IMPERIAL

DEPARTMENT OF MATHEMATICS

GUIDE TO MODULES

SECOND YEAR (BSc/MSci)
2025-2026.

Notes and syllabus details of modules for Mathematics students in their Second Year.
For degree programmes:

G100, G103	MATHEMATICS (BSc, MSci)
G104	MATHEMATICS WITH A YEAR ABROAD (MSci)
G102	MATHEMATICS WITH MATHEMATICAL COMPUTATION
G125	MATHEMATICS (PURE MATHEMATICS)
G1F3	MATHEMATICS WITH APPLIED MATHEMATICS/MATHEMATICAL PHYSICS
G1G3	MATHEMATICS WITH STATISTICS
G1GH	MATHEMATICS WITH STATISTICS FOR FINANCE

Information on the Mathematics modules available to second-year students on the Joint Mathematics and Computing Programmes can be found in this guide.

These notes should be read in conjunction with your undergraduate student handbook and the programme specifications for your year. Some of the information may be subject to alteration.

Updated information will be posted on the Maths Central Blackboard site.

Dr Christopher Hallsworth, Mathematics DUGS
September 2025.

Second Year Programme

In the Second Year, you will continue to build a breadth of understanding in mathematics and study a number of **core** modules. In addition to these, you will be able to select a number of optional modules (**'electives')** to deepen your understanding in specific areas of mathematics.

Electives can be prerequisites for year 3/4 modules: further information about this is given later in this guide.

You are advised to read these notes carefully and to discuss your choice of options with your Personal Tutor. You will be asked to make a preliminary selection of modules before the start of the academic year. However, you will not be committed to your choice of optional modules until the completion of your exam entry information early in term 2.

You will also take an **I-Explore module** in your second year:
Through I-Explore, you will have the chance to deepen your knowledge in a different subject area, chosen from a range of for-credit modules built into your course. All undergraduate programmes offered by the College include one module from I-Explore's wide selection. Further details about choosing your i-Explore module will be provided separately. Further information can be found at:

www.imperial.ac.uk/study/ug/i-explore

Full descriptions of the Mathematics second-year modules are given in this guide.

Choice of electives is dependent on Degree programme. Students on specialist programmes will be required to take some of the optional modules in Year 2 as compulsory electives. Students will take all **core** modules listed below and choose options as follows:

G100: Select one module from Group A and 4 modules from Group B.

G102: Select one module from Group A. The module Principles of Programming is considered core for this Degree coding and must be taken. Select 3 further modules from Group B.

G125: Select one module from Group A. The modules Groups and Rings and Lebesgue Measure and Integration are considered core for this Degree coding and must be taken. Select 2 further modules from Group B.

G1F3: Select one module from Group A. The module Partial Differential Equations in Action is considered core for this Degree coding and must be taken. Select 3 further modules from Group B.

G1G3: Select one module from Group A. The modules Probability for Statistics and Statistical Modelling I are considered core for this Degree coding and must be taken. Select 2 further modules from Group B.

G1GH: Select one module from Group A. The modules Probability for Statistics and Statistical Modelling I are considered core for this Degree coding and must be taken. Select 2 further modules from Group B.

GG31: Select one module from Group A. The modules Probability for Statistics and Statistical Modelling I are considered core for this Degree coding and must be taken. Select 2 further modules from Group B.

G103: Select one module from Group A and 4 modules from Group B.

G104: Select one module from Group A (if you are required to take a language module, this must be taken as your Group A module and will be considered **core**. It will have zero weighting and be worth 7.5 ECTS) and 4 modules from Group B.

In the following table, *weight* refers to the weighting of the module in the second-year overall mark.

Module Title	MATH	Term	Lecturer(s)	Core/ Compulsory	Group	Weight	ECTS
Linear Algebra and Numerical Analysis	50003	1+2	M. Liebeck (T1) S. Olver (T2)	Core		2	10
Analysis 2	50001	1+2	D. Cheraghi (T1) N. Tamam (T2)	Core		2	10
Multi-variable Calculus and Differential Equations	50004	1+2	A.G. Walton (T1) M. Rasmussen (T2)	Core		2	10
Group Research Project	50002	3	T. Bertrand	Core (pass/ fail)		0	5
i-Explore		1, 2	-	Compulsory (pass/ fail)	A	0	5 or 7.5
Groups and Rings	50005	1	A. Skorobogatov	Elective	B	1	5
Lebesgue Measure and Integration	50006	2	A. Menegaki	Elective	B	1	5
Probability for Statistics	50010	1	D. Van Dyk	Elective	B	1	5
Statistical Modelling	50011	2	R. Passeggeri	Elective	B	1	5

