}\right] &+ \frac{f_{\mathrm{body},r}}{\rho}\end{aligned}$$

- azimuthal component of the momentum balance:

$$\begin{aligned}\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right) + u_z\frac{\partial u_\theta}{\partial z} &= \\ -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + \frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{\partial^2 u_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r^2}\right] &+ \frac{f_{\mathrm{body},\theta}}{\rho}\end{aligned}$$

- axial component of the momentum balance:

$$\begin{aligned}\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z} &= \\ -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right] &+ \frac{f_{\mathrm{body},z}}{\rho}\end{aligned}$$

E.4.3 Vector form

Equation of continuity for unsteady flow, variable density, in vector form:

$$0 = \frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \boldsymbol{u})$$

where $\mathbf{u}$ is the velocity field.

Equations of Motion in vector form for unsteady flow: Cauchy form

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u} \;=\; -\frac{1}{\rho}\mathrm{grad}(p) + \frac{1}{\rho}\mathrm{div}(\boldsymbol{\tau}) + \frac{\boldsymbol{f}_{\mathrm{body}}}{\rho}$$

E.4.4 Equations for compressible flows

Isentropic flow relations

$$\frac{p_0}{p} = \left[1 + \frac{\gamma - 1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho_0}{\rho} = \left[1 + \frac{\gamma - 1}{2}M^2\right]^{\frac{1}{\gamma-1}}$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2}M^2$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma + 1}\left(1 + \frac{\gamma - 1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\dot{m}_{\mathrm{max}} = p_0 A^* \sqrt{\frac{\gamma}{RT_0}\left[\frac{2}{\gamma + 1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

Prandtl-Meyer function

$$\nu(\mathrm{Ma}) = \left(\frac{\gamma + 1}{\gamma - 1}\right)^{1/2} \tan^{-1}\left\{\left(\frac{(\gamma + 1)\left(\mathrm{Ma}^2 - 1\right)}{\gamma - 1}\right)^{1/2}\right\} - \tan^{-1}\left\{\left(\mathrm{Ma}^2 - 1\right)^{1/2}\right\}$$

Normal Shock Relations

$$\mathrm{Ma}_2^2 = \frac{1 + \frac{1}{2}(\gamma - 1)\,\mathrm{Ma}_1^2}{\gamma \mathrm{Ma}_1^2 - \frac{1}{2}(\gamma - 1)}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)\,\mathrm{Ma}_1^2}{(\gamma - 1)\,\mathrm{Ma}_1^2 + 2}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{(\gamma + 1)}\left(\mathrm{Ma}_1^2 - 1\right)$$

$$\frac{T_2}{T_1} = \frac{\left[2\gamma \mathrm{Ma}_1^2 - (\gamma - 1)\right]\left[(\gamma - 1)\,\mathrm{Ma}_1^2 + 2\right]}{(\gamma + 1)^2\,\mathrm{Ma}_1^2}$$

E.5 Boundary layer relations

Nomenclature

Streamwise velocity at the edge of the boundary layer	U_∞
Transverse velocity at the edge of the boundary layer	v_∞
Position along a flat plate from the leading edge	x
Length of a flat plate	L
Reynolds number based on x	$Re_x = \frac{\rho U_\infty x}{\mu}$
Reynolds number based on L	$Re_L = \frac{\rho U_\infty L}{\mu}$

Blasius solution

Transverse velocity:	$\frac{v_\infty}{U_\infty} = 0.86\,\mathrm{Re}_x^{-1/2}$
Boundary layer thickness:	$\frac{\delta_{99}}{x} = 4.91\,\mathrm{Re}_x^{-1/2}$
Displacement thickness:	$\frac{\delta_1}{x} = 1.72\,\mathrm{Re}_x^{-1/2}$
Momentum thickness:	$\frac{\delta_2}{x} = 0.664\,\mathrm{Re}_x^{-1/2}$
Friction coefficient:	$C_f = 0.664\,\mathrm{Re}_x^{-1/2}$
Drag coefficient:	$C_D = 1.328\,\mathrm{Re}_L^{-1/2}$

Empirical relations

For a boundary layer that is turbulent from the leading edge

$$Re_L < 5 \times 10^6, \qquad C_D = \frac{0.0722}{Re_L^{0.2}}. \tag{1}$$

$$Re_L > 5 \times 10^6, \qquad C_D = \frac{0.455}{(\log_{10} Re_L)^{2.58}}. \tag{2}$$

If the boundary layer is initially laminar, with transition at $\mathrm{Re}_{\mathrm{crit}}$:

$$C_D = [C_D]_{\mathrm{turbulent}} - \frac{A}{\mathrm{Re}_L}$$

with $[C_D]_{\mathrm{turbulent}}$ calculated, as appropriate, from equations (1) or (2). The constant A is:

$\mathrm{Re}_{\mathrm{crit}}$	3x10^5	5x10^5	10^6	3x10^6
A	1050	1700	3300	8700

E.6 Friction factor for flow in circular pipes (Moody diagram)

Nomenclature

d	pipe diameter
f	Darcy friction factor = $\dfrac{4\tau_w}{\frac{1}{2}\rho V^2} = \dfrac{1}{(L/d)}\dfrac{\triangle P}{\frac{1}{2}\rho V^2}$
L	pipe length
Re	Reynolds Number = $\dfrac{\rho V d}{\mu}$
V	fluid bulk mean velocity
$\triangle P$	frictional pressure drop in length L
ε	roughness height
μ	absolute or dynamic viscosity
ρ	fluid density
τ_{w}	shear stress at pipe wall

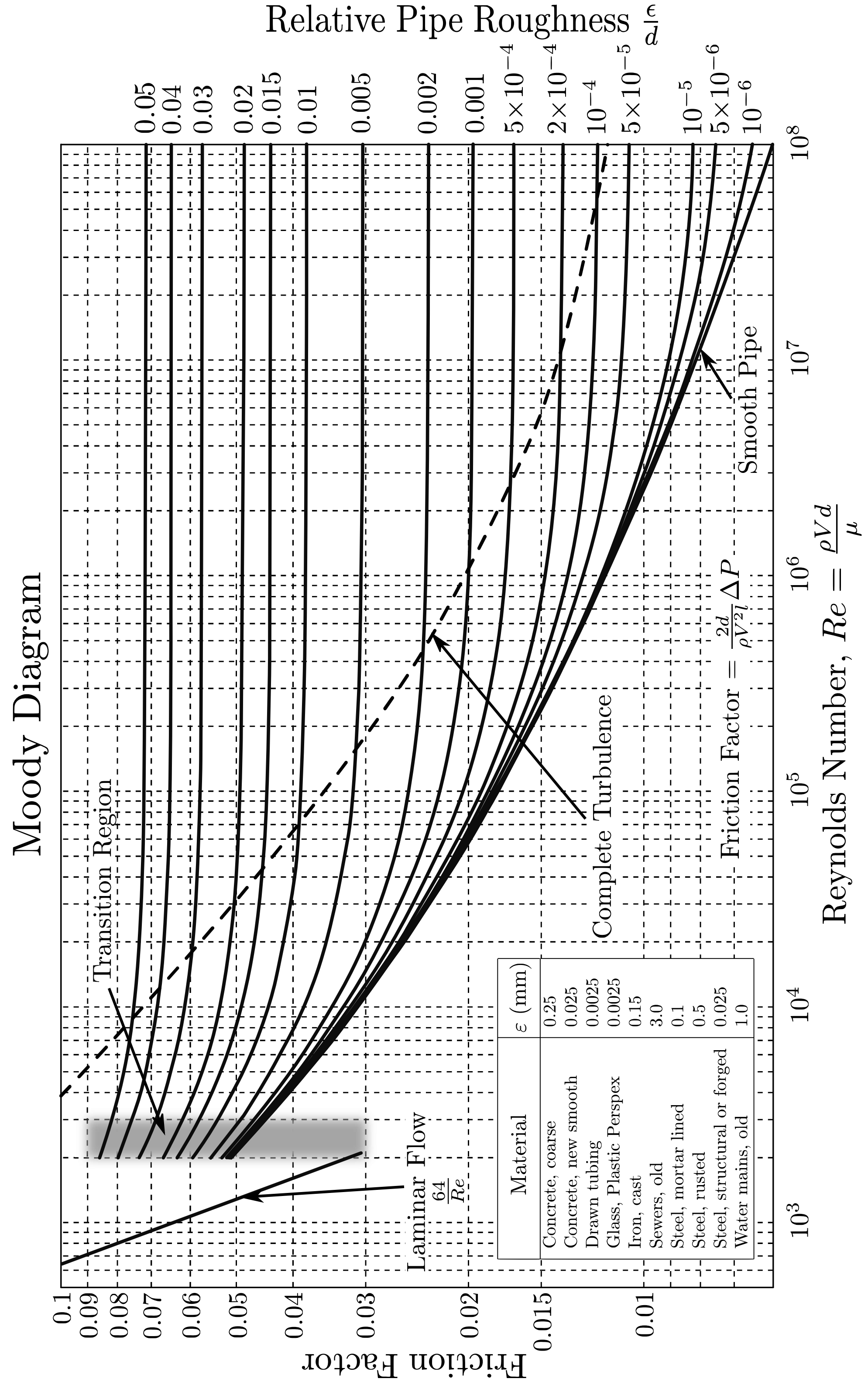

Figure E.1: Moody Diagram; image by S Beck and R Collins, University of Sheffield (2016), Moody Diagram, `https://commons.wikimedia.org/wiki/File:Moody_EN.svg` (licensed under CC BY-SA 4.0)

E.7 Relations between degree of reaction and flow angles in turbomachinery

$$\Lambda = 1 - \frac{1}{2}\frac{C_a}{U}(\tan\alpha_2 + \tan\alpha_3)$$

$$\Lambda = -\frac{1}{2}\frac{C_a}{U}(\tan\beta_2 + \tan\beta_3)$$

E.8 Exergy equations

Exergy equations for closed systems

$$b_{\text{non flow}} = (u - u_0) + P_0(v - v_0) - T_0(s - s_0) + ke + pe$$

$$\Delta b_{\text{non flow}} = (u_2 - u_1) + P_0(v_2 - v_1) - T_0(s_2 - s_1) + \Delta ke + \Delta pe$$

Exergy equations for flow streams

$$b_{\text{flowing fluid}} = (h - h_0) - T_0(s - s_0) + ke + pe$$

$$\Delta b_{\text{flowing fluid}} = (h_2 - h_1) - T_0(s_2 - s_1) + \Delta ke + \Delta pe$$

Exergy transfer by heat transfer

$$B_{\text{heat}} = \sum_k \left(1 - \frac{T_0}{T_k}\right) Q_k$$

Exergy transfer by work

$$B_{\text{work}} = W - P_0(V_2 - V_1)$$

E.9 Perfect gases

Ma	=	Mach number
P, P_0	=	absolute pressure, stagnation pressure
T, T_0	=	absolute temperature, stagnation temperature
v	=	specific volume
ρ, ρ_0	=	density, stagnation density

Over a limited range of temperatures and pressures close to ambient values, the following substances can be assumed to behave as *perfect* gases with properties given by the following relationships:

equation of state	$Pv = RT$
specific heat at constant volume	$C_\mathrm{v} = \dfrac{\mathrm{d}u}{\mathrm{d}T}$ = constant for the particular gas
specific heat at constant pressure	$C_\mathrm{p} = \dfrac{\mathrm{d}h}{\mathrm{d}T}$ = constant for the particular gas
ratio of principal specific heats	$\gamma = \dfrac{C_\mathrm{p}}{C_\mathrm{v}}$ = constant for the particular gas
gas constant	$R = C_\mathrm{p} - C_\mathrm{v}$ = constant for the particular gas

and $R = \dfrac{\overline{R}}{M}$ where $\overline{R}$ = universal gas constant = 8.314 $\mathrm{kJ\,kmol^{-1}\,K^{-1}}$.

Relations for adiabatic processes for an ideal gas

$$Pv^{\gamma} = \text{ const.}$$

$$TP^{\frac{(1-\gamma)}{\gamma}} = \text{ const.}$$

$$Tv^{(\gamma-1)} = \text{ const.}$$

Entropy change equations

$$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2}{v_1}\right)$$

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$$

$$s_2 - s_1 = c_p \ln\left(\frac{v_2}{v_1}\right) + c_v\ln\left(\frac{P_2}{P_1}\right)$$

Cycle efficiency equations for different ideal, air-standard cycles

$$\eta_{\text{th, otto}} = 1 - \frac{1}{r_v^{\gamma-1}} \quad \text{where } r_v = V_1/V_2$$

$$\eta_{\text{th, diesel}} = 1 - \frac{1}{\gamma r_v^{\gamma-1}} \frac{\varphi^\gamma - 1}{\varphi - 1} \quad \text{where } r_v = V_1/V_2 \text{ and } \varphi = V_3/V_2$$

$$\eta_{\text{th,joule}} = 1 - \frac{1}{r_p^{\left(\frac{\gamma-1}{\gamma}\right)}} \quad \text{where } r_P = P_2/P_1$$

Table E.4: Perfect gases (ideal gases with constant specific heats)

Gas	Chemical formula	Molar mass M kg kmol^{-1}	Gas constant R	$\overline{C_\text{p}}$ kJ kg^{-1} K^{-1}	$\overline{C_\text{v}}$	γ
air[1]	—	28.96	0.287	1.01	0.72	1.40
oxygen	O_2	32.00	0.260	0.92	0.66	1.40
nitrogen	N_2	28.01	0.297	1.04	0.74	1.40
atmospheric nitrogen[2]	(AN)	28.17	0.295	1.03	0.74	1.40
carbon dioxide	CO_2	44.01	0.189	0.84	0.65	1.29
carbon monoxide	CO	28.01	0.297	1.04	0.74	1.40
hydrogen	H_2	2.016	4.12	14.31	10.18	1.41
methane	CH_4	16.04	0.518	2.23	1.71	1.30
ethane	C_2H_6	30.07	0.277	1.75	1.47	1.19
helium	He	4.00	2.08	5.20	3.12	1.67

[1] Composition of dry air: 21.0% oxygen, 79.0% atmospheric nitrogen by no. of kmol or by volume; 23.2% oxygen, 76.8% atmospheric nitrogen by mass.

[2] Atmospheric nitrogen contains approx. 1% (by no. of kmol or volume) argon and traces of carbon dioxide and other gases, in addition to nitrogen

Table E.5: Isentropic compressible flow functions for perfect gas with $\gamma = 1.40$

Ma	$\frac{P}{P_0}$	$\frac{\rho}{\rho_0}$	$\frac{T}{T_0}$	Ma	$\frac{P}{P_0}$	$\frac{\rho}{\rho_0}$	$\frac{T}{T_0}$
0	1	1	1	1	0.5283	0.6339	0.8333
0.05	0.9983	0.9988	0.9995	1.02	0.5160	0.6234	0.8278
0.10	0.9930	0.9950	0.9980	1.04	0.5039	0.6129	0.8222
0.15	0.9844	0.9888	0.9955	1.06	0.4919	0.6024	0.8165
				1.08	0.4800	0.5920	0.8108
0.20	0.9725	0.9803	0.9921				
0.22	0.9668	0.9762	0.9904	1.10	0.4684	0.5817	0.8052
0.24	0.9607	0.9718	0.9886	1.15	0.4398	0.5562	0.7908
0.26	0.9541	0.9670	0.9867	1.20	0.4124	0.5311	0.7764
0.28	0.9470	0.9619	0.9846	1.25	0.3861	0.5067	0.7619
				1.30	0.3609	0.4829	0.7474
0.30	0.9395	0.9564	0.9823	1.35	0.3370	0.4598	0.7329
0.32	0.9315	0.9506	0.9799				
0.34	0.9231	0.9445	0.9774	1.40	0.3142	0.4374	0.7184
0.36	0.9143	0.9380	0.9747	1.45	0.2927	0.4158	0.7040
0.38	0.9052	0.9313	0.9719	1.50	0.2724	0.3950	0.6897
				1.55	0.2533	0.3750	0.6754
0.40	0.8956	0.9243	0.9690	1.60	0.2353	0.3557	0.6614
0.42	0.8857	0.9170	0.9659	1.65	0.2184	0.3373	0.6475
0.44	0.8755	0.9094	0.9627				
0.46	0.8650	0.9016	0.9594	1.70	0.2026	0.3197	0.6337
0.48	0.8541	0.8935	0.9559	1.75	0.1878	0.3029	0.6202
				1.80	0.1740	0.2868	0.6068
0.50	0.8430	0.8852	0.9524	1.85	0.1612	0.2715	0.5936
0.52	0.8317	0.8766	0.9487	1.90	0.1492	0.2570	0.5807
0.54	0.8201	0.8679	0.9449	1.95	0.1381	0.2432	0.5680
0.56	0.8082	0.8589	0.9410				
0.58	0.7962	0.8498	0.9370	2.00	0.1278	0.2300	0.5556
				2.10	0.1094	0.2058	0.5313
0.60	0.7840	0.8405	0.9328	2.20	0.09352	0.1841	0.5081
0.62	0.7716	0.8310	0.9286	2.30	0.07997	0.1646	0.4859
0.64	0.7591	0.8213	0.9243	2.40	0.06840	0.1472	0.4647
0.66	0.7465	0.8115	0.9199				
0.68	0.7338	0.8016	0.9153	2.50	0.05853	0.1317	0.4444
				2.60	0.05012	0.1179	0.4252
0.70	0.7209	0.7916	0.9107	2.70	0.04295	0.1056	0.4068
0.72	0.7080	0.7814	0.9061	2.80	0.03685	0.09463	0.3894
0.74	0.6951	0.7712	0.9013	2.90	0.03165	0.08489	0.3729
0.76	0.6821	0.7609	0.8964				
0.78	0.6691	0.7505	0.8915	3.00	0.02722	0.07623	0.3571
				3.20	0.02023	0.06165	0.3281
0.80	0.6560	0.7400	0.8865	3.40	0.01512	0.05009	0.3019
0.82	0.6430	0.7295	0.8815	3.60	0.01138	0.04089	0.2784
0.84	0.6300	0.7189	0.8763	3.80	0.008629	0.03355	0.2572
0.86	0.6170	0.7083	0.8711				
0.88	0.6041	0.6977	0.8659	4.00	0.006586	0.02766	0.2381
				4.50	0.003455	0.01745	0.1980
0.90	0.5913	0.6870	0.8606	5.00	0.001890	0.01134	0.1667
0.92	0.5785	0.6764	0.8552				
0.94	0.5658	0.6658	0.8498	6.00	0.000633	0.005194	0.1220
0.96	0.5532	0.6551	0.8444	8.00	0.000102	0.001414	0.07246
0.98	0.5407	0.6445	0.8389	10.00	0.000024	0.000495	0.04762
1	0.5283	0.6339	0.8333	∞	0	0	0

Table E.6: Ideal (semi-perfect) gas specific enthalpy h (kJ kg^{-1}, 25 řC datum) for combustion calculations on a mass basis

T (řC)	carbon dioxide CO_2	water vapour H_2O	nitrogen N_2	atmos. nitrogen (AN)	oxygen O_2	air —	carbon monoxide CO	hydrogen H_2
0	–22.5	–45.7	–25.6	–25.4	–23.1	–24.8	–25.7	–356.8
25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
100	69.2	139.5	77.5	76.8	70.1	75.2	77.9	1074.5
200	165.3	331.0	182.4	180.9	165.5	177.2	183.5	2517.0
300	266.0	528.9	289.3	286.8	263.1	281.2	291.2	3970.5
400	371.1	733.1	398.0	394.6	362.9	387.1	400.9	5435.3
500	480.7	943.6	508.6	504.3	465.0	494.9	512.7	6911.1
600	594.8	1160.5	621.1	615.8	569.2	604.7	626.5	8398.1
700	713.3	1383.8	735.4	729.2	675.6	716.4	742.5	9896.3
800	836.4	1613.3	851.6	844.4	784.2	830.0	860.4	11405.5
900	963.9	1849.2	969.8	961.5	895.0	945.6	980.5	12925.9
1000	1095.9	2091.5	1089.8	1080.4	1008.0	1063.2	1102.6	14457.5
1100	1232.3	2340.1	1211.6	1201.2	1123.2	1182.6	1226.7	16000.2
1200	1373.3	2595.0	1335.4	1323.9	1240.6	1304.0	1352.9	17554.0
1300	1518.7	2856.3	1461.0	1448.4	1360.2	1427.4	1481.2	19119.0
1400	1668.7	3123.9	1588.5	1574.8	1482.0	1552.7	1611.6	20695.1
1500	1823.1	3397.8	1717.9	1703.1	1606.0	1679.9	1744.0	22282.4

Enthalpy values in Table E.6 have been computed using the approximation

$$C_p = a + bT$$

so that $$h - h_D = \int C_p \, dT = (T - T_D)\left[a + \frac{1}{2}b\,(T + T_D)\right]$$

where datum temperature T_D = 298.15 K and datum enthalpy h_D(at $T = T_D$) = 0 (this is equivalent to using a mean C_p between 298.15 K (25 řC) and T). The values of the constants are tabulated below; T must be in K, giving h in kJ kg^{-1}. The magnitudes of the maximum and mean errors in h refer to the range 0 to 1500 řC.

	carbon dioxide CO_2	water vapour H_2O	nitrogen N_2	atmos. nitrogen (AN)	oxygen O_2	air —	carbon monoxide CO	hydrogen H_2
a	0.772	1.647	0.970	0.962	0.861	0.938	0.969	13.95
$b \times 10^6$ ($\mathrm{kJ\,kg^{-1}\,K^{-2}}$)	448	634	188	186	220	194	206	1114
max. error	4.0	0.7	0.5	0.6	1.3	0.4	0.5	0.7
mean. error (%)	1.4	0.2	0.2	0.2	0.5	0.2	0.2	0.3

Table E.7: Molar Enthalpy of Formation h_f^0 ($\mathrm{kJ\,kmol^{-1}}$ at 25 řC and 1 atmosphere) as gas or vapour (g), except where indicated as solid (s) or liquid (l).

carbon as graphite	C(s)	0
hydrogen	H_2	0
methane	CH_4	–74 850
ethane	C_2H_6	–84 680
propane	C_3H_8	–103 850
n-octane	C_8H_{18}(l)	–249 950
ethanol	C_2H_5OH(l)	–277 690
hydrogen peroxide	H_2O_2	–136 310
carbon dioxide	CO_2	–393 520
water vapour	H_2O(g)	–241 820
liquid water	H_2O(l)	–285 830
nitrogen	N_2	0
atmospheric nitrogen	(AN)	0
oxygen	O_2	0
air	—	0
carbon monoxide	CO	–110 530

Table E.8: Ideal gas molar enthalpy h (kJ kmol^{-1}, 25 řC datum)

T (řC)	carbon dioxide CO_2	water vapour H_2O	nitrogen N_2	atmos. nitrogen (AN)	oxygen O_2	air[a] —	carbon monoxide CO	hydrogen H_2
0	–990	–823	–717	–715	–739	–719	–720	–719
25	0	0	0	0	0	0	0	0
100	3 045	2 513	2 170	2 164	2 244	2 179	2 181	2 166
200	7 276	5 964	5 110	5 096	5 297	5 133	5 139	5 074
300	11 705	9 528	8 103	8 080	8 420	8 143	8 155	8 005
400	16 332	13 207	11 148	11 117	11 614	11 210	11 229	10 957
500	21 155	17 001	14 245	14 205	14 878	14 333	14 360	13 933
600	26 176	20 908	17 396	17 347	18 213	17 512	17 550	16 931
700	31 393	24 930	20 599	20 540	21 618	20 747	20 796	19 951
800	36 808	29 066	23 855	23 786	25 094	24 038	24 101	22 994
900	42 420	33 316	27 163	27 085	28 640	27 385	27 463	26 059
1000	48 229	37 680	30 524	30 436	32 256	30 789	30 883	29 146
1100	54 236	42 159	33 938	33 839	35 943	34 249	34 361	32 256
1200	60 439	46 751	37 404	37 295	39 700	37 765	37 896	35 389
1300	66 840	51 458	40 923	40 803	43 527	41 337	41 489	38 544
1400	73 438	56 280	44 495	44 363	47 425	44 965	45 140	41 721
1500	80 233	61 215	48 119	47 976	51 393	48 650	48 848	44 921

[a]N.B. In a reaction equation, $n_{ox}(O_2 + 3.762N_2)$ represents $4.762n_{ox}$ equivalent kmol of air

E.10 Heating (or calorific) values of fuels

M = molar mass

$\overline{C_p}$ = mean constant-pressure specific heat for use near 25 řC Abbreviations:

$\overline{\rho}$ = approximate density

GCV	gross calorific value	(= HHV = HCV)	
HHV	higher heating value	(=GCV = HCV =	negative of enthalpy of combustion with liquid H_20 in products)
LHV	lower heating value	(=NCV = LCV =	negative of enthalpy of combustion with vapour H_20 in products)
NCV	net calorific value	(=LHV = LCV)	

Table E.9: Heating (or calorific) values of gas fuels at 25 řC.

Gas	M kg kmol^{-1}	HHV or GCV kJ kg^{-1}	LHV or NCV kJ kg^{-1}	$\overline{C_p}$ kJ kg^{-1} K^{-1}
hydrogen	2.016	141 800	119 980	14.31
methane	16.04	55 500	50 020	2.23
ethane	30.07	51 880	47 490	1.75
propane	44.10	50 350	46 360	1.67
n-butane	58.12	49 500	45 720	1.68
n-pentane	72.15	49 020	45 360	1.67
n-hexane	86.18	48 680	45 110	1.66
carbon monoxide	28.01	10 100	10 100	1.04
typical North Sea gas[a]	17.05	53 510	48 290	2.15

[a] molar composition: CH_4 94.4%, C_2H_6 3.0%, N_2 1.5%, other gases 1.1%. Elemental composition by mass: C 73.26%, H 23.90%, O 0.38%, N 2.46%.

Table E.10: Heating (or calorific) values of liquid fuels at 25 řC.

Liquid	Approx. elemental composition by mass (%)	$\overline{\rho}$ kg m^{-3}	HHV or GCV kJ kg^{-1}	LHV or NCV kJ kg^{-1}	$\overline{C_p}$ kJ kg^{-1} K^{-1}
n-octane C_8H_{18}	C84.1 H15.9	703	47 890	44 420	2.11
methanol CH_3OH	C37.5 H12.6 O49.9	790	22 690	19 960	2.51
petrol (gasoline)	C85.0 H15.0	740	46 900	43 630	2.06
kerosine	C86.1 H13.9	770	46 140	43 100	2.02
distillate fuel oil	C86.8 H13.2	820	45 600	42 720	1.95

E.11 Properties of R134a refrigerant

R134a or HFC134a is a hydrofluorocarbon refrigerant (1,1,1,2-tetrafluoroethane, CH_2FCF_3) with zero ozone-depleting potential, although it has some global warming potential. It is a substitute for the chlorofluorocarbon refrigerant R12 (banned under the Montreal Protocol 1984) in domestic refrigeration and freezing applications and in the coolers of air conditioning plant.

T, T_{sat} = temperature, at saturation

P = absolute pressure

v = specific volume, $m^3\,kg^{-1}$

h = specific enthalpy, $kJ\,kg^{-1}$

s = specific entropy, $kJ\,kg^{-1}\,K^{-1}$

Table E.11: Saturated Refrigerant 134a — Temperature (−60°C to critical point)

T (°C)	P bar (abs)	v_f $m^3\,kg^{-1}$	v_g $m^3\,kg^{-1}$	h_f $kJ\,kg^{-1}$	h_g $kJ\,kg^{-1}$	s_f $kJ\,kg^{-1}\,K^{-1}$	s_g $kJ\,kg^{-1}\,K^{-1}$
−60	0.1587	0.0006795	1.0808	22.92	261.17	0.6828	1.8005
−50	0.2942	0.0006923	0.6064	35.33	267.44	0.7396	1.7798
−40	0.5118	0.0007060	0.3609	47.88	273.71	0.7946	1.7632
−35	0.6612	0.0007132	0.2838	54.22	276.84	0.8214	1.7562
−30	0.8436	0.0007206	0.2257	60.60	279.95	0.8479	1.7500
−25	1.064	0.0007284	0.1814	67.03	283.05	0.8740	1.7445
−20	1.327	0.0007364	0.1473	73.51	286.13	0.8998	1.7397
−15	1.639	0.0007448	0.1206	80.05	289.18	0.9252	1.7354
−10	2.005	0.0007536	0.09954	88.64	292.20	0.9504	1.7316
−5	2.432	0.0007627	0.08279	93.29	295.19	0.9753	1.7283
0	2.925	0.0007723	0.06933	100.00	298.10	1.0000	1.7254
5	3.492	0.0007823	0.05842	106.78	301.02	1.0244	1.7228
10	4.139	0.0007929	0.04951	113.62	303.86	1.0486	1.7205
15	4.873	0.0008041	0.04218	120.54	306.64	1.0727	1.7185
20	5.702	0.0008160	0.03610	127.54	309.35	1.0965	1.7167
25	6.634	0.0008285	0.03102	134.61	311.97	1.1202	1.7150
30	7.675	0.0008419	0.02676	141.77	314.49	1.1437	1.7134
35	8.835	0.0008563	0.02315	149.02	316.91	1.1671	1.7119
40	10.12	0.0008718	0.02008	156.37	319.20	1.1904	1.7104
45	11.55	0.0008886	0.01746	163.83	321.34	1.2136	1.7087
50	13.11	0.0009068	0.01520	171.41	323.31	1.2368	1.7069
60	16.73	0.0009493	0.01154	187.00	326.60	1.2833	1.7024
80	26.21	0.001077	0.006516	221.07	328.95	1.3798	1.6852
101	40.55	0.001964	0.001964	289.40	289.40	1.5609	1.5609

Table E.12: Saturated Refrigerant 134a — Pressure (0.2 bar to critical point)

P bar (abs)	T řC	v_f $m^3\,kg^{-1}$	v_g $m^3\,kg^{-1}$	h_f $kJ\,kg^{-1}$	h_g $kJ\,kg^{-1}$	s_f $kJ\,kg^{-1}\,K^{-1}$	s_g $kJ\,kg^{-1}\,K^{-1}$
0.2	−56.38	0.0006841	0.8703	27.40	263.44	0.7035	1.7925
0.4	−44.58	0.0006996	0.4547	42.11	270.84	0.7696	1.7703
0.6	−36.93	0.0007104	0.3109	51.77	275.64	0.8111	1.7588
0.8	−31.11	0.0007189	0.2373	59.18	279.26	0.8420	1.7513
1.0	−26.36	0.0007262	0.1923	65.28	282.21	0.8669	1.7460
1.2	−22.31	0.0007327	0.1620	70.51	284.71	0.8879	1.7418
1.4	−18.75	0.0007385	0.1400	75.14	286.89	0.9062	1.7385
1.6	−15.58	0.0007438	0.1234	79.29	288.83	0.9223	1.7358
2.0	−10.06	0.0007534	0.09978	86.53	292.16	0.9500	1.7316
2.5	−4.26	0.0007641	0.08062	94.28	295.62	0.9790	1.7278
3.0	0.70	0.0007737	0.06766	100.95	298.54	1.0034	1.7250
3.5	5.07	0.0007825	0.05829	106.87	301.06	1.0247	1.7228
4.0	8.98	0.0007907	0.05119	112.22	303.29	1.0437	1.7210
5.0	15.80	0.0008060	0.04113	121.66	307.08	1.0765	1.7182
6.0	21.66	0.0008201	0.03431	129.88	310.23	1.1044	1.7161
8.0	31.45	0.0008460	0.02565	143.86	315.21	1.1505	1.7130
10.0	39.55	0.0008703	0.02034	155.70	319.00	1.1883	1.7105
12.0	46.50	0.0008939	0.01675	166.09	321.95	1.2206	1.7082
15.0	55.45	0.0009289	0.01308	179.83	325.23	1.2621	1.7046
20.0	67.72	0.0009896	0.009318	199.58	328.37	1.3197	1.6975
30.0	86.38	0.001144	0.005306	233.52	327.42	1.4135	1.6747
40.55	101.00	0.001964	0.001964	289.35	289.35	1.5609	1.5609

Table E.13: Superheated Refrigerant 134a (0.2 bar to 1 bar)

T(řC)	0.2 bar abs ($T_{sat} = -56.38$ řC)			0.4 bar abs ($T_{sat} = -44.58$ řC)			0.6 bar abs ($T_{sat} = -36.93$ řC)			0.8 bar abs ($T_{sat} = -31.11$ řC)			1.0 bar abs ($T_{sat} = -26.36$ řC)			T(řC)
	v	*h*	*s*	*v*	*h*	*s*	*v*	*h*	*s*	*v*	*h*	*s*	*v*	*h*	*s*	
Sat.	0.8703	263.44	1.7925	0.4547	270.84	1.7703	0.3109	275.64	1.7588	0.2373	279.26	1.7513	0.1923	282.21	1.7460	Sat.
−50	0.8975	267.92	1.8129													−50
−40	0.9398	275.08	1.8443	0.4647	274.21	1.7849										−40
−30	0.9818	282.41	1.8750	0.4864	281.67	1.8162	0.3212	280.90	1.7808	0.2386	280.13	1.7549				−30
−20	1.0236	289.91	1.9053	0.5079	289.27	1.8469	0.3359	288.61	1.8119	0.2499	287.95	1.7864	0.1982	287.27	1.7662	−20
−10	1.0652	297.59	1.9350	0.5291	297.03	1.8769	0.3504	296.46	1.8422	0.2610	295.88	1.8171	0.2073	295.30	1.7973	−10
0	1.107	305.43	1.9643	0.5502	304.94	1.9064	0.3647	304.45	1.8720	0.2719	303.94	1.8472	0.2162	303.43	1.8276	0
10	1.148	313.45	1.9931	0.5712	313.02	1.9355	0.3789	312.58	1.9013	0.2827	312.14	1.8767	0.2250	311.69	1.8573	10
20	1.189	321.64	2.0215	0.5921	321.26	1.9641	0.3930	320.87	1.9300	0.2935	320.47	1.9056	0.2337	320.08	1.8864	20
30	1.231	330.01	2.0496	0.6129	329.66	1.9922	0.4071	329.31	1.9584	0.3041	328.96	1.9341	0.2423	328.60	1.9150	30
40	1.272	338.54	2.0773	0.6337	338.22	2.0200	0.4210	337.91	1.9863	0.3147	337.59	1.9621	0.2509	337.27	1.9432	40
50				0.6544	346.95	2.0475	0.4350	346.67	2.0138	0.3252	346.38	1.9897	0.2594	346.09	1.9709	50
60				0.6751	355.84	2.0746	0.4489	355.58	2.0410	0.3357	355.32	2.0169	0.2679	355.05	1.9982	60
70							0.4627	364.66	2.0678	0.3462	364.41	2.0438	0.2763	364.17	2.0251	70
80													0.2847	373.44	2.0518	80

Table E.14: Superheated Refrigerant 134a (1.5 bar to 4 bar)

T(řC)	1.5 bar abs ($T_{sat} = -17.13$ řC)			2.0 bar abs ($T_{sat} = -10.06$ řC)			2.5 bar abs ($T_{sat} = -4.26$ řC)			3.0 bar abs ($T_{sat} = 0.70$ řC)			3.5 bar abs ($T_{sat} = 8.98$ řC)			T(řC)
	v	*h*	*s*	*v*	*h*	*s*	*v*	*h*	*s*	*v*	*h*	*s*	*v*	*h*	*s*	
Sat.	0.1311	287.89	1.7371	0.0998	292.16	1.7316	0.0806	295.62	1.7278	0.0677	298.54	1.7250	0.0512	303.29	1.7210	Sat.
−10	0.1357	293.79	1.7599	0.0998	292.22	1.7318										−10
0	0.1419	302.13	1.7910	0.1047	300.77	1.7638	0.0824	299.37	1.7416							0
10	0.1481	310.55	1.8212	0.1095	309.37	1.7947	0.0864	308.15	1.7732	0.0709	306.90	1.7550	0.0515	304.25	1.7244	10
20	0.1541	319.07	1.8508	0.1142	318.03	1.8247	0.0902	316.97	1.8038	0.0742	315.88	1.7862	0.0542	313.59	1.7568	20
30	0.1600	327.70	1.8798	0.1187	326.78	1.8541	0.0940	325.84	1.8336	0.0775	324.88	1.8164	0.0568	322.89	1.7880	30
40	0.1658	336.46	1.9082	0.1232	335.64	1.8828	0.0977	334.80	1.8627	0.0806	333.95	1.8458	0.0593	332.18	1.8181	40
50	0.1716	345.36	1.9362	0.1277	344.61	1.9110	0.1013	343.86	1.8911	0.0837	343.09	1.8745	0.0617	341.51	1.8475	50
60	0.1773	354.39	1.9637	0.1321	353.71	1.9388	0.1049	353.03	1.9191	0.0868	352.33	1.9027	0.0641	350.91	1.8761	60
70	0.1831	363.56	1.9908	0.1364	362.94	1.9661	0.1085	362.31	1.9465	0.0898	361.68	1.9303	0.0665	360.38	1.9041	70
80	0.1887	372.87	2.0176	0.1408	372.30	1.9930	0.1120	371.73	1.9736	0.0928	371.14	1.9575	0.0688	369.96	1.9316	80
90	0.1944	382.34	2.0440	0.1451	381.81	2.0195	0.1155	381.27	2.0002	0.0957	380.73	1.9843	0.0710	379.64	1.9586	90
100	0.2000	391.95	2.0701	0.1493	391.45	2.0457	0.1189	390.95	2.0265	0.0986	390.45	2.0107	0.0733	389.44	1.9853	100
110				0.1536	401.24	2.0716	0.1224	400.77	2.0525	0.1015	400.31	2.0368	0.0755	399.36	2.0115	110
120										0.1044	410.30	2.0625	0.0777	409.41	2.0374	120
130													0.0799	419.59	2.0630	130

Table E.15: Superheated Refrigerant 134a (5 bar to 12 bar)

	5 bar abs (T_{sat} = 15.80 řC)			6 bar abs (T_{sat} = 21.66 řC)			8 bar abs (T_{sat} = 31.45 řC)			10 bar abs (T_{sat} = 39.55 řC)			12 bar abs (T_{sat} = 46.50 řC)			
T(řC)	*v*	*h*	*s*	*v*	*h*	*s*	*v*	*h*	*s*	*v*	*h*	*s*	*v*	*h*	*s*	T(řC)
Sat.	0.0411	307.08	1.7182	0.0343	310.23	1.7161	0.0256	315.21	1.7130	0.0203	319.00	1.7105	0.0167	321.95	1.7082	Sat.
20	0.0421	311.16	1.7322													20
30	0.0443	320.79	1.7645	0.0360	318.56	1.7440										30
40	0.0465	330.34	1.7955	0.0379	328.41	1.7760	0.0270	324.24	1.7422	0.0204	319.51	1.7121				40
50	0.0485	339.88	1.8255	0.0397	338.18	1.8067	0.0286	334.56	1.7747	0.0218	330.58	1.7469	0.0172	326.10	1.7211	50
60	0.0505	349.44	1.8546	0.0414	347.93	1.8364	0.0300	344.74	1.8057	0.0231	341.30	1.7796	0.0184	337.53	1.7559	60
70	0.0524	359.06	1.8831	0.0431	357.69	1.8653	0.0314	354.85	1.8356	0.0243	351.82	1.8107	0.0195	348.56	1.7886	70
80	0.0543	368.75	1.9109	0.0447	367.51	1.8935	0.0327	364.94	1.8646	0.0254	362.23	1.8406	0.0205	359.36	1.8196	80
90	0.0562	378.53	1.9382	0.0463	377.39	1.9210	0.0339	375.05	1.8928	0.0265	372.60	1.8696	0.0215	370.04	1.8494	90
100	0.0581	388.41	1.9651	0.0479	387.36	1.9481	0.0352	385.21	1.9204	0.0276	382.97	1.8978	0.0224	380.65	1.8782	100
110	0.0599	398.40	1.9915	0.0495	397.42	1.9747	0.0364	395.43	1.9475	0.0286	393.38	1.9253	0.0234	391.26	1.9063	110
120	0.0617	408.51	2.0175	0.0510	407.60	2.0010	0.0376	405.75	1.9740	0.0296	403.84	1.9522	0.0242	401.89	1.9337	120
130	0.0635	418.74	2.0432	0.0525	417.89	2.0268	0.0388	416.15	2.0002	0.0306	414.38	1.9787	0.0251	412.57	1.9605	130
140	0.0653	429.11	2.0686	0.0540	428.30	2.0523	0.0400	426.67	2.0259	0.0316	425.01	2.0048	0.0259	423.32	1.9868	140
150							0.0412	437.30	2.0514	0.0325	435.73	2.0304	0.0268	434.15	2.0127	150