Classical Mechanics	50020	2	C. Ford	Elective	B	1	5
Partial Differential Equations in Action	50008	2	T. Bertrand	Elective	B	1	5
Principles of Programming	50009	1	D. Ham	Elective	B	1	5

Choosing your electives

You are strongly advised to consider your workload when choosing your elective modules and balance the electives (and your i-Explore module) between terms 1 and 2 so that you do not become overloaded.

Personal Tutorials and other academic support

In Year 2 the role of the Personal Tutor changes from more of an academic tutor to a guide and support for your module choices and future options. You will meet your Personal Tutor individually for meetings at least twice a term during Meet Your Personal Tutor Weeks. You are welcome to also contact your Personal Tutor at different times of the term with queries. You should discuss with your Personal Tutor your academic progress and any concerns and questions. Personal Tutors are also there to support you with wellbeing concerns you may have and to sign-post you to Departmental or College support services if need be.

In Year 2, Academic support is provided through problems classes and office hours. Students are greatly encouraged to set up their own small study groups to support each other by working together on lecture notes and non-assessed problem sheets.

Examinations and marks

For details of second-year main exams, resit opportunities, computation of module marks and year weightings, please consult the undergraduate handbook for your year group.

Progression requirements

Progression Requirements for three-year degrees
G100, G102, G125, G1F3, G1G3, G1GH and GG31:
In order to progress to the next level of study, you must have passed all modules (equivalent to 60 ECTS) in the current level of study at first attempt, at resit or by an allowed (dependent on degree programme) compensated pass.
The overall weighted average for each year must be 40%, including where a module(s) has been compensated, in order for you to progress to the next year of the programme.

MSci Degrees: The MSci is an undergraduate 'Masters' degree with a final year at the level of a taught postgraduate MSc programme. On successful completion, a degree title on the

lines of ‘Master in Science (incorporating Bachelor’s level study)’ is awarded. The department offers two MSci degrees – G103 and G104.
Very occasionally, circumstances may require the Department to graduate an MSci student with a BSc.

Progression Requirements for G103 and G104:
G103: In order to progress to the next level of study, you must have passed all modules (equivalent to 60 ECTS) in the current level of study at first attempt, at resit or by a compensated pass. Additionally, the overall weighted average mark for the year, including where a module(s) has been compensated, must normally be at least as follows:

year 1: 40 percent
year 2: 60 percent
year 3: 58 percent.

A student who fails to meet the above threshold in year 2 may remain on the G103 programme if they have a year 2 overall weighted average mark of at least 58 percent. However, they will normally be required to achieve an overall weighted average mark of at least 60 percent in year 3. A student who is not permitted to remain on G103 for year 3 will be transferred to a BSc degree.

G104: In order to progress to the next level of study, you must have passed all modules (equivalent to 60 ECTS) in the current level of study at first attempt, at resit or by a compensated pass. Additionally, the overall aggregate mark for the year, including where a module(s) has been compensated, must normally be at least as follows:

year 1: 40 percent
year 2: 60 percent

Satisfactory completion of a language requirement (Level 3 or above, as determined by the College’s Centre of Languages, Culture and Communication) will normally be required for students spending their year abroad in a non-English speaking country. This will include in most cases, students being required to take and pass language modules at the College’s Centre for Languages, Culture and Communication (or its equivalent elsewhere) in Years 1 and 2 Language modules taken do not count for degree classification and are instead for pass/fail credit.

A student who is not permitted to remain on G104 for year 3 will be transferred to a BSc or MSci Mathematics degree.

Full progression information is available in the Programme Specifications:
http://www.imperial.ac.uk/staff/tools-and-reference/quality-assurance-enhancement/programme-information/programme-specifications/

Degree Changes

Students are able to change between three-year mathematics degree programmes (or dropping down from a four-year to a three-year programme) by submitting a Degree Change request and (if appropriate) ensuring that they comply with the requirements for any specialist coding module options.

Students wishing to move to the G103 programme (after the first year) must be able to comply with the Year 2 and 3 mark requirements.