Table E.16: Superheated Refrigerant 134a (16 bar to 30 bar)

	15 bar abs ($T_{sat} = 55.45$ řC)			18 bar abs ($T_{sat} = 63.13$ řC)			22 bar abs ($T_{sat} = 71.96$ řC)			26 bar abs ($T_{sat} = 79.62$ řC)			30 bar abs ($T_{sat} = 86.38$ řC)			
T(řC)	*v*	*h*	*s*	*v*	*h*	*s*	*v*	*h*	*s*	*v*	*h*	*s*	*v*	*h*	*s*	T(řC)
Sat.	0.0131	325.23	1.7046	0.0106	327.42	1.7006	0.00826	328.95	1.6941	0.00659	328.99	1.6858	0.00531	327.42	1.6747	Sat.
60	0.0136	331.03	1.7222													60
70	0.0147	343.14	1.7580	0.0113	336.80	1.7282										70
80	0.0156	354.70	1.7912	0.0123	349.46	1.7646	0.00909	341.13	1.7290	0.00664	329.72	1.6878				80
90	0.0165	365.94	1.8225	0.0131	361.45	1.7981	0.00993	354.66	1.7668	0.00763	346.45	1.7346	0.00575	335.43	1.6969	90
100	0.0173	376.99	1.8526	0.0139	373.05	1.8296	0.0107	367.28	1.8011	0.00840	360.68	1.7732	0.00665	352.83	1.7442	100
110	0.0181	387.94	1.8815	0.0146	384.42	1.8596	0.0113	379.37	1.8331	0.00905	373.80	1.8079	0.00734	367.52	1.7830	110
120	0.0189	398.85	1.9096	0.0153	395.67	1.8886	0.0120	391.17	1.8634	0.00965	386.31	1.8402	0.00793	381.00	1.8178	120
130	0.0196	409.76	1.9371	0.0159	406.85	1.9167	0.0125	402.78	1.8926	0.0102	398.45	1.8707	0.00846	393.83	1.8500	130
140	0.0203	420.71	1.9639	0.0165	418.02	1.9441	0.0131	414.30	1.9208	0.0107	410.38	1.8999	0.00894	406.27	1.8805	140
150	0.0210	431.71	1.9902	0.0171	429.21	1.9708	0.0136	425.77	1.9483	0.0112	422.19	1.9281	0.00940	418.47	1.9096	150

E.12 Transport properties of air, water and steam

C_p	=	specific heat at constant pressure	$kJ\,kg^{-1}\,K^{-1}$
C_v	=	specific heat at constant volume	$kJ\,kg^{-1}\,K^{-1}$
k	=	thermal conductivity	$W\,m^{-1}\,K^{-1}$
T	=	temperature	K or řC
x	=	definition	$kJ\,kg^{-1}\,K^{-1}$
x	=	definition	$kJ\,kg^{-1}\,K^{-1}$
α	=	thermal diffusivity	$m^2\,s^{-1}$
μ	=	absolute viscosity	$kg\,m^{-1}\,s^{-1}$ (= $N\,s\,m^{-2}$)
ν	=	kinematic viscosity	$m^2\,s^{-1}$
ρ	=	density	$kg\,m^{-3}$

Table E.17: Transport properties of dry air at atmospheric pressure

Temperature T (řC)	Density ρ ($\mathrm{kg\,m^{-3}}$)	Specific heat at constant pressure C_p ($\mathrm{J\,kg^{-1}\,K^{-1}}$)	Absolute (or dynamic) viscosity μ ($\mathrm{kg\,m^{-1}\,s^{-1}}$)	Kinematic viscosity $\nu = \frac{\mu}{\rho}$ ($\mathrm{m^2\,s^{-1}}$)	Thermal conductivity k ($\mathrm{W\,m^{-1}\,K^{-1}}$)	Thermal diffusivity α ($\mathrm{m^2\,s^{-1}}$)	Prandtl number $\mathrm{Pr} = \frac{\mu C_p}{k}$	$\frac{g\beta}{\alpha\nu} = \frac{\mathrm{GrPr}}{L^3 \Delta T}$ ($\mathrm{m^{-3}\,K^{-1}}$)	Temperature T (řC)
−180	3.72	1035	6.50×10^{-6}	1.75×10^{-6}	0.0076	1.9×10^{-6}	0.92	3.2×10^{10}	−180
−100	2.04	1010	1.16×10^{-5}	5.69×10^{-6}	0.016	7.6×10^{-6}	0.75	1.3×10^{9}	−100
−50	1.582	1006	1.45×10^{-5}	9.17×10^{-6}	0.020	1.30×10^{-5}	0.72	3.67×10^{8}	−50
0	1.293	1006	1.71×10^{-5}	1.32×10^{-5}	0.024	1.84×10^{-5}	0.72	1.48×10^{8}	0
10	1.247	1006	1.76×10^{-5}	1.41×10^{-5}	0.025	1.96×10^{-5}	0.72	1.25×10^{8}	10
20	1.205	1006	1.81×10^{-5}	1.50×10^{-5}	0.025	2.08×10^{-5}	0.72	1.07×10^{8}	20
30	1.165	1006	1.86×10^{-5}	1.60×10^{-5}	0.026	2.23×10^{-5}	0.72	9.07×10^{7}	30
60	1.060	1008	2.00×10^{-5}	1.89×10^{-5}	0.028	2.74×10^{-5}	0.70	5.71×10^{7}	60
100	0.946	1011	2.18×10^{-5}	2.30×10^{-5}	0.032	3.28×10^{-5}	0.70	3.48×10^{7}	100
200	0.746	1025	2.58×10^{-5}	3.46×10^{-5}	0.039	5.19×10^{-5}	0.68	9.53×10^{6}	200
300	0.616	1045	2.95×10^{-5}	4.79×10^{-5}	0.045	7.17×10^{-5}	0.68	4.96×10^{6}	300
500	0.456	1093	3.58×10^{-5}	7.85×10^{-5}	0.056	1.14×10^{-4}	0.70	1.42×10^{6}	500
1000	0.277	1185	4.82×10^{-5}	1.74×10^{-4}	0.076	2.42×10^{-4}	0.72	1.8×10^{5}	1000

Table E.18: Transport properties of saturated water and steam

Temp.	Specific volume		Specific heat at constant pressure		Thermal conductivity		Absolute (or dynamic) viscosity		Prandtl number $\frac{\mu C_p}{k}$		Temp.
T	v_f	v_g	C_{pf}	C_{pg}	k_f	k_g	μ_f	μ_g	Pr_f	Pr_g	T
(řC)	($m^3\,kg^{-1}$)	($m^3\,kg^{-1}$)	($J\,kg^{-1}\,K^{-1}$)	($J\,kg^{-1}\,K^{-1}$)	($W\,m^{-1}\,K^{-1}$)	($W\,m^{-1}\,K^{-1}$)	($kg\,m^{-1}\,s^{-1}$)	($kg\,m^{-1}\,s^{-1}$)			(řC)
0.01	0.001000	206.0	4217	1854	0.569	0.0173	1.755×10^{-3}	8.8×10^{-6}	13.02	0.942	0.01
10	0.001000	106.3	4193	1860	0.587	0.0185	1.301×10^{-3}	9.1×10^{-6}	9.29	0.915	10
20	0.001002	57.78	4182	1866	0.603	0.0191	1.002×10^{-3}	9.4×10^{-6}	6.95	0.918	20
30	0.001004	32.90	4179	1875	0.618	0.0198	7.97×10^{-4}	9.7×10^{-6}	5.39	0.923	30
40	0.001008	19.53	4179	1885	0.632	0.0204	6.51×10^{-4}	1.01×10^{-5}	4.31	0.930	40
50	0.001012	12.04	4181	1899	0.643	0.0210	5.44×10^{-4}	1.04×10^{-5}	3.53	0.939	50
60	0.001017	7.674	4185	1915	0.653	0.0217	4.62×10^{-4}	1.07×10^{-5}	2.96	0.947	60
70	0.001023	5.045	4190	1936	0.662	0.0224	4.00×10^{-4}	1.11×10^{-5}	2.53	0.956	70
80	0.001029	3.409	4197	1962	0.670	0.0231	3.50×10^{-4}	1.14×10^{-5}	2.19	0.966	80
90	0.001036	2.362	4205	1992	0.676	0.0240	3.11×10^{-4}	1.17×10^{-5}	1.93	0.976	90
100	0.001043	1.674	4216	2028	0.681	0.0249	2.78×10^{-4}	1.21×10^{-5}	1.723	0.986	100
125	0.001065	0.7709	4254	2147	0.687	0.0272	2.19×10^{-4}	1.33×10^{-5}	1.358	1.047	125
150	0.001090	0.3929	4310	2314	0.687	0.0300	1.80×10^{-4}	1.44×10^{-5}	1.133	1.110	150
175	0.001121	0.2168	4389	2542	0.679	0.0334	1.53×10^{-4}	1.56×10^{-5}	0.990	1.185	175
200	0.001156	0.1273	4497	2843	0.665	0.0375	1.33×10^{-4}	1.67×10^{-5}	0.902	1.270	200
225	0.001199	0.07846	4648	3238	0.644	0.0427	1.182×10^{-4}	1.79×10^{-5}	0.853	1.36	225
250	0.001251	0.05011	4867	3772	0.616	0.0495	1.065×10^{-4}	1.91×10^{-5}	0.841	1.45	250
275	0.001317	0.03278	5202	4561	0.582	0.0587	9.72×10^{-5}	2.02×10^{-5}	0.869	1.56	275
300	0.001404	0.02167	5762	5863	0.541	0.0719	8.97×10^{-5}	2.14×10^{-5}	0.955	1.74	300
325	0.001528	0.01419	6861	8440	0.493	0.0929	7.90×10^{-5}	2.30×10^{-5}	1.100	2.09	325
350	0.001740	0.008812	10100	17150	0.437	0.1343	6.48×10^{-5}	2.58×10^{-5}	1.50	3.29	350
360	0.001894	0.006962	14600	25100	0.400	0.168	5.82×10^{-5}	2.75×10^{-5}	2.11	3.89	360
374	0.003106	0.003106	∞	∞	0.24	0.24	4.5×10^{-5}	4.5×10^{-5}	∞	∞	374

E.13 Approximate physical properties

ρ	=	density	$\mathrm{kg\,m^{-3}}$
C_p	=	specific heat at constant pressure	$\mathrm{J\,kg^{-1}\,K^{-1}}$
k	=	thermal conductivity	$\mathrm{W\,m^{-1}\,K^{-1}}$
μ	=	absolute or dynamic viscosity	$\mathrm{kg\,m^{-1}\,s^{-1}}$ or $\mathrm{N\,s\,m^{-2}}$
σ	=	surface tension	$\mathrm{N\,m^{-1}}$

Table E.19: Approximate physical properties at 20řC, 1 bar.

Gases[a]	ρ	C_p	k	μ	
air	1.19	1 010	0.025	1.81×10^{-5}	
oxygen	1.31	910	0.026	2.03×10^{-5}	
nitrogen	1.15	1 040	0.026	1.76×10^{-5}	
carbon dioxide	1.80	8 40	0.017	1.47×10^{-5}	
hydrogen	0.083	14 300	0.18	0.88×10^{-5}	
helium	0.164	5 230	0.14	1.96×10^{-5}	
Liquids	ρ	C_p	k	μ	σ
water	1000	4 190	0.60	1.00×10^{-3}	0.073
mercury	13 600	140	8.7	1.55×10^{-3}	0.51
ethanol	790	2 860	0.19	1.20×10^{-3}	0.022
R134a (25řC)	1 210	1430	0.080	0.21×10^{-3}	
Solids	ρ	C_p	k	Notes	
mild steel	7 850	460	52		
stainless steel	7810	460	16	18% Ni 8% Cr	
aluminium alloy	2720	880	170	Duralumin	
copper	8 950	380	400		
brass	8 410	380	120	30% Zn	
polyethylene	1 000	2 100	0.5	moderately high density	
expanded polystyrene	25		0.04	board	
concrete	2 400	900	1.0	moderately dense	
brick	1 800	750	0.6	common building brick	
wood	500	2 500	0.15	pine, dry	
glass	2 500	800	1.0	window	

[a] NB. Constant C_p values in Table E.4 are averages over a range of temperature. $C_p(T)$ relationships used in Table E.6 are fits over a different temperature range. Neither will necessarily agree well with the 20řC values tabulated here.

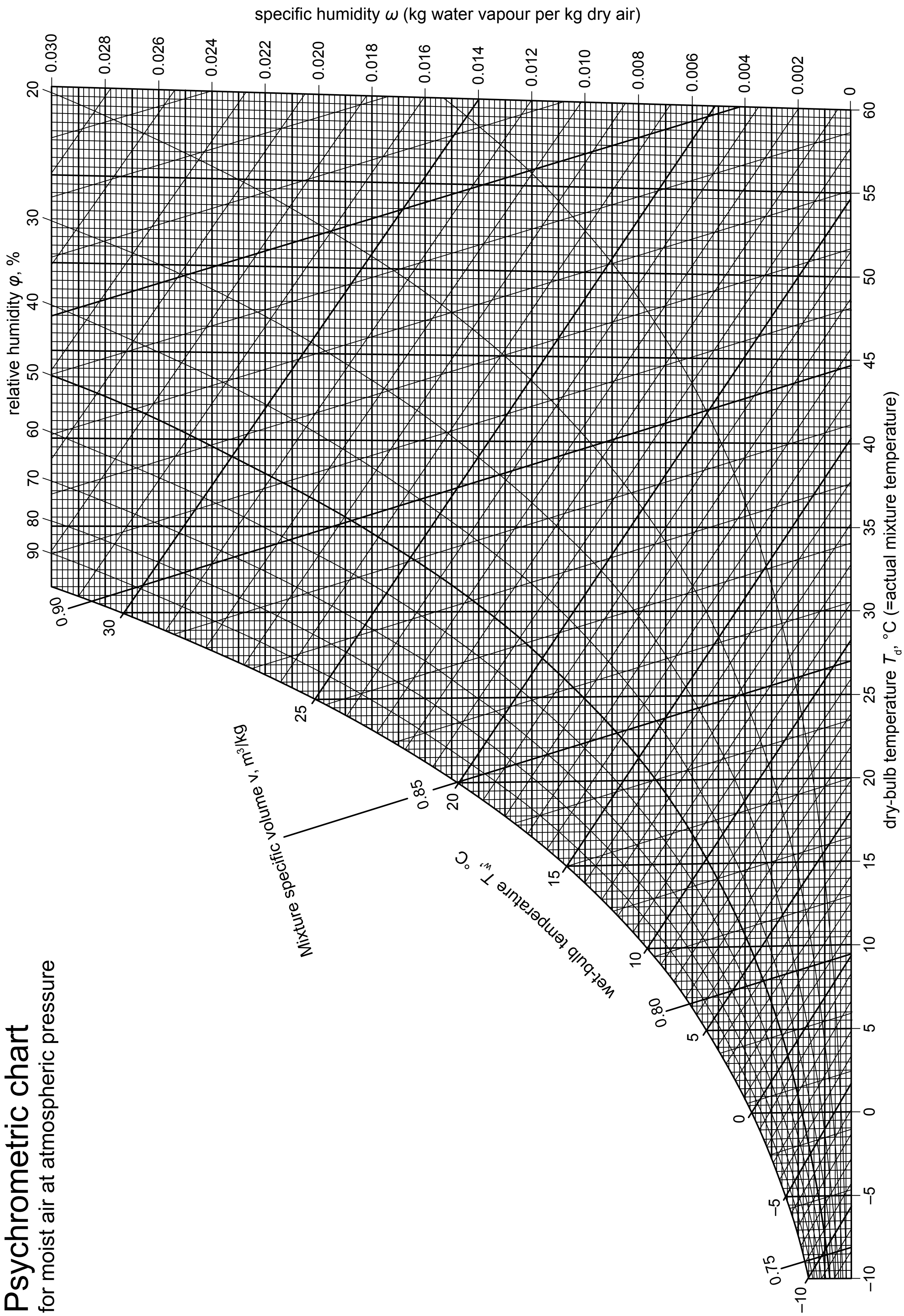

Figure E.2: Psychrometric Chart

E.14 Thermodynamic property tables for water/steam (IAPWS-IF97 formulation)

h	=	specific enthalpy	$\mathrm{kJ\,kg^{-1}}$
P	=	absolute pressure	bar (1 bar = 10^5 Pa = 10^5 $\mathrm{N\,m^{-2}}$)
s	=	specific entropy	$\mathrm{kJ\,kg^{-1}\,K^{-1}}$
T	=	temperature	řC
u	=	specific internal energy	$\mathrm{kJ\,kg^{-1}}$
v	=	specific volume	$\mathrm{m^3\,kg^{-1}}$

Subscripts:

f	=	saturated liquid state
fg	=	change between saturated liquid and saturated vapour states at constant pressure ($h_{fg} = h_g - h_f$)
g	=	saturated vapour state
sat	=	saturation state

Properties in these tables were evaluated from the Industrial Formulation 1997 of the International Association for the Properties of Water and Steam (IAPWS). There are small differences between these values and those in use in the Department of Mechanical Engineering up to 2003 (evaluated from the IAPWS 1984 Formulation for Scientific and General Use and 1986 Supplementary Release on Saturation Properties).

Triple point:	$T = 0.01$ řC	$P = 0.006117$ bar
Critical point:	$T = 373.95$ řC	$P = 220.64$ bar

u and s are chosen to be zero for saturated liquid at the triple point.

Linear interpolation in the tables is not advisable near the critical point.

Table E.20: Saturated water and steam — Temperature (triple point to 100 řC)

T řC	P bar (abs)	v_f $m^3\,kg^{-1}$	v_g $m^3\,kg^{-1}$	u_f $kJ\,kg^{-1}$	u_g $kJ\,kg^{-1}$	h_f $kJ\,kg^{-1}$	h_{fg} $kJ\,kg^{-1}$	h_g $kJ\,kg^{-1}$	s_f $kJ\,kg^{-1}\,K^{-1}$	s_g $kJ\,kg^{-1}\,K^{-1}$
0.01	0.006117	0.001000	206.0	0	2374.9	0.000612	2500.9	2500.9	0	9.155
1	0.006571	0.001000	192.4	4.18	2376.3	4.177	2498.6	2502.7	0.0153	9.129
2	0.007060	0.001000	179.8	8.39	2377.7	8.392	2496.2	2504.6	0.0306	9.103
3	0.007581	0.001000	168.0	12.60	2379.0	12.60	2493.8	2506.4	0.0459	9.076
4	0.008135	0.001000	157.1	16.81	2380.4	16.81	2491.4	2508.2	0.0611	9.051
5	0.008726	0.001000	147.0	21.02	2381.8	21.02	2489.1	2510.1	0.0763	9.025
6	0.009354	0.001000	137.6	25.22	2383.2	25.22	2486.7	2511.9	0.0913	8.999
7	0.01002	0.001000	128.9	29.42	2384.5	29.43	2484.3	2513.7	0.1064	8.974
8	0.01073	0.001000	120.8	33.62	2385.9	33.63	2481.9	2515.6	0.1213	8.949
9	0.01148	0.001000	113.3	37.82	2387.3	37.82	2479.6	2517.4	0.1362	8.924
10	0.01228	0.001000	106.3	42.02	2388.7	42.02	2477.2	2519.2	0.1511	8.900
11	0.01313	0.001000	99.79	46.21	2390.0	46.22	2474.8	2521.1	0.1659	8.876
12	0.01403	0.001001	93.72	50.41	2391.4	50.41	2472.5	2522.9	0.1806	8.851
13	0.01498	0.001001	88.07	54.60	2392.8	54.60	2470.1	2524.7	0.1953	8.828
14	0.01599	0.001001	82.80	58.79	2394.1	58.79	2467.7	2526.5	0.2099	8.804
15	0.01706	0.001001	77.88	62.98	2395.5	62.98	2465.4	2528.4	0.2245	8.780
16	0.01819	0.001001	73.29	67.17	2396.9	67.17	2463.0	2530.2	0.2390	8.757
17	0.01938	0.001001	69.01	71.36	2398.3	71.36	2460.6	2532.0	0.2534	8.734
18	0.02065	0.001001	65.00	75.55	2399.6	75.55	2458.3	2533.8	0.2678	8.711
19	0.02198	0.001002	61.26	79.73	2401.0	79.73	2455.9	2535.7	0.2822	8.689
20	0.02339	0.001002	57.76	83.92	2402.4	83.92	2453.5	2537.5	0.2965	8.666
21	0.02488	0.001002	54.49	88.10	2403.7	88.10	2451.2	2539.3	0.3108	8.644
22	0.02645	0.001002	51.42	92.29	2405.1	92.29	2448.8	2541.1	0.3250	8.622
23	0.02811	0.001003	48.55	96.47	2406.4	96.47	2446.4	2542.9	0.3391	8.600
24	0.02986	0.001003	45.86	100.7	2407.8	100.7	2444.1	2544.7	0.3532	8.578
25	0.03170	0.001003	43.34	104.8	2409.2	104.8	2441.7	2546.5	0.3673	8.557
26	0.03364	0.001003	40.98	109.0	2410.5	109.0	2439.3	2548.4	0.3813	8.535
27	0.03568	0.001004	38.76	113.2	2411.9	113.2	2437.0	2550.2	0.3952	8.514
28	0.03783	0.001004	36.68	117.4	2413.2	117.4	2434.6	2552.0	0.4091	8.493
29	0.04009	0.001004	34.72	121.6	2414.6	121.6	2432.2	2553.8	0.4230	8.473
30	0.04247	0.001004	32.88	125.7	2415.9	125.7	2429.8	2555.6	0.4368	8.452
32	0.04759	0.001005	29.53	134.1	2418.7	134.1	2425.1	2559.2	0.4643	8.411
34	0.05325	0.001006	26.56	142.5	2421.4	142.5	2420.3	2562.8	0.4916	8.372
36	0.05948	0.001006	23.93	150.8	2424.0	150.8	2415.6	2566.4	0.5187	8.332
38	0.06632	0.001007	21.60	159.2	2426.7	159.2	2410.8	2570.0	0.5457	8.294
40	0.07384	0.001008	19.52	167.5	2429.4	167.5	2406.0	2573.5	0.5724	8.256
42	0.08209	0.001009	17.67	175.9	2432.1	175.9	2401.2	2577.1	0.5990	8.218
44	0.09112	0.001009	16.01	184.2	2434.8	184.3	2396.4	2580.7	0.6255	8.182
46	0.1010	0.001010	14.54	192.6	2437.4	192.6	2391.6	2584.2	0.6517	8.145
48	0.1118	0.001011	13.21	201.0	2440.1	201.0	2386.8	2587.8	0.6778	8.110
50	0.1235	0.001012	12.03	209.3	2442.8	209.3	2382.0	2591.3	0.7038	8.075
52	0.1363	0.001013	10.96	217.7	2445.4	217.7	2377.1	2594.8	0.7296	8.040
54	0.1502	0.001014	10.01	226.0	2448.0	226.1	2372.3	2598.4	0.7552	8.007
56	0.1653	0.001015	9.145	234.4	2450.7	234.4	2367.4	2601.9	0.7807	7.973
58	0.1817	0.001016	8.369	242.8	2453.3	242.8	2362.6	2605.4	0.8060	7.940
60	0.1995	0.001017	7.668	251.1	2455.9	251.2	2357.7	2608.8	0.8312	7.908
62	0.2187	0.001018	7.034	259.5	2458.5	259.5	2352.8	2612.3	0.8563	7.876
64	0.2394	0.001019	6.460	267.9	2461.1	267.9	2347.9	2615.8	0.8811	7.845
66	0.2618	0.001020	5.940	276.2	2463.7	276.3	2343.0	2619.2	0.9059	7.814
68	0.2860	0.001022	5.468	284.6	2466.3	284.6	2338.0	2622.7	0.9305	7.784
70	0.3120	0.001023	5.040	293.0	2468.9	293.0	2333.1	2626.1	0.9550	7.754
72	0.3400	0.001024	4.650	301.4	2471.4	301.4	2328.1	2629.5	0.9793	7.725
74	0.3701	0.001025	4.295	309.7	2474.0	309.8	2323.1	2632.9	1.004	7.696
76	0.4024	0.001026	3.971	318.1	2476.5	318.2	2318.1	2636.3	1.028	7.667
78	0.4370	0.001028	3.675	326.5	2479.0	326.6	2313.1	2639.7	1.052	7.639
80	0.4741	0.001029	3.405	334.9	2481.6	334.9	2308.1	2643.0	1.075	7.611
82	0.5139	0.001030	3.158	343.3	2484.1	343.3	2303.0	2646.4	1.099	7.584
84	0.5564	0.001032	2.932	351.7	2486.6	351.7	2297.9	2649.7	1.123	7.557
86	0.6017	0.001033	2.724	360.1	2489.0	360.1	2292.8	2653.0	1.146	7.530
88	0.6502	0.001035	2.534	368.5	2491.5	368.6	2287.7	2656.3	1.169	7.504
90	0.7018	0.001036	2.359	376.9	2494.0	377.0	2282.6	2659.5	1.193	7.478
92	0.7568	0.001037	2.198	385.3	2496.4	385.4	2277.4	2662.8	1.216	7.453
94	0.8154	0.001039	2.050	393.7	2498.8	393.8	2272.2	2666.0	1.239	7.427
96	0.8777	0.001040	1.914	402.1	2501.2	402.2	2267.0	2669.2	1.262	7.403
98	0.9439	0.001042	1.788	410.6	2503.6	410.7	2261.7	2672.4	1.284	7.378
100	1.0142	0.001043	1.672	419.0	2506.0	419.1	2256.5	2675.6	1.307	7.354

Table E.21: Saturated water and steam — Pressure (triple point to 2 bar)

P bar (abs)	T řC	v_f $m^3\,kg^{-1}$	v_g $m^3\,kg^{-1}$	u_f $kJ\,kg^{-1}$	u_g $kJ\,kg^{-1}$	h_f $kJ\,kg^{-1}$	h_{fg} $kJ\,kg^{-1}$	h_g $kJ\,kg^{-1}$	s_f $kJ\,kg^{-1}\,K^{-1}$	s_g $kJ\,kg^{-1}\,K^{-1}$
0.006117	0.01	0.001000	206.0	0	2374.9	0.000611783	2500.9	2500.9	0	9.155
0.008	3.76	0.001000	159.6	15.81	2380.1	15.809	2492.0	2507.8	0.0575	9.057
0.010	6.97	0.001000	129.2	29.30	2384.5	29.298	2484.4	2513.7	0.1059	8.975
0.012	9.65	0.001000	108.7	40.57	2388.2	40.569	2478.0	2518.6	0.1460	8.908
0.014	11.97	0.001001	93.90	50.28	2391.4	50.28	2472.5	2522.8	0.1802	8.852
0.016	14.01	0.001001	82.75	58.83	2394.2	58.84	2467.7	2526.6	0.2101	8.804
0.018	15.84	0.001001	74.01	66.49	2396.7	66.49	2463.4	2529.9	0.2366	8.761
0.020	17.50	0.001001	66.99	73.43	2398.9	73.43	2459.5	2532.9	0.2606	8.723
0.022	19.01	0.001002	61.21	79.79	2401.0	79.79	2455.9	2535.7	0.2824	8.688
0.024	20.41	0.001002	56.38	85.65	2402.9	85.66	2452.6	2538.2	0.3024	8.657
0.026	21.72	0.001002	52.27	91.11	2404.7	91.11	2449.5	2540.6	0.3210	8.628
0.028	22.94	0.001002	48.73	96.20	2406.4	96.20	2446.6	2542.8	0.3382	8.601
0.030	24.08	0.001003	45.66	100.99	2407.9	100.99	2443.9	2544.9	0.3543	8.577
0.035	26.67	0.001003	39.47	111.83	2411.4	111.84	2437.7	2549.6	0.3907	8.521
0.040	28.96	0.001004	34.79	121.40	2414.5	121.40	2432.3	2553.7	0.4224	8.473
0.045	31.01	0.001005	31.13	129.98	2417.3	129.98	2427.4	2557.4	0.4507	8.431
0.050	32.88	0.001005	28.19	137.76	2419.8	137.77	2423.0	2560.8	0.4763	8.394
0.060	36.16	0.001006	23.73	151.49	2424.3	151.49	2415.2	2566.7	0.5209	8.329
0.070	39.00	0.001007	20.53	163.36	2428.1	163.37	2408.4	2571.8	0.5591	8.275
0.080	41.51	0.001008	18.10	173.84	2431.4	173.85	2402.4	2576.2	0.5925	8.227
0.090	43.76	0.001009	16.20	183.25	2434.5	183.26	2397.0	2580.3	0.6223	8.186
0.100	45.81	0.001010	14.67	191.80	2437.2	191.81	2392.1	2583.9	0.6492	8.149
0.11	47.68	0.001011	13.41	199.65	2439.7	199.66	2387.6	2587.2	0.6737	8.115
0.12	49.42	0.001012	12.36	206.90	2442.0	206.91	2383.4	2590.3	0.6963	8.085
0.13	51.04	0.001013	11.46	213.65	2444.1	213.66	2379.5	2593.1	0.7172	8.057
0.14	52.55	0.001013	10.69	220.0	2446.1	220.0	2375.8	2595.8	0.7366	8.031
0.15	53.97	0.001014	10.02	225.9	2448.0	225.9	2372.4	2598.3	0.7548	8.007
0.16	55.31	0.001015	9.431	231.5	2449.8	231.6	2369.1	2600.7	0.7720	7.985
0.17	56.59	0.001015	8.909	236.9	2451.4	236.9	2366.0	2602.9	0.7882	7.964
0.18	57.80	0.001016	8.443	241.9	2453.0	241.9	2363.1	2605.0	0.8035	7.944
0.19	58.95	0.001017	8.025	246.8	2454.5	246.8	2360.2	2607.0	0.8181	7.925
0.20	60.06	0.001017	7.648	251.4	2456.0	251.4	2357.5	2608.9	0.8320	7.907
0.22	62.13	0.001018	6.994	260.1	2458.7	260.1	2352.5	2612.6	0.8579	7.874
0.24	64.05	0.001019	6.446	268.1	2461.2	268.1	2347.8	2615.9	0.8818	7.844
0.26	65.84	0.001020	5.979	275.6	2463.5	275.6	2343.4	2619.0	0.9040	7.817
0.28	67.52	0.001021	5.578	282.6	2465.7	282.6	2339.2	2621.8	0.9246	7.791
0.30	69.10	0.001022	5.229	289.2	2467.7	289.2	2335.3	2624.6	0.9439	7.767
0.35	72.68	0.001024	4.525	304.2	2472.3	304.3	2326.4	2630.7	0.9876	7.715
0.40	75.86	0.001026	3.993	317.5	2476.3	317.6	2318.5	2636.1	1.026	7.669
0.45	78.71	0.001028	3.576	329.5	2479.9	329.6	2311.3	2640.9	1.060	7.629
0.50	81.32	0.001030	3.240	340.4	2483.2	340.5	2304.7	2645.2	1.091	7.593
0.55	83.71	0.001032	2.964	350.5	2486.2	350.5	2298.7	2649.2	1.119	7.561
0.60	85.93	0.001033	2.732	359.8	2488.9	359.8	2293.0	2652.9	1.145	7.531
0.65	87.99	0.001035	2.535	368.5	2491.5	368.5	2287.7	2656.2	1.169	7.504
0.70	89.93	0.001036	2.365	376.6	2493.9	376.7	2282.7	2659.4	1.192	7.479
0.75	91.76	0.001037	2.217	384.3	2496.1	384.4	2278.0	2662.4	1.213	7.456
0.80	93.49	0.001038	2.087	391.6	2498.2	391.6	2273.5	2665.2	1.233	7.434
0.85	95.13	0.001040	1.972	398.5	2500.2	398.5	2269.3	2667.8	1.252	7.413
0.90	96.69	0.001041	1.869	405.0	2502.1	405.1	2265.2	2670.3	1.269	7.394
0.95	98.18	0.001042	1.777	411.3	2503.8	411.4	2261.3	2672.7	1.286	7.376
1.00	99.61	0.001043	1.694	417.3	2505.5	417.4	2257.5	2674.9	1.303	7.359
1.01325	99.97	0.001043	1.673	418.9	2506.0	419.0	2256.5	2675.5	1.307	7.354
1.1	102.3	0.001045	1.550	428.7	2508.7	428.8	2250.4	2679.2	1.333	7.327
1.2	104.8	0.001047	1.428	439.2	2511.6	439.3	2243.8	2683.1	1.361	7.298
1.3	107.1	0.001049	1.325	449.0	2514.3	449.1	2237.5	2686.6	1.387	7.271
1.4	109.3	0.001051	1.237	458.2	2516.9	458.4	2231.6	2690.0	1.411	7.246
1.5	111.4	0.001053	1.159	466.9	2519.2	467.1	2226.0	2693.1	1.434	7.223
1.6	113.3	0.001054	1.091	475.2	2521.4	475.3	2220.7	2696.0	1.455	7.201
1.7	115.1	0.001056	1.031	483.0	2523.5	483.2	2215.6	2698.8	1.475	7.181
1.8	116.9	0.001058	0.9775	490.5	2525.5	490.7	2210.7	2701.4	1.494	7.162
1.9	118.6	0.001059	0.9293	497.6	2527.3	497.8	2206.1	2703.9	1.513	7.144
2.0	120.2	0.001061	0.8857	504.5	2529.1	504.7	2201.6	2706.2	1.530	7.127

Table E.22: Saturated water and steam — Pressure (2 bar to 45 bar)

P bar (abs)	T řC	v_f $m^3\,kg^{-1}$	v_g $m^3\,kg^{-1}$	u_f $kJ\,kg^{-1}$	u_g $kJ\,kg^{-1}$	h_f $kJ\,kg^{-1}$	h_{fg} $kJ\,kg^{-1}$	h_g $kJ\,kg^{-1}$	s_f $kJ\,kg^{-1}\,K^{-1}$	s_g $kJ\,kg^{-1}\,K^{-1}$
2.0	120.2	0.001061	0.8857	504.5	2529.1	504.7	2201.6	2706.2	1.530	7.127
2.2	123.3	0.001063	0.8101	517.4	2532.4	517.6	2193.0	2710.6	1.563	7.095
2.4	126.1	0.001066	0.7467	529.4	2535.4	529.6	2185.0	2714.6	1.593	7.066
2.6	128.7	0.001068	0.6928	540.6	2538.2	540.9	2177.4	2718.3	1.621	7.039
2.8	131.2	0.001071	0.6463	551.2	2540.8	551.5	2170.3	2721.7	1.647	7.015
3.0	133.5	0.001073	0.6058	561.1	2543.2	561.5	2163.4	2724.9	1.672	6.992
3.2	135.7	0.001075	0.5702	570.6	2545.4	570.9	2156.9	2727.9	1.695	6.970
3.4	137.8	0.001078	0.5387	579.6	2547.5	580.0	2150.7	2730.6	1.717	6.950
3.6	139.9	0.001080	0.5105	588.2	2549.5	588.6	2144.7	2733.3	1.738	6.931
3.8	141.8	0.001082	0.4852	596.4	2551.3	596.8	2138.9	2735.7	1.758	6.913
4.0	143.6	0.001084	0.4624	604.3	2553.1	604.7	2133.3	2738.1	1.777	6.895
4.2	145.4	0.001085	0.4417	611.9	2554.8	612.3	2127.9	2740.3	1.795	6.879
4.4	147.1	0.001087	0.4227	619.2	2556.4	619.7	2122.7	2742.4	1.812	6.863
4.6	148.7	0.001089	0.4054	626.2	2557.9	626.7	2117.6	2744.4	1.829	6.849
4.8	150.3	0.001091	0.3895	633.0	2559.3	633.6	2112.7	2746.3	1.845	6.834
5.0	151.8	0.001093	0.3748	639.6	2560.7	640.2	2107.9	2748.1	1.861	6.821
5.5	155.5	0.001097	0.3426	655.3	2563.9	655.9	2096.5	2752.3	1.897	6.789
6.0	158.8	0.001101	0.3156	669.8	2566.8	670.5	2085.6	2756.1	1.931	6.759
6.5	162.0	0.001104	0.2926	683.5	2569.4	684.2	2075.4	2759.6	1.963	6.732
7.0	165.0	0.001108	0.2728	696.4	2571.8	697.1	2065.6	2762.7	1.992	6.707
7.5	167.8	0.001111	0.2555	708.6	2574.0	709.4	2056.3	2765.6	2.020	6.684
8.0	170.4	0.001115	0.2403	720.1	2576.0	721.0	2047.3	2768.3	2.046	6.662
8.5	172.9	0.001118	0.2269	731.2	2577.9	732.1	2038.6	2770.8	2.071	6.641
9.0	175.4	0.001121	0.2149	741.7	2579.7	742.7	2030.3	2773.0	2.094	6.621
9.5	177.7	0.001124	0.2041	751.8	2581.3	752.9	2022.3	2775.2	2.117	6.603
10	179.9	0.001127	0.1943	761.6	2582.8	762.7	2014.4	2777.1	2.138	6.585
11	184.1	0.001133	0.1774	780.0	2585.5	781.2	1999.5	2780.7	2.179	6.552
12	188.0	0.001139	0.1632	797.1	2587.9	798.5	1985.3	2783.8	2.216	6.522
13	191.6	0.001144	0.1512	813.3	2590.0	814.8	1971.7	2786.5	2.251	6.494
14	195.0	0.001149	0.1408	828.5	2591.8	830.1	1958.8	2788.9	2.284	6.468
15	198.3	0.001154	0.1317	843.0	2593.5	844.7	1946.3	2791.0	2.315	6.443
16	201.4	0.001159	0.1237	856.8	2594.9	858.6	1934.3	2792.9	2.344	6.420
17	204.3	0.001163	0.1167	869.9	2596.2	871.9	1922.6	2794.5	2.371	6.398
18	207.1	0.001168	0.1104	882.5	2597.3	884.6	1911.4	2796.0	2.398	6.378
19	209.8	0.001172	0.1047	894.6	2598.3	896.8	1900.4	2797.3	2.423	6.358
20	212.4	0.001177	0.09958	906.3	2599.2	908.6	1889.8	2798.4	2.447	6.339
21	214.9	0.001181	0.09493	917.5	2600.0	920.0	1879.4	2799.4	2.470	6.321
22	217.3	0.001185	0.09070	928.4	2600.7	931.0	1869.2	2800.2	2.492	6.304
23	219.6	0.001189	0.08681	938.9	2601.3	941.6	1859.3	2800.9	2.514	6.287
24	221.8	0.001193	0.08324	949.1	2601.8	952.0	1849.6	2801.5	2.534	6.271
25	224.0	0.001197	0.07995	959.0	2602.2	962.0	1840.1	2802.0	2.554	6.256
26	226.1	0.001201	0.07690	968.6	2602.5	971.7	1830.7	2802.5	2.574	6.241
27	228.1	0.001205	0.07407	978.0	2602.8	981.2	1821.5	2802.8	2.593	6.227
28	230.1	0.001209	0.07143	987.1	2603.0	990.5	1812.5	2803.0	2.611	6.213
29	232.0	0.001213	0.06897	996.0	2603.2	999.5	1803.6	2803.2	2.628	6.199
30	233.9	0.001217	0.06666	1004.7	2603.3	1008.4	1794.9	2803.3	2.646	6.186
31	235.7	0.001220	0.06450	1013.2	2603.3	1017.0	1786.3	2803.3	2.662	6.173
32	237.5	0.001224	0.06247	1021.5	2603.3	1025.5	1777.8	2803.2	2.679	6.160
33	239.2	0.001228	0.06056	1029.7	2603.3	1033.7	1769.4	2803.1	2.695	6.148
34	240.9	0.001231	0.05876	1037.6	2603.2	1041.8	1761.1	2803.0	2.710	6.136
35	242.6	0.001235	0.05706	1045.5	2603.0	1049.8	1753.0	2802.7	2.725	6.125
36	244.2	0.001239	0.05545	1053.1	2602.9	1057.6	1744.9	2802.5	2.740	6.113
37	245.8	0.001242	0.05392	1060.6	2602.6	1065.2	1736.9	2802.1	2.755	6.102
38	247.3	0.001246	0.05247	1068.0	2602.4	1072.8	1729.0	2801.8	2.769	6.091
39	248.9	0.001249	0.05109	1075.3	2602.1	1080.2	1721.2	2801.4	2.783	6.080
40	250.4	0.001253	0.04978	1082.4	2601.8	1087.4	1713.5	2800.9	2.797	6.070
41	251.8	0.001256	0.04853	1089.4	2601.4	1094.6	1705.8	2800.4	2.810	6.059
42	253.3	0.001259	0.04733	1096.3	2601.1	1101.6	1698.2	2799.9	2.823	6.049
43	254.7	0.001263	0.04619	1103.1	2600.6	1108.6	1690.7	2799.3	2.836	6.039
44	256.1	0.001266	0.04510	1109.8	2600.2	1115.4	1683.2	2798.7	2.849	6.029
45	257.4	0.001270	0.04406	1116.4	2599.7	1122.1	1675.9	2798.0	2.861	6.020

Table E.23: Saturated water and steam — Pressure (45 bar to critical point)

P bar (abs)	T řC	v_f $m^3\,kg^{-1}$	v_g $m^3\,kg^{-1}$	u_f $kJ\,kg^{-1}$	u_g $kJ\,kg^{-1}$	h_f $kJ\,kg^{-1}$	h_{fg} $kJ\,kg^{-1}$	h_g $kJ\,kg^{-1}$	s_f $kJ\,kg^{-1}\,K^{-1}$	s_g $kJ\,kg^{-1}\,K^{-1}$
45	257.4	0.001270	0.04406	1116.4	2599.7	1122.1	1675.9	2798.0	2.861	6.020
46	258.8	0.001273	0.04306	1122.9	2599.2	1128.8	1668.5	2797.3	2.874	6.010
47	260.1	0.001276	0.04210	1129.3	2598.7	1135.3	1661.2	2796.6	2.886	6.001
48	261.4	0.001280	0.04118	1135.7	2598.2	1141.8	1654.0	2795.8	2.898	5.992
49	262.7	0.001283	0.04030	1141.9	2597.6	1148.2	1646.8	2795.0	2.909	5.983
50	263.9	0.001286	0.03945	1148.1	2597.0	1154.5	1639.7	2794.2	2.921	5.974
51	265.2	0.001290	0.03863	1154.2	2596.4	1160.7	1632.7	2793.4	2.932	5.965
52	266.4	0.001293	0.03784	1160.2	2595.7	1166.9	1625.6	2792.5	2.943	5.956
53	267.6	0.001296	0.03708	1166.1	2595.1	1173.0	1618.6	2791.6	2.954	5.948
54	268.8	0.001300	0.03635	1172.0	2594.4	1179.0	1611.7	2790.7	2.965	5.939
55	270.0	0.001303	0.03564	1177.8	2593.7	1184.9	1604.8	2789.7	2.976	5.931
56	271.1	0.001306	0.03496	1183.5	2593.0	1190.8	1597.9	2788.7	2.986	5.922
57	272.3	0.001309	0.03430	1189.2	2592.2	1196.6	1591.1	2787.7	2.997	5.914
58	273.4	0.001313	0.03366	1194.8	2591.5	1202.4	1584.3	2786.7	3.007	5.906
59	274.5	0.001316	0.03305	1200.3	2590.7	1208.1	1577.6	2785.6	3.017	5.898
60	275.6	0.001319	0.03245	1205.8	2589.9	1213.7	1570.8	2784.6	3.027	5.890
62	277.7	0.001326	0.03131	1216.6	2588.2	1224.9	1557.5	2782.3	3.047	5.874
64	279.8	0.001332	0.03024	1227.3	2586.5	1235.8	1544.2	2780.0	3.067	5.859
66	281.9	0.001339	0.02923	1237.7	2584.7	1246.5	1531.1	2777.6	3.085	5.844
68	283.9	0.001345	0.02828	1247.9	2582.8	1257.1	1518.1	2775.1	3.104	5.829
70	285.8	0.001352	0.02738	1258.0	2580.9	1267.4	1505.1	2772.6	3.122	5.815
72	287.7	0.001358	0.02653	1267.9	2578.9	1277.7	1492.3	2769.9	3.140	5.800
74	289.6	0.001365	0.02572	1277.6	2576.9	1287.7	1479.5	2767.2	3.157	5.786
76	291.4	0.001371	0.02495	1287.2	2574.8	1297.6	1466.8	2764.4	3.174	5.772
78	293.2	0.001378	0.02422	1296.7	2572.6	1307.4	1454.1	2761.5	3.191	5.758
80	295.0	0.001385	0.02353	1306.0	2570.4	1317.1	1441.5	2758.6	3.208	5.745
82	296.7	0.001391	0.02286	1315.2	2568.1	1326.6	1429.0	2755.6	3.224	5.731
84	298.4	0.001398	0.02223	1324.3	2565.8	1336.0	1416.5	2752.5	3.240	5.718
86	300.1	0.001405	0.02163	1333.3	2563.4	1345.3	1404.0	2749.4	3.256	5.705
88	301.7	0.001411	0.02105	1342.1	2560.9	1354.5	1391.6	2746.2	3.271	5.692
90	303.3	0.001418	0.02049	1350.9	2558.4	1363.7	1379.2	2742.9	3.287	5.679
92	304.9	0.001425	0.01996	1359.6	2555.9	1372.7	1366.9	2739.5	3.302	5.666
94	306.5	0.001432	0.01945	1368.1	2553.3	1381.6	1354.5	2736.1	3.317	5.653
96	308.0	0.001439	0.01896	1376.6	2550.6	1390.4	1342.2	2732.6	3.331	5.641
98	309.5	0.001446	0.01849	1385.0	2547.9	1399.2	1329.9	2729.1	3.346	5.628
100	311.0	0.001453	0.01803	1393.3	2545.1	1407.9	1317.6	2725.5	3.360	5.616
105	314.6	0.001470	0.01697	1413.8	2538.0	1429.3	1286.9	2716.1	3.396	5.585
110	318.1	0.001489	0.01599	1433.9	2530.5	1450.3	1256.1	2706.4	3.430	5.555
115	321.4	0.001507	0.01510	1453.6	2522.6	1470.9	1225.3	2696.2	3.464	5.524
120	324.7	0.001526	0.01427	1473.0	2514.4	1491.3	1194.3	2685.6	3.496	5.494
125	327.8	0.001546	0.01350	1492.1	2505.7	1511.5	1163.0	2674.5	3.529	5.464
130	330.9	0.001566	0.01279	1511.0	2496.7	1531.4	1131.5	2662.9	3.561	5.434
135	333.8	0.001588	0.01212	1529.8	2487.2	1551.2	1099.6	2650.8	3.592	5.404
140	336.7	0.001610	0.01149	1548.3	2477.2	1570.9	1067.2	2638.1	3.623	5.373
145	339.5	0.001633	0.01090	1566.8	2466.8	1590.5	1034.3	2624.8	3.654	5.342
150	342.2	0.001657	0.01034	1585.3	2455.8	1610.2	1000.7	2610.9	3.684	5.311
155	344.8	0.001682	0.009811	1603.8	2444.1	1629.9	966.4	2596.2	3.715	5.279
160	347.4	0.001710	0.009308	1622.3	2431.9	1649.7	931.1	2580.8	3.746	5.246
165	349.9	0.001738	0.008828	1641.0	2418.9	1669.7	894.9	2564.6	3.776	5.213
170	352.3	0.001769	0.008369	1660.0	2405.1	1690.0	857.4	2547.4	3.808	5.179
175	354.7	0.001803	0.007927	1679.2	2390.4	1710.8	818.4	2529.1	3.839	5.143
180	357.0	0.001839	0.007499	1698.9	2374.6	1732.0	777.5	2509.5	3.872	5.106
185	359.3	0.001880	0.007082	1719.2	2357.4	1754.0	734.4	2488.4	3.905	5.066
190	361.5	0.001925	0.006673	1740.3	2338.6	1776.9	688.5	2465.4	3.940	5.025
195	363.6	0.001977	0.006267	1762.5	2317.8	1801.1	638.9	2440.0	3.976	4.980
200	365.7	0.002039	0.005858	1786.3	2294.2	1827.1	584.3	2411.4	4.015	4.930
205	367.8	0.002114	0.005438	1812.6	2266.7	1855.9	522.3	2378.2	4.059	4.874
210	369.8	0.002212	0.004988	1842.9	2232.8	1889.4	448.1	2337.5	4.109	4.806
215	371.8	0.002360	0.004463	1882.1	2186.2	1932.8	349.4	2282.2	4.175	4.717
220	373.7	0.002750	0.003577	1961.4	2085.5	2021.9	142.3	2164.2	4.311	4.531
220.64	**373.95**	0.003106	0.003106	2019.0	2019.0	2087.5	0	2087.5	4.412	4.412

Table E.24: Subcooled water and Superheated Steam (triple point to 0.1 bar)

T(řC)	0.006117 bar (T_{sat} = 0.01 řC)				0.01 bar (T_{sat} = 7.0 řC)				0.05 bar (T_{sat} = 32.9 řC)				0.1 bar (T_{sat} = 45.8 řC)				T(řC)
	v	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	
0.01	206.0	2374.9	2500.9	9.155	0.001000	0.000007	0.001007	0.0000	0.001000	0.000081	0.005082	0.0000	0.001000	0.000174	0.01018	0.000001	0.01
20	221.1	2403.1	2538.4	9.288	135.2	2403.0	2538.2	9.060	0.001002	83.92	83.92	0.2965	0.001002	83.92	83.93	0.2965	20
40	236.2	2431.3	2575.8	9.411	144.5	2431.2	2575.7	9.184	28.85	2430.1	2574.4	8.438	0.001008	167.5	167.5	0.5724	40
60	251.3	2459.5	2613.3	9.527	153.7	2459.5	2613.2	9.300	30.71	2458.7	2612.3	8.555	15.34	2457.8	2611.2	8.233	60
80	266.4	2487.9	2650.8	9.637	163.0	2487.8	2650.8	9.410	32.57	2487.3	2650.1	8.666	16.27	2486.7	2649.3	8.344	80
100	281.5	2516.4	2688.6	9.741	172.2	2516.3	2688.5	9.514	34.42	2516.0	2688.0	8.770	17.20	2515.5	2687.4	8.449	100
120	296.6	2545.0	2726.5	9.840	181.4	2545.0	2726.4	9.613	36.27	2544.7	2726.1	8.869	18.12	2544.3	2725.6	8.548	120
140	311.7	2573.9	2764.6	9.934	190.7	2573.9	2764.5	9.707	38.12	2573.6	2764.2	8.964	19.05	2573.3	2763.8	8.643	140
160	326.8	2602.9	2802.8	10.02	199.9	2602.9	2802.8	9.798	39.97	2602.7	2802.6	9.055	19.98	2602.5	2802.2	8.734	160
180	341.9	2632.2	2841.3	10.11	209.1	2632.2	2841.3	9.885	41.82	2632.0	2841.1	9.141	20.90	2631.8	2840.8	8.821	180
200	357.0	2661.6	2880.0	10.20	218.4	2661.6	2880.0	9.968	43.66	2661.5	2879.8	9.225	21.83	2661.3	2879.6	8.905	200
220	372.1	2691.3	2918.9	10.28	227.6	2691.3	2918.9	10.05	45.51	2691.2	2918.8	9.306	22.75	2691.1	2918.6	8.986	220
240	387.2	2721.3	2958.1	10.35	236.8	2721.2	2958.1	10.13	47.36	2721.1	2957.9	9.384	23.67	2721.0	2957.8	9.063	240
260	402.3	2751.4	2997.5	10.43	246.1	2751.4	2997.5	10.20	49.20	2751.3	2997.3	9.459	24.60	2751.2	2997.2	9.139	260
280	417.4	2781.8	3037.1	10.50	255.3	2781.8	3037.1	10.27	51.05	2781.7	3037.0	9.532	25.52	2781.6	3036.8	9.212	280
300	432.5	2812.4	3077.0	10.57	264.5	2812.4	3077.0	10.35	52.90	2812.4	3076.9	9.603	26.45	2812.3	3076.7	9.283	300
320	447.6	2843.3	3117.1	10.64	273.7	2843.3	3117.1	10.41	54.75	2843.3	3117.0	9.672	27.37	2843.2	3116.9	9.351	320
340	462.6	2874.5	3157.4	10.71	283.0	2874.5	3157.4	10.48	56.59	2874.4	3157.4	9.738	28.29	2874.3	3157.3	9.418	340
360	477.7	2905.9	3198.1	10.77	292.2	2905.8	3198.1	10.55	58.44	2905.8	3198.0	9.804	29.22	2905.7	3197.9	9.484	360
380	492.8	2937.5	3238.9	10.84	301.4	2937.5	3238.9	10.61	60.28	2937.4	3238.9	9.867	30.14	2937.4	3238.8	9.547	380
400	507.9	2969.4	3280.1	10.90	310.7	2969.4	3280.1	10.67	62.13	2969.4	3280.0	9.929	31.06	2969.3	3279.9	9.609	400
420	523.0	3001.6	3321.5	10.96	319.9	3001.6	3321.5	10.73	63.98	3001.5	3321.4	9.990	31.99	3001.5	3321.3	9.670	420
440	538.1	3034.0	3363.2	11.02	329.1	3034.0	3363.1	10.79	65.82	3034.0	3363.1	10.05	32.91	3033.9	3363.0	9.729	440
460	553.2	3066.7	3405.1	11.08	338.4	3066.7	3405.1	10.85	67.67	3066.7	3405.0	10.11	33.83	3066.6	3405.0	9.787	460
480	568.3	3099.7	3447.3	11.13	347.6	3099.7	3447.3	10.91	69.52	3099.7	3447.2	10.16	34.76	3099.6	3447.2	9.844	480
500	583.4	3132.9	3489.8	11.19	356.8	3132.9	3489.8	10.96	71.36	3132.9	3489.7	10.22	35.68	3132.9	3489.7	9.900	500
520	598.5	3166.5	3532.5	11.24	366.1	3166.5	3532.5	11.02	73.21	3166.4	3532.5	10.27	36.60	3166.4	3532.4	9.954	520
540	613.6	3200.3	3575.6	11.30	375.3	3200.3	3575.6	11.07	75.06	3200.2	3575.5	10.33	37.53	3200.2	3575.5	10.01	540
560	628.7	3234.4	3618.9	11.35	384.5	3234.4	3618.9	11.12	76.90	3234.3	3618.8	10.38	38.45	3234.3	3618.8	10.06	560
580	643.7	3268.7	3662.5	11.40	393.7	3268.7	3662.5	11.18	78.75	3268.7	3662.4	10.43	39.37	3268.7	3662.4	10.11	580
600	658.8	3303.4	3706.3	11.45	403.0	3303.4	3706.3	11.23	80.59	3303.3	3706.3	10.48	40.30	3303.3	3706.3	10.16	600
640	689.0	3373.5	3794.9	11.55	421.4	3373.5	3794.9	11.33	84.29	3373.5	3794.9	10.58	42.14	3373.4	3794.9	10.26	640
680	719.2	3444.8	3884.7	11.65	439.9	3444.8	3884.7	11.42	87.98	3444.7	3884.6	10.68	43.99	3444.7	3884.6	10.36	680
720	749.4	3517.2	3975.5	11.74	458.4	3517.2	3975.5	11.51	91.67	3517.2	3975.5	10.77	45.84	3517.1	3975.5	10.45	720
760	779.6	3590.7	4067.5	11.83	476.8	3590.7	4067.5	11.61	95.36	3590.7	4067.5	10.86	47.68	3590.7	4067.5	10.54	760
800	809.7	3665.4	4160.7	11.92	495.3	3665.4	4160.7	11.69	99.06	3665.4	4160.6	10.95	49.53	3665.3	4160.6	10.63	800

Table E.25: Subcooled water and Superheated Steam (0.1 bar to 1 atmosphere)

	0.2 bar (T_{sat} = 60.1 řC)				0.5 bar (T_{sat} = 81.3 řC)				1 bar (T_{sat} = 99.6 řC)				1.01325 bar (T_{sat} = 100.0 řC)				
T(řC)	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	T(řC)
0.01	0.001000	0.000360	0.02036	0.00000	0.001000	0.000915	0.05092	0.00000	0.001000	0.001841	0.1019	0.00001	0.001000	0.001865	0.1032	0.00001	0.01
20	0.001002	83.92	83.94	0.296	0.001002	83.91	83.96	0.2965	0.001002	83.91	84.01	0.2965	0.001002	83.91	84.01	0.2965	20
40	0.001008	167.5	167.6	0.572	0.001008	167.5	167.6	0.5724	0.001008	167.5	167.6	0.5724	0.001008	167.5	167.6	0.5724	40
60	0.001017	251.1	251.2	0.831	0.001017	251.1	251.2	0.8312	0.001017	251.1	251.2	0.8312	0.001017	251.1	251.2	0.8312	60
80	8.118	2485.3	2647.7	8.020	0.001029	334.9	335.0	1.075	0.001029	334.9	335.0	1.075	0.001029	334.9	335.0	1.075	80
100	8.586	2514.5	2686.2	8.126	3.419	2511.5	2682.4	7.695	1.696	2506.2	2675.8	7.361	1.673	2506.0	2675.6	7.355	100
120	9.052	2543.6	2724.6	8.227	3.608	2541.3	2721.7	7.798	1.793	2537.3	2716.6	7.468	1.770	2537.2	2716.5	7.461	120
140	9.517	2572.7	2763.1	8.322	3.796	2570.9	2760.7	7.895	1.889	2567.8	2756.7	7.567	1.864	2567.7	2756.6	7.561	140
160	9.98	2602.0	2801.6	8.413	3.983	2600.5	2799.7	7.987	1.984	2598.0	2796.4	7.661	1.958	2597.9	2796.3	7.655	160
180	10.44	2631.4	2840.3	8.500	4.170	2630.2	2838.7	8.075	2.079	2628.1	2836.0	7.750	2.051	2628.1	2835.9	7.744	180
200	10.91	2661.0	2879.1	8.584	4.356	2660.0	2877.8	8.159	2.172	2658.2	2875.5	7.836	2.144	2658.2	2875.4	7.829	200
220	11.37	2690.8	2918.2	8.665	4.542	2689.9	2917.0	8.240	2.266	2688.4	2915.0	7.917	2.236	2688.4	2915.0	7.911	220
240	11.83	2720.8	2957.4	8.743	4.728	2720.0	2956.4	8.319	2.360	2718.7	2954.7	7.996	2.329	2718.7	2954.6	7.990	240
260	12.30	2751.0	2996.9	8.818	4.913	2750.3	2996.0	8.394	2.453	2749.2	2994.4	8.072	2.421	2749.1	2994.4	8.066	260
280	12.76	2781.4	3036.6	8.892	5.099	2780.8	3035.8	8.468	2.546	2779.8	3034.4	8.146	2.512	2779.8	3034.4	8.140	280
300	13.22	2812.1	3076.5	8.962	5.284	2811.6	3075.8	8.539	2.639	2810.7	3074.5	8.217	2.604	2810.6	3074.5	8.211	300
320	13.68	2843.0	3116.7	9.031	5.469	2842.5	3116.0	8.608	2.732	2841.7	3114.9	8.286	2.696	2841.7	3114.9	8.280	320
340	14.14	2874.2	3157.1	9.098	5.654	2873.7	3156.5	8.675	2.825	2873.0	3155.5	8.354	2.788	2873.0	3155.4	8.347	340
360	14.61	2905.6	3197.7	9.164	5.840	2905.2	3197.2	8.740	2.917	2904.5	3196.2	8.419	2.879	2904.5	3196.2	8.413	360
380	15.07	2937.3	3238.6	9.227	6.025	2936.9	3238.1	8.804	3.010	2936.3	3237.3	8.483	2.971	2936.2	3237.2	8.477	380
400	15.53	2969.2	3279.8	9.289	6.209	2968.8	3279.3	8.866	3.103	2968.3	3278.5	8.545	3.062	2968.3	3278.5	8.539	400
420	15.99	3001.4	3321.2	9.350	6.394	3001.1	3320.8	8.927	3.195	3000.5	3320.1	8.606	3.154	3000.5	3320.0	8.600	420
440	16.45	3033.8	3362.9	9.409	6.579	3033.5	3362.5	8.986	3.288	3033.0	3361.8	8.665	3.245	3033.0	3361.8	8.659	440
460	16.92	3066.5	3404.8	9.467	6.764	3066.3	3404.5	9.044	3.381	3065.8	3403.9	8.723	3.336	3065.8	3403.8	8.717	460
480	17.38	3099.5	3447.1	9.524	6.949	3099.3	3446.7	9.101	3.473	3098.8	3446.2	8.780	3.428	3098.8	3446.1	8.774	480
500	17.84	3132.8	3489.6	9.580	7.134	3132.6	3489.2	9.156	3.566	3132.2	3488.7	8.836	3.519	3132.1	3488.7	8.830	500
520	18.30	3166.3	3532.3	9.634	7.319	3166.1	3532.0	9.211	3.658	3165.7	3531.5	8.891	3.610	3165.7	3531.5	8.885	520
540	18.76	3200.1	3575.4	9.688	7.503	3199.9	3575.1	9.265	3.751	3199.6	3574.6	8.944	3.701	3199.6	3574.6	8.938	540
560	19.22	3234.2	3618.7	9.741	7.688	3234.0	3618.4	9.317	3.843	3233.7	3618.0	8.997	3.793	3233.7	3618.0	8.991	560
580	19.69	3268.6	3662.3	9.792	7.873	3268.4	3662.1	9.369	3.936	3268.1	3661.6	9.049	3.884	3268.1	3661.6	9.043	580
600	20.15	3303.2	3706.2	9.843	8.058	3303.1	3706.0	9.420	4.028	3302.8	3705.6	9.100	3.975	3302.8	3705.6	9.094	600
640	21.07	3373.4	3794.8	9.942	8.427	3373.2	3794.6	9.519	4.213	3373.0	3794.3	9.199	4.158	3373.0	3794.2	9.193	640
680	21.99	3444.7	3884.6	10.04	8.797	3444.5	3884.4	9.615	4.398	3444.3	3884.1	9.295	4.340	3444.3	3884.1	9.289	680
720	22.92	3517.1	3975.4	10.13	9.166	3517.0	3975.3	9.709	4.582	3516.7	3975.0	9.389	4.522	3516.7	3975.0	9.383	720
760	23.84	3590.6	4067.4	10.22	9.535	3590.5	4067.3	9.800	4.767	3590.3	4067.0	9.480	4.705	3590.3	4067.0	9.474	760
800	24.76	3665.3	4160.6	10.31	9.905	3665.2	4160.4	9.888	4.952	3665.0	4160.2	9.568	4.887	3665.0	4160.2	9.562	800

Table E.26: Subcooled water and Superheated Steam (2 bar to 8 bar)

	2 bar (T_{sat} = 120.2 řC)				4 bar (T_{sat} = 143.6 řC)				6 bar (T_{sat} = 158.8 řC)				8 bar (T_{sat} = 170.4 řC)				
T(řC)	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	T(řC)
0.01	0.001000	0.003688	0.2037	0.00001	0.001000	0.007365	0.4074	0.00003	0.001000	0.0110	0.6110	0.00004	0.001000	0.0147	0.8145	0.00005	0.01
20	0.001002	83.91	84.11	0.2965	0.001002	83.89	84.29	0.2964	0.001002	83.88	84.48	0.2964	0.001001	83.87	84.67	0.2963	20
40	0.001008	167.5	167.7	0.5724	0.001008	167.5	167.9	0.5723	0.001008	167.5	168.1	0.5722	0.001008	167.4	168.2	0.5721	40
60	0.001017	251.1	251.3	0.8311	0.001017	251.1	251.5	0.8310	0.001017	251.0	251.6	0.8309	0.001017	251.0	251.8	0.8308	60
80	0.001029	334.9	335.1	1.075	0.001029	334.8	335.2	1.075	0.001029	334.8	335.4	1.075	0.001029	334.7	335.5	1.075	80
100	0.001043	419.0	419.2	1.307	0.001043	418.9	419.3	1.307	0.001043	418.8	419.5	1.307	0.001043	418.8	419.6	1.306	100
120	0.001060	503.6	503.8	1.528	0.001060	503.5	503.9	1.528	0.001060	503.4	504.1	1.527	0.001060	503.4	504.2	1.527	120
140	0.9353	2561.3	2748.3	7.231	0.001080	588.8	589.2	1.739	0.001080	588.7	589.4	1.739	0.001079	588.6	589.5	1.739	140
160	0.9843	2592.8	2789.7	7.329	0.4839	2581.6	2775.2	6.983	0.3167	2569.0	2759.0	6.766	0.001102	674.8	675.7	1.943	160
180	1.033	2623.9	2830.4	7.421	0.5094	2614.9	2818.6	7.081	0.3347	2605.2	2806.0	6.872	0.2472	2594.7	2792.4	6.715	180
200	1.081	2654.7	2870.8	7.508	0.5343	2647.3	2861.0	7.172	0.3521	2639.4	2850.7	6.968	0.2609	2631.1	2839.8	6.818	200
220	1.128	2685.4	2911.0	7.591	0.5589	2679.1	2902.7	7.259	0.3690	2672.6	2894.0	7.058	0.2740	2665.7	2885.0	6.911	220
240	1.175	2716.1	2951.2	7.671	0.5831	2710.7	2944.0	7.341	0.3857	2705.2	2936.6	7.143	0.2869	2699.4	2928.9	6.999	240
260	1.222	2746.9	2991.4	7.748	0.6072	2742.2	2985.1	7.419	0.4021	2737.4	2978.6	7.223	0.2995	2732.4	2972.0	7.081	260
280	1.269	2777.8	3031.7	7.822	0.6311	2773.7	3026.1	7.495	0.4183	2769.4	3020.4	7.300	0.3119	2765.1	3014.6	7.159	280
300	1.316	2808.8	3072.1	7.894	0.6549	2805.2	3067.1	7.568	0.4344	2801.4	3062.1	7.374	0.3242	2797.6	3056.9	7.235	300
320	1.363	2840.1	3112.7	7.964	0.6786	2836.8	3108.2	7.638	0.4504	2833.4	3103.7	7.445	0.3363	2830.0	3099.1	7.307	320
340	1.410	2871.5	3153.4	8.031	0.7022	2868.5	3149.4	7.706	0.4663	2865.5	3145.3	7.514	0.3484	2862.4	3141.1	7.377	340
360	1.456	2903.1	3194.4	8.097	0.7257	2900.4	3190.7	7.773	0.4822	2897.6	3187.0	7.581	0.3604	2894.9	3183.2	7.444	360
380	1.503	2935.0	3235.6	8.161	0.7492	2932.5	3232.2	7.837	0.4980	2930.0	3228.8	7.646	0.3724	2927.4	3225.3	7.510	380
400	1.549	2967.1	3277.0	8.223	0.7726	2964.8	3273.9	7.900	0.5137	2962.5	3270.7	7.710	0.3843	2960.1	3267.6	7.573	400
420	1.596	2999.5	3318.6	8.284	0.7960	2997.3	3315.7	7.961	0.5294	2995.2	3312.8	7.771	0.3961	2993.0	3309.9	7.635	420
440	1.642	3032.0	3360.5	8.344	0.8194	3030.1	3357.8	8.021	0.5451	3028.1	3355.1	7.831	0.4080	3026.1	3352.5	7.696	440
460	1.689	3064.9	3402.6	8.402	0.8428	3063.0	3400.1	8.080	0.5608	3061.2	3397.7	7.890	0.4198	3059.3	3395.2	7.755	460
480	1.735	3098.0	3445.0	8.459	0.8661	3096.3	3442.7	8.137	0.5764	3094.5	3440.4	7.948	0.4316	3092.8	3438.1	7.813	480
500	1.781	3131.3	3487.6	8.515	0.8894	3129.7	3485.5	8.193	0.5920	3128.1	3483.3	8.004	0.4433	3126.5	3481.2	7.869	500
520	1.828	3165.0	3530.5	8.570	0.9126	3163.5	3528.5	8.248	0.6076	3162.0	3526.5	8.059	0.4551	3160.4	3524.5	7.924	520
540	1.874	3198.9	3573.7	8.624	0.9359	3197.5	3571.8	8.302	0.6232	3196.0	3569.9	8.113	0.4668	3194.6	3568.0	7.979	540
560	1.920	3233.0	3617.1	8.676	0.9591	3231.7	3615.4	8.355	0.6387	3230.4	3613.6	8.166	0.4785	3229.0	3611.8	8.032	560
580	1.967	3267.5	3660.8	8.728	0.9824	3266.2	3659.2	8.407	0.6542	3265.0	3657.5	8.218	0.4902	3263.7	3655.8	8.084	580
600	2.013	3302.2	3704.8	8.779	1.006	3301.0	3703.2	8.458	0.6698	3299.8	3701.7	8.269	0.5019	3298.6	3700.1	8.135	600
640	2.106	3372.4	3793.6	8.879	1.052	3371.4	3792.2	8.558	0.7008	3370.3	3790.8	8.369	0.5252	3369.3	3789.4	8.235	640
680	2.198	3443.8	3883.4	8.975	1.098	3442.9	3882.2	8.654	0.7318	3441.9	3881.0	8.466	0.5485	3441.0	3879.7	8.332	680
720	2.291	3516.3	3974.4	9.068	1.145	3515.4	3973.3	8.748	0.7627	3514.6	3972.2	8.560	0.5717	3513.7	3971.1	8.426	720
760	2.383	3589.9	4066.5	9.159	1.191	3589.1	4065.5	8.839	0.7937	3588.3	4064.5	8.651	0.5950	3587.6	4063.5	8.517	760
800	2.476	3664.7	4159.8	9.248	1.237	3663.9	4158.9	8.927	0.8246	3663.2	4157.9	8.739	0.6182	3662.5	4157.0	8.606	800

Table E.27: Subcooled water and Superheated Steam (10 bar to 40 bar)

T(řC)	10 bar (T_{sat} = 179.9 řC)				20 bar (T_{sat} = 212.4 řC)				30 bar (T_{sat} = 233.9 řC)				40 bar (T_{sat} = 250.4 řC)				T(řC)
	v	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	
0.01	0.001000	0.01827	1.018	0.00007	0.000999	0.03601	2.034	0.00013	0.000999	0.05321	3.049	0.00019	0.000998	0.06989	4.063	0.00024	0.01
20	0.001001	83.86	84.86	0.2963	0.001001	83.80	85.80	0.2961	0.001000	83.74	86.74	0.2959	0.001000	83.68	87.68	0.2957	20
40	0.001007	167.4	168.4	0.5720	0.001007	167.3	169.3	0.5717	0.001007	167.2	170.2	0.5713	0.001006	167.1	171.1	0.5709	40
60	0.001017	251.0	252.0	0.8307	0.001016	250.8	252.8	0.8302	0.001016	250.6	253.7	0.8296	0.001015	250.4	254.5	0.8291	60
80	0.001029	334.7	335.7	1.075	0.001028	334.4	336.5	1.074	0.001028	334.2	337.3	1.073	0.001027	334.0	338.1	1.073	80
100	0.001043	418.7	419.8	1.306	0.001042	418.4	420.5	1.306	0.001042	418.2	421.3	1.305	0.001041	417.9	422.0	1.304	100
120	0.001060	503.3	504.3	1.527	0.001059	502.9	505.1	1.526	0.001059	502.6	505.8	1.525	0.001058	502.2	506.5	1.524	120
140	0.001079	588.5	589.6	1.739	0.001079	588.1	590.3	1.738	0.001078	587.7	590.9	1.737	0.001077	587.3	591.6	1.736	140
160	0.001102	674.7	675.8	1.942	0.001101	674.2	676.4	1.941	0.001100	673.7	677.0	1.940	0.001100	673.2	677.6	1.939	160
180	0.1944	2583.0	2777.4	6.586	0.001127	761.4	763.7	2.138	0.001126	760.8	764.2	2.137	0.001125	760.2	764.7	2.135	180
200	0.2060	2622.3	2828.3	6.695	0.001156	850.3	852.6	2.330	0.001155	849.5	853.0	2.329	0.001154	848.8	853.4	2.327	200
220	0.2170	2658.6	2875.6	6.793	0.1022	2617.3	2821.7	6.387	0.001189	940.3	943.8	2.517	0.001188	939.3	944.1	2.515	220
240	0.2276	2693.4	2921.0	6.884	0.1085	2660.2	2877.2	6.497	0.06823	2619.9	2824.6	6.228	0.001228	1032.7	1037.6	2.700	240
260	0.2379	2727.4	2965.2	6.968	0.1144	2699.7	2928.5	6.595	0.07289	2667.8	2886.4	6.346	0.05178	2630.1	2837.2	6.138	260
280	0.2480	2760.7	3008.7	7.048	0.1200	2737.1	2977.2	6.685	0.07716	2710.7	2942.2	6.449	0.05549	2680.9	2902.9	6.259	280
300	0.2580	2793.7	3051.7	7.125	0.1255	2773.2	3024.3	6.769	0.08118	2750.8	2994.3	6.541	0.05887	2726.2	2961.7	6.364	300
320	0.2678	2826.5	3094.4	7.198	0.1308	2808.5	3070.2	6.847	0.08502	2789.1	3044.2	6.627	0.06202	2768.2	3016.3	6.458	320
340	0.2776	2859.3	3136.9	7.268	0.1360	2843.2	3115.3	6.922	0.08874	2826.2	3092.4	6.707	0.06502	2808.1	3068.1	6.544	340
360	0.2873	2892.1	3179.4	7.337	0.1411	2877.6	3159.9	6.994	0.09235	2862.4	3139.5	6.782	0.06790	2846.5	3118.1	6.624	360
380	0.2970	2924.9	3221.9	7.403	0.1462	2911.8	3204.2	7.063	0.09590	2898.1	3185.8	6.854	0.07070	2883.9	3166.7	6.699	380
400	0.3066	2957.8	3264.4	7.467	0.1512	2945.8	3248.2	7.129	0.09938	2933.4	3231.6	6.923	0.07343	2920.6	3214.4	6.771	400
420	0.3162	2990.8	3307.0	7.529	0.1562	2979.8	3292.2	7.193	0.1028	2968.5	3277.0	6.990	0.07611	2956.9	3261.4	6.840	420
440	0.3257	3024.1	3349.8	7.590	0.1611	3013.9	3336.1	7.256	0.1062	3003.5	3322.1	7.054	0.07874	2992.9	3307.9	6.906	440
460	0.3352	3057.5	3392.7	7.649	0.1660	3048.1	3380.0	7.317	0.1096	3038.5	3367.2	7.116	0.08134	3028.7	3354.0	6.970	460
480	0.3447	3091.1	3435.7	7.707	0.1708	3082.3	3424.0	7.376	0.1129	3073.4	3412.1	7.177	0.08390	3064.4	3400.0	7.032	480
500	0.3541	3124.9	3479.0	7.764	0.1757	3116.7	3468.1	7.433	0.1162	3108.5	3457.0	7.236	0.08644	3100.1	3445.8	7.092	500
520	0.3635	3158.9	3522.5	7.819	0.1805	3151.3	3512.3	7.490	0.1195	3143.6	3502.0	7.293	0.08896	3135.8	3491.6	7.150	520
540	0.3730	3193.2	3566.2	7.874	0.1853	3186.0	3556.6	7.545	0.1227	3178.8	3547.0	7.349	0.09146	3171.5	3537.3	7.207	540
560	0.3824	3227.7	3610.1	7.927	0.1901	3221.0	3601.2	7.599	0.1260	3214.2	3592.2	7.404	0.09394	3207.4	3583.1	7.263	560
580	0.3917	3262.4	3654.2	7.980	0.1949	3256.1	3645.8	7.652	0.1292	3249.7	3637.4	7.458	0.09640	3243.3	3628.9	7.317	580
600	0.4011	3297.4	3698.6	8.031	0.200	3291.5	3690.7	7.704	0.1324	3285.5	3682.8	7.510	0.09886	3279.4	3674.8	7.370	600
640	0.4198	3368.2	3788.0	8.131	0.209	3362.9	3781.1	7.805	0.1389	3357.5	3774.1	7.612	0.1037	3352.1	3767.0	7.474	640
680	0.4385	3440.0	3878.5	8.228	0.219	3435.2	3872.3	7.903	0.1452	3430.4	3866.1	7.711	0.1086	3425.5	3859.8	7.573	680
720	0.4572	3512.8	3970.0	8.322	0.228	3508.5	3964.4	7.998	0.1516	3504.1	3958.8	7.806	0.1134	3499.7	3953.2	7.669	720
760	0.4758	3586.8	4062.5	8.413	0.237	3582.8	4057.5	8.090	0.1579	3578.8	4052.5	7.899	0.1182	3574.8	4047.4	7.762	760
800	0.4944	3661.8	4156.1	8.502	0.247	3658.1	4151.6	8.179	0.1642	3654.5	4147.0	7.989	0.1229	3650.8	4142.5	7.852	800

Table E.28: Subcooled water and Superheated Steam (50 bar to 80 bar)

T(řC)	50 bar (T_{sat} = 263.9 řC)				60 bar (T_{sat} = 275.6 řC)				70 bar (T_{sat} = 285.8 řC)				80 bar (T_{sat} = 295.0 řC)				T(řC)
	v	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	
0.01	0.000998	0.08604	5.074	0.00029	0.000997	0.10167	6.085	0.00034	0.000997	0.11679	7.094	0.00038	0.000996	0.13139	8.101	0.00042	0.01
20	0.001000	83.62	88.61	0.2955	0.000999	83.55	89.55	0.2952	0.000999	83.49	90.48	0.2950	0.000998	83.43	91.42	0.2948	20
40	0.001006	166.9	172.0	0.5705	0.001005	166.8	172.8	0.5701	0.001005	166.7	173.7	0.5697	0.001004	166.6	174.6	0.5693	40
60	0.001015	250.3	255.3	0.8286	0.001014	250.1	256.2	0.8280	0.001014	249.9	257.0	0.8275	0.001014	249.7	257.8	0.8270	60
80	0.001027	333.8	338.9	1.072	0.001026	333.5	339.7	1.071	0.001026	333.3	340.5	1.071	0.001025	333.1	341.3	1.070	80
100	0.001041	417.6	422.8	1.303	0.001040	417.3	423.5	1.302	0.001040	417.0	424.3	1.302	0.001039	416.7	425.0	1.301	100
120	0.001058	501.9	507.2	1.523	0.001057	501.5	507.9	1.523	0.001057	501.2	508.6	1.522	0.001056	500.8	509.3	1.521	120
140	0.001077	586.8	592.2	1.734	0.001076	586.4	592.9	1.733	0.001076	586.0	593.5	1.732	0.001075	585.6	594.2	1.731	140
160	0.001099	672.7	678.1	1.938	0.001098	672.1	678.7	1.936	0.001097	671.6	679.3	1.935	0.001097	671.1	679.9	1.934	160
180	0.001124	759.6	765.2	2.134	0.001123	759.0	765.7	2.133	0.001122	758.4	766.2	2.131	0.001122	757.8	766.8	2.130	180
200	0.001153	848.0	853.8	2.325	0.001152	847.3	854.2	2.324	0.001151	846.6	854.6	2.322	0.001150	845.9	855.1	2.321	200
220	0.001187	938.4	944.4	2.513	0.001186	937.6	944.7	2.511	0.001184	936.7	945.0	2.509	0.001183	935.8	945.3	2.507	220
240	0.001227	1031.5	1037.7	2.698	0.001225	1030.4	1037.8	2.696	0.001224	1029.3	1037.9	2.694	0.001222	1028.3	1038.0	2.692	240
260	0.001275	1128.4	1134.8	2.884	0.001273	1127.0	1134.6	2.881	0.001271	1125.6	1134.5	2.879	0.001269	1124.2	1134.3	2.876	260
280	0.04227	2646.7	2858.1	6.091	0.03320	2606.0	2805.2	5.928	0.001331	1227.0	1236.3	3.066	0.001328	1225.2	1235.8	3.063	280
300	0.04535	2698.9	2925.6	6.211	0.03619	2668.3	2885.5	6.070	0.02949	2633.4	2839.8	5.934	0.02428	2592.1	2786.4	5.794	300
320	0.04813	2745.5	2986.2	6.315	0.03878	2720.9	2953.5	6.187	0.03201	2693.8	2917.9	6.067	0.02684	2663.6	2878.4	5.951	320
340	0.05073	2788.7	3042.4	6.408	0.04114	2768.1	3014.9	6.289	0.03423	2745.9	2985.5	6.180	0.02899	2721.9	2953.9	6.077	340
360	0.05319	2829.7	3095.6	6.493	0.04334	2811.9	3072.0	6.380	0.03626	2793.2	3047.0	6.278	0.03092	2773.3	3020.6	6.184	360
380	0.05555	2869.1	3146.8	6.573	0.04542	2853.6	3126.1	6.465	0.03816	2837.3	3104.4	6.368	0.03268	2820.3	3081.8	6.279	380
400	0.05784	2907.4	3196.6	6.648	0.04742	2893.6	3178.2	6.543	0.03996	2879.4	3159.1	6.450	0.03435	2864.5	3139.3	6.366	400
420	0.06007	2945.0	3245.3	6.719	0.04936	2932.6	3228.8	6.617	0.04169	2919.9	3211.8	6.527	0.03593	2906.8	3194.2	6.446	420
440	0.06225	2982.0	3293.3	6.788	0.05124	2970.9	3278.3	6.688	0.04337	2959.4	3263.0	6.600	0.03745	2947.6	3247.3	6.522	440
460	0.06439	3018.7	3340.7	6.853	0.05308	3008.5	3327.0	6.755	0.04500	2998.1	3313.1	6.669	0.03893	2987.5	3298.9	6.593	460
480	0.06650	3055.2	3387.7	6.917	0.05489	3045.9	3375.2	6.820	0.04659	3036.3	3362.5	6.736	0.04036	3026.6	3349.5	6.661	480
500	0.06858	3091.6	3434.5	6.978	0.05667	3082.9	3422.9	6.882	0.04816	3074.1	3411.3	6.800	0.04177	3065.2	3399.4	6.726	500
520	0.07064	3127.8	3481.1	7.037	0.05843	3119.8	3470.4	6.943	0.04970	3111.7	3459.6	6.861	0.04315	3103.5	3448.6	6.789	520
540	0.07268	3164.1	3527.5	7.095	0.06016	3156.7	3517.6	7.002	0.05121	3149.1	3507.6	6.921	0.04450	3141.5	3497.5	6.850	540
560	0.07470	3200.4	3574.0	7.152	0.06188	3193.5	3564.7	7.059	0.05271	3186.4	3555.4	6.979	0.04584	3179.3	3546.0	6.909	560
580	0.07671	3236.8	3620.4	7.207	0.06358	3230.3	3611.8	7.115	0.05420	3223.7	3603.1	7.036	0.04716	3217.0	3594.3	6.966	580
600	0.07870	3273.3	3666.8	7.260	0.06526	3267.2	3658.8	7.169	0.05566	3261.0	3650.6	7.091	0.04846	3254.7	3642.4	7.022	600
640	0.08265	3346.7	3759.9	7.365	0.06860	3341.2	3752.8	7.275	0.05857	3335.7	3745.6	7.197	0.05104	3330.1	3738.4	7.130	640
680	0.08657	3420.6	3853.5	7.465	0.07190	3415.7	3847.1	7.376	0.06143	3410.8	3840.8	7.299	0.05357	3405.8	3834.4	7.232	680
720	0.09045	3495.3	3947.6	7.562	0.07517	3490.9	3941.9	7.473	0.06426	3486.4	3936.2	7.397	0.05607	3481.9	3930.5	7.331	720
760	0.09431	3570.8	4042.4	7.655	0.07842	3566.8	4037.3	7.567	0.06706	3562.7	4032.2	7.492	0.05855	3558.6	4027.0	7.427	760
800	0.09815	3647.1	4137.9	7.746	0.08164	3643.4	4133.3	7.658	0.06985	3639.7	4128.7	7.584	0.06101	3636.0	4124.0	7.519	800

Table E.29: Subcooled water and Superheated Steam (90 bar to 140 bar)

T(řC)	90 bar ($T_{sat} = 303.3$ řC)				100 bar ($T_{sat} = 311.0$ řC)				120 bar ($T_{sat} = 324.7$ řC)				140 bar ($T_{sat} = 336.7$ řC)				T(řC)
	v	u	h	s	v	u	h	s	v	u	h	s	v	u	h	s	
0.01	0.000996	0.1455	9.107	0.00046	0.000995	0.1591	10.11	0.00049	0.000994	0.1847	12.12	0.00055	0.000993	0.2084	14.11	0.00059	0.01
20	0.000998	83.37	92.35	0.2946	0.000997	83.31	93.29	0.2944	0.000996	83.19	95.15	0.2939	0.000996	83.07	97.01	0.2935	20
40	0.001004	166.5	175.5	0.5689	0.001003	166.3	176.4	0.5685	0.001003	166.1	178.1	0.5678	0.001002	165.9	179.9	0.5670	40
60	0.001013	249.6	258.7	0.8265	0.001013	249.4	259.5	0.8259	0.001012	249.1	261.2	0.8249	0.001011	248.7	262.9	0.8238	60
80	0.001025	332.9	342.1	1.070	0.001024	332.6	342.9	1.069	0.001023	332.2	344.5	1.068	0.001023	331.7	346.1	1.066	80
100	0.001039	416.4	425.8	1.300	0.001038	416.2	426.5	1.299	0.001038	415.6	428.1	1.298	0.001037	415.1	429.6	1.296	100
120	0.001055	500.5	510.0	1.520	0.001055	500.2	510.7	1.519	0.001054	499.5	512.1	1.517	0.001053	498.8	513.5	1.516	120
140	0.001074	585.2	594.8	1.730	0.001074	584.8	595.5	1.729	0.001073	583.9	596.8	1.727	0.001071	583.1	598.1	1.725	140
160	0.001096	670.7	680.5	1.933	0.001095	670.2	681.1	1.932	0.001094	669.2	682.3	1.929	0.001093	668.2	683.5	1.927	160
180	0.001121	757.2	767.3	2.129	0.001120	756.6	767.8	2.127	0.001118	755.4	768.9	2.125	0.001117	754.3	769.9	2.122	180
200	0.001149	845.1	855.5	2.319	0.001148	844.4	855.9	2.318	0.001146	843.0	856.8	2.315	0.001144	841.6	857.7	2.312	200
220	0.001182	934.9	945.6	2.506	0.001181	934.1	945.9	2.504	0.001179	932.4	946.5	2.500	0.001176	930.7	947.2	2.497	220
240	0.001221	1027.2	1038.2	2.690	0.001219	1026.1	1038.3	2.688	0.001216	1024.0	1038.6	2.683	0.001213	1022.0	1039.0	2.679	240
260	0.001267	1122.8	1134.2	2.873	0.001265	1121.5	1134.1	2.871	0.001262	1118.8	1134.0	2.866	0.001258	1116.3	1133.9	2.861	260
280	0.001325	1223.4	1235.3	3.059	0.001323	1221.6	1234.8	3.056	0.001317	1218.1	1233.9	3.050	0.001312	1214.8	1233.1	3.044	280
300	0.001402	1331.6	1344.3	3.253	0.001398	1329.1	1343.1	3.248	0.001390	1324.3	1340.9	3.240	0.001382	1319.6	1339.0	3.231	300
320	0.02271	2629.5	2833.9	5.835	0.01927	2589.9	2782.7	5.713	0.001494	1442.4	1460.3	3.444	0.001480	1435.2	1455.9	3.432	320
340	0.02486	2695.8	2919.6	5.977	0.02149	2667.2	2882.1	5.878	0.01621	2598.9	2793.5	5.672	0.01200	2504.4	2672.4	5.429	340
360	0.02672	2752.0	2992.5	6.094	0.02333	2729.3	2962.6	6.007	0.01812	2678.4	2895.9	5.837	0.01423	2617.2	2816.4	5.661	360
380	0.02840	2802.4	3058.0	6.196	0.02495	2783.6	3033.1	6.117	0.01971	2742.6	2979.1	5.966	0.01587	2696.1	2918.3	5.819	380
400	0.02996	2849.1	3118.8	6.287	0.02644	2833.0	3097.4	6.214	0.02111	2798.6	3051.9	6.076	0.01724	2760.9	3002.2	5.946	400
420	0.03144	2893.2	3176.1	6.371	0.02783	2879.2	3157.5	6.302	0.02239	2849.6	3118.2	6.173	0.01846	2817.7	3076.1	6.054	420
440	0.03284	2935.5	3231.1	6.450	0.02915	2923.1	3214.6	6.383	0.02358	2897.1	3180.1	6.261	0.01958	2869.5	3143.6	6.150	440
460	0.03420	2976.6	3284.4	6.523	0.03041	2965.4	3269.5	6.459	0.02471	2942.3	3238.8	6.343	0.02062	2918.0	3206.7	6.237	460
480	0.03551	3016.7	3336.3	6.593	0.03163	3006.6	3322.9	6.531	0.02579	2985.8	3295.2	6.418	0.02160	2964.1	3266.5	6.318	480
500	0.03680	3056.2	3387.3	6.660	0.03281	3046.9	3375.1	6.599	0.02683	3028.0	3350.0	6.490	0.02255	3008.4	3324.1	6.393	500
520	0.03805	3095.1	3437.6	6.724	0.03397	3086.6	3426.3	6.665	0.02784	3069.3	3403.4	6.558	0.02345	3051.5	3379.8	6.464	520
540	0.03928	3133.7	3487.2	6.786	0.03510	3125.9	3476.9	6.728	0.02882	3109.9	3455.8	6.624	0.02433	3093.5	3434.2	6.532	540
560	0.04049	3172.1	3536.5	6.846	0.03621	3164.8	3526.9	6.789	0.02978	3150.0	3507.4	6.686	0.02519	3134.9	3487.5	6.597	560
580	0.04168	3210.3	3585.4	6.904	0.03730	3203.5	3576.5	6.847	0.03072	3189.8	3558.4	6.747	0.02602	3175.7	3540.1	6.659	580
600	0.04286	3248.4	3634.2	6.960	0.03838	3242.1	3625.8	6.905	0.03165	3229.2	3609.0	6.806	0.02684	3216.1	3591.9	6.719	600
640	0.04518	3324.6	3731.2	7.069	0.04049	3319.0	3723.9	7.014	0.03346	3307.6	3709.2	6.918	0.02844	3296.1	3694.3	6.834	640
680	0.04746	3400.8	3828.0	7.173	0.04257	3395.8	3821.5	7.119	0.03523	3385.7	3808.5	7.024	0.03000	3375.5	3795.4	6.942	680
720	0.04971	3477.4	3924.8	7.272	0.04461	3472.9	3919.0	7.219	0.03697	3463.8	3907.5	7.126	0.03151	3454.6	3895.8	7.045	720
760	0.05193	3554.5	4021.9	7.368	0.04663	3550.4	4016.7	7.316	0.03868	3542.2	4006.4	7.223	0.03301	3533.9	3995.9	7.144	760
800	0.05413	3632.2	4119.4	7.461	0.04862	3628.5	4114.7	7.409	0.04037	3621.0	4105.4	7.318	0.03448	3613.4	4096.0	7.239	800

Table E.30: Subcooled water and Superheated Steam (160 bar to 220 bar)

T(řC)	160 bar (T_{sat} = 347.4 řC)				180 bar (T_{sat} = 357.0 řC)				200 bar (T_{sat} = 365.7 řC)				220 bar (T_{sat} = 373.7 řC)				T(řC)
	v	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	
0.01	0.000992	0.2302	16.11	0.00061	0.000991	0.2501	18.09	0.00063	0.000990	0.2680	20.08	0.00062	0.000989	0.2842	22.05	0.00061	0.01
20	0.000995	82.95	98.87	0.2930	0.000994	82.83	100.7	0.2926	0.000993	82.71	102.6	0.2921	0.000992	82.59	104.4	0.2916	20
40	0.001001	165.6	181.7	0.5662	0.001000	165.4	183.4	0.5654	0.000999	165.2	185.2	0.5646	0.000998	165.0	186.9	0.5639	40
60	0.001010	248.4	264.5	0.8228	0.001009	248.1	266.2	0.8218	0.001008	247.7	267.9	0.8207	0.001008	247.4	269.6	0.8197	60
80	0.001022	331.3	347.6	1.065	0.001021	330.9	349.2	1.064	0.001020	330.4	350.8	1.062	0.001019	330.0	352.4	1.061	80
100	0.001036	414.5	431.1	1.295	0.001035	414.0	432.6	1.293	0.001034	413.4	434.1	1.292	0.001033	412.9	435.6	1.290	100
120	0.001052	498.1	515.0	1.514	0.001051	497.5	516.4	1.512	0.001050	496.8	517.8	1.510	0.001049	496.2	519.2	1.509	120
140	0.001070	582.3	599.5	1.723	0.001069	581.5	600.8	1.721	0.001068	580.7	602.1	1.719	0.001067	580.0	603.4	1.718	140
160	0.001091	667.3	684.7	1.925	0.001090	666.3	685.9	1.923	0.001089	665.4	687.2	1.920	0.001087	664.5	688.4	1.918	160
180	0.001115	753.2	771.0	2.120	0.001114	752.0	772.1	2.117	0.001112	750.9	773.2	2.115	0.001111	749.8	774.2	2.112	180
200	0.001143	840.3	858.6	2.309	0.001141	838.9	859.5	2.306	0.001139	837.6	860.4	2.303	0.001137	836.3	861.3	2.300	200
220	0.001174	929.0	947.8	2.494	0.001172	927.4	948.5	2.490	0.001170	925.8	949.2	2.487	0.001168	924.3	949.9	2.484	220
240	0.001211	1019.9	1039.3	2.675	0.001208	1018.0	1039.7	2.671	0.001205	1016.0	1040.1	2.668	0.001203	1014.1	1040.6	2.664	240
260	0.001254	1113.7	1133.8	2.856	0.001251	1111.3	1133.8	2.851	0.001247	1108.9	1133.8	2.847	0.001244	1106.5	1133.9	2.842	260
280	0.001307	1211.5	1232.5	3.038	0.001302	1208.4	1231.8	3.032	0.001298	1205.3	1231.3	3.026	0.001293	1202.4	1230.8	3.021	280
300	0.001375	1315.2	1337.2	3.224	0.001368	1311.0	1335.6	3.216	0.001361	1306.9	1334.1	3.209	0.001355	1303.0	1332.8	3.202	300
320	0.001467	1428.5	1451.9	3.420	0.001456	1422.2	1448.4	3.409	0.001445	1416.4	1445.3	3.399	0.001435	1410.9	1442.5	3.390	320
340	0.001616	1561.4	1587.3	3.645	0.001591	1550.1	1578.7	3.625	0.001569	1540.1	1571.5	3.608	0.001551	1531.2	1565.3	3.593	340
360	0.01106	2538.7	2715.6	5.462	0.004624	2145.8	2233.9	4.658	0.002365	1834.4	1876.1	4.101	0.002047	1768.2	1803.9	3.987	360
380	0.01288	2642.2	2848.3	5.668	0.01042	2577.4	2764.9	5.505	0.008258	2494.0	2659.2	5.314	0.004125	2183.2	2279.4	4.700	380
400	0.01428	2719.0	2947.5	5.818	0.01191	2671.8	2886.3	5.688	0.009950	2617.8	2816.8	5.552	0.008255	2554.2	2735.8	5.405	400
420	0.01548	2783.2	3030.9	5.940	0.01312	2745.7	2981.9	5.828	0.01120	2704.5	2928.5	5.716	0.009588	2659.0	2869.9	5.602	420
440	0.01655	2840.2	3105.0	6.045	0.01417	2808.9	3064.0	5.945	0.01225	2775.3	3020.3	5.847	0.01065	2739.3	2973.6	5.749	440
460	0.01753	2892.5	3173.0	6.139	0.01512	2865.6	3137.7	6.047	0.01317	2837.2	3100.6	5.958	0.01156	2807.2	3061.6	5.871	460
480	0.01845	2941.5	3236.7	6.225	0.01599	2917.9	3205.7	6.138	0.01401	2893.2	3173.4	6.056	0.01238	2867.5	3139.9	5.976	480
500	0.01932	2988.1	3297.3	6.305	0.01681	2967.1	3269.7	6.222	0.01479	2945.3	3241.2	6.145	0.01314	2922.8	3211.8	6.070	500
520	0.02016	3033.1	3355.6	6.379	0.01759	3014.1	3330.7	6.300	0.01553	2994.6	3305.2	6.226	0.01384	2974.5	3279.0	6.156	520
540	0.02096	3076.7	3412.1	6.449	0.01833	3059.5	3389.5	6.373	0.01623	3041.8	3366.4	6.303	0.01451	3023.7	3342.9	6.236	540
560	0.02174	3119.4	3467.3	6.516	0.01905	3103.6	3446.6	6.443	0.01690	3087.5	3425.6	6.374	0.01514	3071.0	3404.1	6.310	560
580	0.02250	3161.4	3521.4	6.581	0.01975	3146.8	3502.4	6.509	0.01755	3132.0	3483.0	6.443	0.01575	3116.8	3463.4	6.381	580
600	0.02324	3202.8	3574.6	6.642	0.02043	3189.3	3557.0	6.572	0.01818	3175.5	3539.2	6.508	0.01635	3161.6	3521.2	6.448	600
640	0.02467	3284.5	3679.2	6.759	0.02174	3272.7	3664.0	6.692	0.01940	3260.7	3648.7	6.630	0.01748	3248.6	3633.2	6.573	640
680	0.02607	3365.2	3782.2	6.870	0.02301	3354.7	3768.9	6.804	0.02056	3344.2	3755.5	6.745	0.01856	3333.5	3742.0	6.690	680
720	0.02742	3445.4	3884.1	6.975	0.02424	3436.0	3872.3	6.911	0.02169	3426.6	3860.5	6.853	0.01961	3417.2	3848.6	6.799	720
760	0.02875	3525.5	3985.5	7.075	0.02544	3517.1	3975.0	7.012	0.02279	3508.6	3964.4	6.955	0.02063	3500.1	3953.8	6.903	760
800	0.03006	3605.7	4086.6	7.171	0.02662	3598.1	4077.2	7.109	0.02387	3590.4	4067.7	7.053	0.02162	3582.6	4058.2	7.002	800

Table E.31: Supercritical steam (250 bar to 500 bar)

	250 bar				300 bar				400 bar				500 bar				
T(řC)	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*T*(řC)
0.01	0.000988	0.3049	25	0.00056	0.000986	0.3308	29.9	0.00043	0.000981	0.3511	39.6	-0.00008	0.000977	0.3323	49.17	-0.00087	0.01
20	0.000991	82.41	107.18	0.2909	0.000989	82.12	111.8	0.2897	0.000985	81.52	120.9	0.2872	0.00098	80.93	130	0.2845	20
40	0.000997	164.6	189.5	0.5627	0.000995	164.1	193.9	0.5607	0.000991	163	202.6	0.5568	0.000987	161.9	211.3	0.5528	40
60	0.001006	246.9	272.1	0.8181	0.001004	246.1	276.2	0.8156	0.001	244.6	284.6	0.8105	0.000996	243.1	292.9	0.8054	60
80	0.001018	329.4	354.8	1.059	0.001016	328.3	358.8	1.056	0.001011	326.3	366.8	1.05	0.001007	324.4	374.7	1.044	80
100	0.001031	412.1	437.9	1.288	0.001029	410.8	441.7	1.284	0.001024	408.3	449.3	1.277	0.00102	405.9	456.9	1.27	100
120	0.001047	495.2	521.4	1.506	0.001045	493.6	525	1.502	0.00104	490.6	532.2	1.494	0.001035	487.7	539.4	1.486	120
140	0.001065	578.8	605.4	1.715	0.001062	576.9	608.8	1.71	0.001057	573.3	615.6	1.701	0.001052	569.8	622.4	1.692	140
160	0.001085	663.1	690.2	1.915	0.001082	660.8	693.3	1.91	0.001076	656.5	699.6	1.899	0.00107	652.4	706	1.889	160
180	0.001108	748.2	775.9	2.108	0.001105	745.5	778.7	2.102	0.001098	740.5	784.4	2.091	0.001091	735.6	790.2	2.079	180
200	0.001135	834.4	862.7	2.296	0.00113	831.2	865.1	2.289	0.001122	825.2	870.1	2.276	0.001115	819.6	875.3	2.263	200
220	0.001164	921.9	951.1	2.479	0.001159	918.2	953	2.471	0.00115	911.1	957.1	2.456	0.001141	904.4	961.5	2.441	220
240	0.001199	1011.3	1041.3	2.658	0.001193	1006.8	1042.6	2.649	0.001181	998.4	1045.6	2.632	0.001171	990.5	1049	2.615	240
260	0.001239	1103.1	1134.1	2.835	0.001231	1097.6	1134.6	2.825	0.001217	1087.4	1136.1	2.805	0.001204	1078.1	1138.3	2.786	260
280	0.001287	1198.1	1230.2	3.012	0.001277	1191.3	1229.6	3	0.001259	1178.8	1229.1	2.976	0.001243	1167.5	1229.7	2.954	280
300	0.001346	1297.4	1331.1	3.192	0.001332	1288.7	1328.7	3.176	0.001308	1273.1	1325.4	3.147	0.001288	1259.3	1323.7	3.121	300
320	0.001421	1403.2	1438.7	3.376	0.001401	1391.5	1433.5	3.355	0.001368	1371.3	1426	3.319	0.001341	1354.2	1421.2	3.289	320
340	0.001526	1519.3	1557.5	3.573	0.001493	1502.3	1547.1	3.544	0.001443	1474.8	1532.5	3.496	0.001405	1452.8	1523.1	3.457	340
360	0.001858	1714.6	1750.4	3.897	0.001715	1663.3	1704.5	3.811	0.001581	1605.1	1660.1	3.713	0.001507	1568.4	1637.2	3.651	360
380	0.002749	1987.1	2052.8	4.35	0.002165	1867.2	1922	4.145	0.001805	1757.9	1816.6	3.962	0.001656	1698.3	1768.3	3.863	380
400	0.005284	2372.9	2510.3	5.03	0.002937	2093.3	2180.3	4.518	0.002102	1913.8	1987.5	4.213	0.001838	1829.3	1908.6	4.072	400
420	0.007579	2580	2769.4	5.42	0.004566	2371.7	2512.3	4.999	0.002518	2077.6	2174.1	4.475	0.002065	1962.6	2056.8	4.282	420
440	0.008697	2679.6	2897.1	5.601	0.006228	2562	2748.9	5.342	0.003138	2254.3	2381.2	4.761	0.002355	2099.8	2213.5	4.496	440
460	0.009617	2758.8	2999.2	5.743	0.007193	2668	2883.8	5.528	0.004149	2447.4	2613.3	5.084	0.002741	2242.4	2379.6	4.72	460
480	0.01042	2826.6	3087.1	5.861	0.007992	2752.2	2992	5.674	0.00495	2579.2	2777.2	5.305	0.003277	2391.7	2556.8	4.957	480
500	0.01114	2887.4	3165.9	5.964	0.00869	2824.1	3084.8	5.796	0.005625	2681.7	2906.7	5.475	0.003889	2528	2722.5	5.176	500
520	0.01181	2943.2	3238.5	6.057	0.00932	2888.1	3167.7	5.901	0.006213	2766.9	3015.4	5.613	0.004417	2636.5	2857.4	5.348	520
540	0.01244	2995.7	3306.6	6.142	0.009899	2946.7	3243.7	5.996	0.00674	2841.1	3110.7	5.732	0.004896	2728.4	2973.2	5.492	540
560	0.01303	3045.6	3371.3	6.22	0.01044	3001.6	3314.8	6.083	0.007221	2907.8	3196.7	5.837	0.005332	2808.8	3075.4	5.617	560
580	0.01359	3093.6	3433.5	6.294	0.01095	3053.6	3382.3	6.163	0.007669	2969.3	3276	5.931	0.005734	2881	3167.7	5.726	580
600	0.01414	3140.2	3493.7	6.364	0.01144	3103.5	3446.9	6.237	0.008089	3026.9	3350.4	6.017	0.006109	2947.2	3252.6	5.825	600
640	0.01518	3230.2	3609.7	6.494	0.01237	3198.9	3569.9	6.375	0.008869	3134.1	3488.8	6.172	0.006796	3067.4	3407.2	5.998	640
680	0.01617	3317.4	3721.5	6.614	0.01324	3290.1	3687.2	6.501	0.009589	3234	3617.6	6.31	0.007422	3176.9	3548	6.149	680
720	0.01711	3402.8	3830.6	6.726	0.01406	3378.6	3800.5	6.617	0.01026	3329.4	3740	6.436	0.008004	3279.5	3679.6	6.284	720
760	0.01803	3487.2	3937.9	6.832	0.01486	3465.5	3911.3	6.727	0.01091	3421.7	3857.9	6.552	0.008552	3377.4	3805	6.408	760
800	0.01892	3570.9	4044	6.932	0.01563	3551.4	4020.2	6.83	0.01152	3511.9	3972.8	6.661	0.009074	3472.3	3926	6.523	800

Table E.32: Supercritical steam (600 bar to 1000 bar)

	600 bar				700 bar				800 bar				1000 bar				
T(řC)	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*T*(řC)
0.01	0.000972	0.2777	58.63	-0.00193	0.000968	0.1902	67.97	-0.00323	0.000964	0.0725	77.22	-0.00476	0.000957	-0.2439	95.43	-0.00844	0.01
20	0.000977	80.35	138.9	0.2818	0.000973	79.76	147.9	0.279	0.000969	79.19	156.7	0.2761	0.000962	78.04	174.2	0.27	20
40	0.000983	160.9	219.9	0.5489	0.00098	159.9	228.4	0.5449	0.000976	158.9	237	0.5409	0.000969	157	253.9	0.5329	40
60	0.000992	241.6	301.1	0.8004	0.000989	240.2	309.4	0.7955	0.000985	238.8	317.6	0.7905	0.000978	236.2	334	0.7808	60
80	0.001003	322.5	382.7	1.038	0.000999	320.7	390.6	1.032	0.000996	318.9	398.5	1.026	0.000988	315.5	414.4	1.015	80
100	0.001016	403.5	464.5	1.263	0.001012	401.3	472.1	1.257	0.001008	399.1	479.8	1.25	0.001	395	495	1.237	100
120	0.00103	484.8	546.7	1.478	0.001026	482.1	554	1.47	0.001022	479.5	561.3	1.463	0.001014	474.6	576	1.449	120
140	0.001047	566.5	629.3	1.683	0.001042	563.3	636.2	1.674	0.001037	560.2	643.2	1.666	0.001028	554.4	657.2	1.65	140
160	0.001065	648.5	712.4	1.879	0.00106	644.8	718.9	1.87	0.001054	641.2	725.5	1.861	0.001045	634.4	738.9	1.843	160
180	0.001085	731	796.2	2.068	0.001079	726.7	802.2	2.058	0.001074	722.5	808.4	2.048	0.001063	714.7	820.9	2.028	180
200	0.001108	814.2	880.7	2.251	0.001101	809.1	886.2	2.239	0.001095	804.3	891.8	2.228	0.001083	795.3	903.5	2.207	200
220	0.001133	898.2	966.2	2.428	0.001125	892.3	971	2.415	0.001118	886.7	976.1	2.402	0.001104	876.3	986.7	2.379	220
240	0.001161	983.2	1052.8	2.6	0.001152	976.3	1056.9	2.586	0.001143	969.8	1061.3	2.572	0.001128	957.9	1070.7	2.546	240
260	0.001193	1069.4	1141	2.769	0.001182	1061.4	1144.1	2.752	0.001172	1053.9	1147.7	2.737	0.001154	1040.2	1155.6	2.708	260
280	0.001229	1157.3	1231	2.934	0.001216	1147.8	1232.9	2.916	0.001204	1139.1	1235.4	2.898	0.001183	1123.3	1241.6	2.866	280
300	0.00127	1247	1323.3	3.098	0.001254	1235.9	1323.7	3.077	0.00124	1225.7	1324.9	3.057	0.001215	1207.4	1328.9	3.022	300
320	0.001318	1339.2	1418.3	3.261	0.001298	1325.9	1416.7	3.237	0.00128	1313.8	1416.3	3.214	0.00125	1292.7	1417.7	3.174	320
340	0.001374	1434.2	1516.7	3.424	0.001349	1418.1	1512.5	3.395	0.001327	1403.8	1509.9	3.369	0.00129	1379.1	1508.1	3.324	340
360	0.001457	1541.1	1623.1	3.604	0.001419	1519.1	1613.7	3.565	0.001388	1500.4	1607.4	3.532	0.001339	1469.6	1600.6	3.477	360
380	0.001569	1657.8	1740.6	3.794	0.001509	1627.2	1722.8	3.742	0.001464	1602.5	1710.7	3.7	0.001398	1563.5	1696.5	3.632	380
400	0.001701	1775.1	1864.6	3.982	0.001613	1735.6	1836.8	3.916	0.001549	1704.7	1818	3.863	0.001462	1657.5	1795.2	3.782	400
420	0.001856	1893.4	1994.1	4.167	0.001731	1844.5	1955	4.086	0.001645	1807.1	1928.5	4.023	0.001532	1751.3	1896	3.929	420
440	0.002042	2013.3	2128.8	4.352	0.001868	1954.2	2076.9	4.255	0.001754	1909.9	2041.9	4.181	0.001609	1845.1	1998.7	4.073	440
460	0.00227	2135.4	2268.6	4.539	0.002028	2065	2202.2	4.422	0.001877	2013.2	2158	4.337	0.001694	1939.1	2103.3	4.214	460
480	0.002555	2260.1	2413.7	4.73	0.002219	2177	2330.9	4.591	0.00202	2117.2	2276.4	4.492	0.001789	2033.2	2209.4	4.354	480
500	0.002922	2388	2564.6	4.926	0.002449	2290.5	2462.9	4.761	0.002186	2222	2397.1	4.647	0.001895	2127.6	2316.8	4.491	500
520	0.003361	2512.3	2713.9	5.119	0.002732	2405.5	2598.3	4.933	0.002382	2327.5	2519.8	4.801	0.002015	2222.1	2425.3	4.627	520
540	0.003762	2617.2	2842.9	5.28	0.003067	2518.7	2733.4	5.103	0.002616	2433.9	2644.8	4.957	0.00215	2316.8	2534.8	4.762	540
560	0.004142	2709.6	2958.1	5.42	0.003378	2617.6	2854.1	5.25	0.002885	2538.2	2769	5.11	0.002305	2411.7	2645.3	4.896	560
580	0.004499	2792.1	3062	5.543	0.003683	2707.3	2965.1	5.382	0.003135	2631.2	2882	5.244	0.002484	2506.8	2756.8	5.031	580
600	0.004834	2866.9	3157	5.653	0.003975	2789.3	3067.5	5.5	0.003384	2717.4	2988.1	5.367	0.002672	2597.8	2865.1	5.158	600
640	0.005447	3000.4	3327.3	5.844	0.00452	2934.8	3251.2	5.706	0.003861	2872.1	3181	5.583	0.003026	2761.2	3063.8	5.381	640
680	0.006003	3119.6	3479.8	6.007	0.005018	3063.1	3414.4	5.881	0.004306	3008.5	3352.9	5.768	0.003376	2907.7	3245.3	5.575	680
720	0.006518	3229.5	3620.6	6.152	0.005478	3180.2	3563.6	6.034	0.00472	3132.1	3509.7	5.929	0.003712	3041.7	3413	5.748	720
760	0.007	3333.2	3753.2	6.283	0.005909	3289.5	3703.1	6.172	0.005108	3246.7	3655.3	6.073	0.004031	3165.5	3568.6	5.901	760
800	0.007457	3432.7	3880.2	6.403	0.006317	3393.6	3835.8	6.298	0.005476	3355.2	3793.3	6.204	0.004336	3281.6	3715.2	6.04	800

F Periodic Table of the Elements

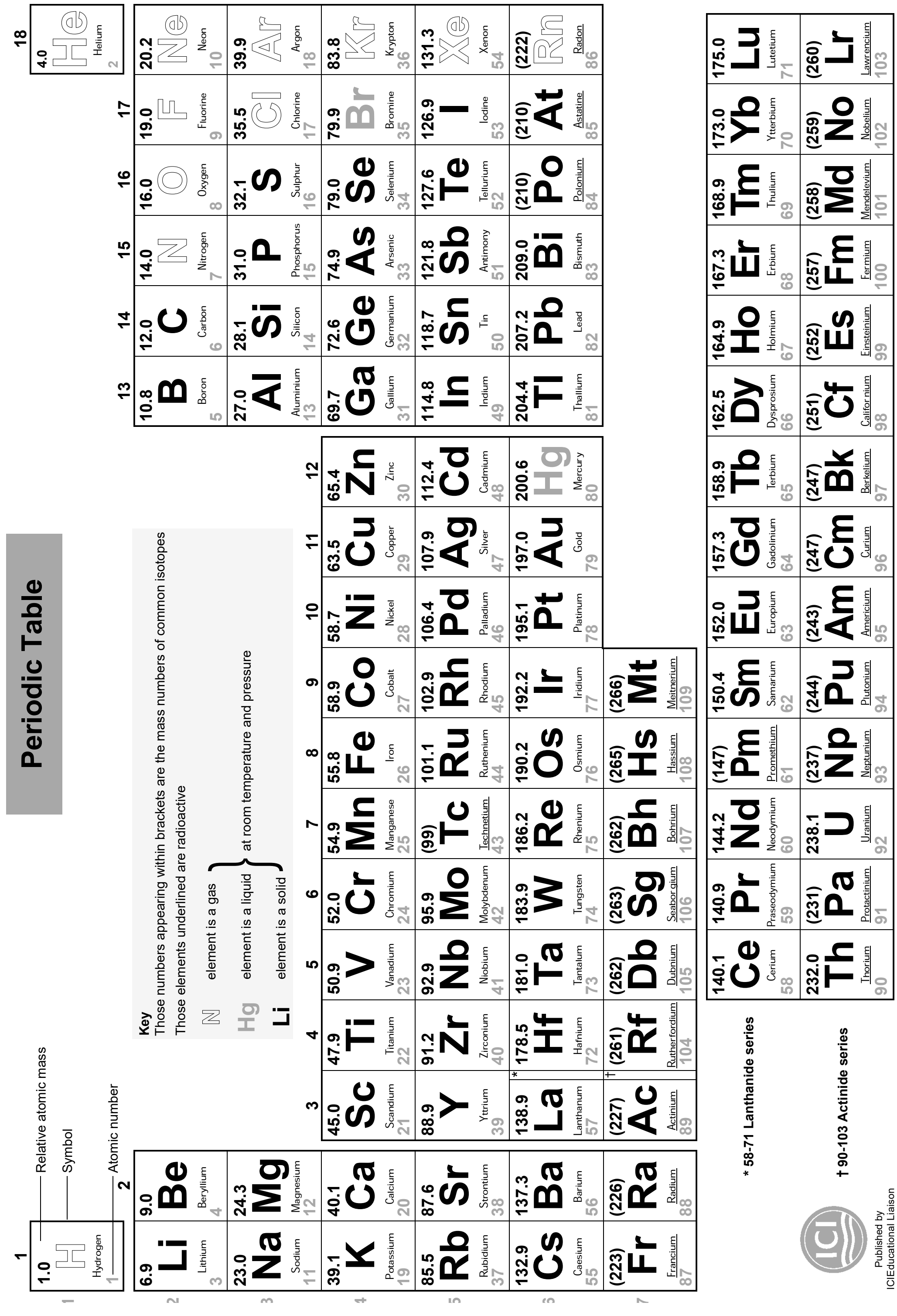

Figure F.1: Periodic Table of the Elements

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Acknowledgements

The Editor wishes to express his sincere gratitude to all of the many Mechanical Engineering Department staff, past and present, who have helped to compile this work.

The Elastic constants of materials on page 55 are from Granta EduPack 2022.

Michells table on page 69 was adapted from Barber, J. R.. Elasticity, Kluwer Academic Publishers, 2001. ProQuest Ebook Central, `https://ebookcentral.proquest.com/lib/imperial/detail.action?docID=3035937`.

Perfect gas properties in Table E.4 are based on data from *Technical Data on Fuel* (7th edition), ed. J.W. Rose and J.R. Cooper, Scottish Academic Press, 1977.

Enthalpies of ideal (but not perfect) gases in Table E.6 are approximate values obtained from fitting quadratic relationships to enthalpy-temperature data from Rose and Cooper, *Technical Data on Fuel*; the coefficients of the corresponding linear specific heat relationships and the mean and maximum errors are also given. This allows a choice between tabular and analytical methods for solving combustion problems, with consistent results. The particular form of functional relationship is one which enables combustion product temperature to be found without iteration. Tables E.9 and E.10, for heating (or calorific) values (negative of enthalpies of combustion) of simple compounds and some typical fuels, is based on data from the same source.

The Moody diagram of Figure E.1 is published under the Creative Commons Licence (CC BY-SA 4.0) by S Beck and R Collins, University of Sheffield. The original Moody diagram was published in L. F. Moody, 'Friction Factors for Pipe Flow', Trans. ASME, vol. 66, no. 8, 1944, p.671.

The psychrometric chart in Figure E.2, for standard atmospheric pressure, is based on that published in 1970 by the Institution of Heating and Ventilating Engineers.

The R134a data of Tables E.11 to E.16 have been condensed from much more detailed tables in the ICI Chemicals & Polymers Ltd publication *Thermodynamic Properties of KLEA134a*. Other physical properties of R134a, together with the equations from which the thermodynamic properties were computed, are in *Physical Property Data KLEA134a*, ICI Chemicals & Polymers Ltd, Runcorn, 1993. The refrigerants business ICI Klea was acquired in 2001 by INEOS Fluor.

The table of coefficients for conduction E.3 is vased on data from Bergman, T. L. et al. Incroperas Principles of Heat and Mass Transfer. Eighth edition / Theodore L. Bergman, Adrienne S. Lavine, Frank P. Incropera and David P. DeWitt. Hoboken, NJ: John Wiley & Sons, Inc., 2017. Print.

The periodic table F.1 was originally published by ICI education

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Character samples

The typescript was prepared in $\LaTeX\,2_\varepsilon$ using *XeLaTeX* and set in Arev. The Arev font samples on this page show standard alphanumeric characters and the Greek symbols used in formulae.

abcdefghijklmnopqrstuvwxyz

abcdefghijklmnopqrstuvwxyz

ABCDEFGHIJKLMNOPQRSTUVWXYZ

ABCDEFGHIJKLMNOPQRSTUVWXYZ

01234567890

Symbol	**Upper-case**	**Lower-case**
alpha	A	α
beta	B	β
gamma	Γ	γ
delta	Δ	δ
epsilon	E	ϵ ε
zeta	Z	ζ
eta	H	η
theta	Θ	θ ϑ
iota	I	ι
kappa	K	κ
lambda	Λ	λ
mu	M	μ
nu	N	ν
xi	Ξ	ξ
omicron	O	o
pi	Π	π
rho	P	ρ ϱ
sigma	Σ	σ
tau	T	τ
upsilon	Υ	υ
phi	Φ	ϕ φ
chi	X	χ
psi	Ψ	ψ
omega	Ω	ω