Students may be able to transfer into G104 if they can satisfy the Department of their language skills (if wishing to go to a partner institution in Europe). Normally such transfers will be considered at the end of the First Year of study. Students must also meet the normal G104 Year 2 mark requirements in addition to the language requirements.

International students on a Tier-4 visa are advised to consult the International Student Support Office prior to making ANY degree change as you may be required to apply for a new visa (outside of the UK).

All degree transfer requests should normally be made by 31st of March in the year of the transfer, but transfers can in principle be made in any of years 1,2 or 3.

Third Year Programmes

In the Third Year, students choose all of their modules and will take between 60 and 62.5 ECTS. At least 52.5 ECTS must come from 3rd year (level 6) Mathematics modules. The remaining ECTS can come from 2nd year Mathematics electives not taken in year 2, a further Mathematics year 3 module, a module from another Department (subject to approval by DUGS) or certain CLCC or Business School modules. Specialist Degrees will require students to take a certain number of modules from a specified subset of available modules. Full information on these requirements are available in the Programme Specifications for each degree programme. Students may seek advice from specialist staff at set Office Hours and their Personal Tutors on module choices which align with their individual interests and strengths.

Prerequisites for year 3 and 4 modules

In the following, we list the year 3 modules which the elective is:

- *Essential for:* meaning that the y3 module will assume full familiarity with the elective;
- *Useful for:* meaning that the y3 module will make some use of the ideas and results in the elective.

Please note that this is provisional, as the full list of year 3/4 modules for 2024-25 has not yet been finalised.

Groups and Rings:
Essential for: All Algebra and Discrete Mathematics modules, all Number Theory modules. Some Geometry modules (Algebraic Curves; Algebraic Topology; Algebraic Geometry).

Useful for: All Pure Mathematics modules.

Lebesgue Measure and Integration:
Essential for: All Pure Analysis modules; Geometry of curves and surfaces.

Useful for: All pure mathematics modules. Dynamical Systems; Statistics modules. Mathematical Finance; Function Spaces and Applications; Advanced Topics in PDEs; modules involving stochastic analysis; Random Dynamical Systems and Ergodic Theory.

Probability for Statistics:
Essential for: Applied Probability, Spatial Statistics, Time Series, Introduction to Statistical Learning, Statistical Theory, Stochastic Simulation, Statistical Modelling, Applied Statistical Inference, Survival Analysis

Useful for: Probability Theory; Dynamical Systems; Random Dynamical Systems and Ergodic Theory.

Statistical Modelling:
Essential for: Introduction to Statistical Learning, Statistical Theory, Survival Analysis, Applied Statistical Inference.

Useful for: Spatial Statistics, Time Series, Stochastic Simulation.

Classical Mechanics:
Useful for: All mathematical physics modules.

PDEs in Action:
Useful for: Introduction to PDEs, All fluid mechanics modules, Mathematical Biology, Finite elements, Computational PDEs, Tensor Calculus and General Relativity, Quantum Mechanics 1 and 2, Special Relativity and Electromagnetism, Asymptotic Analysis.

Principles of Programming:
Useful for: All applied numerical modules, Finite elements, Scientific Computing.

Module Descriptions: Core Modules

MATH50001 Analysis 2

Brief description: This is a continuation of the Analysis I module where you learn about similar concepts in more general settings. The first part of the module looks at higher-dimensional derivatives, leading to the inverse and implicit function theorems. You will also learn about metric and topological spaces as generalisations of n dimensional spaces, and limiting behaviour of sequences in such spaces. The second part of the module is about functions of one complex variable, and the analysis of such functions with infinite power series expansions.

ECTS and term: 10 ECTS (core); terms 1 and 2.

Learning Outcomes:
On successful completion of this module, you will be able to:
– work with higher-dimensional derivatives as linear maps and relate these to partial derivates and Jacobian matrices;
– discuss a variety of examples of metric and topological spaces, and describe convergence of sequences in such spaces,
- define continuity, compactness and connectedness in terms of open (or closed) sets in general metric spaces,
- employ the analogues of completeness of real numbers in metric spaces to derive solutions for functional equations,
- explain aspects of convergence of sequences of functions, such as uniform convergence and point-wise convergence,
-- prove the Banach fixed point theorem and apply it in a variety of settings,
– work with a range of examples of functions of a single complex variable
-- use Cauchy's Integral Formulas and its consequences,
-- determine the local behaviour of holomorphic functions, including types of singularities, and residues,

-- use contour integration to evaluate difficult real integrals,
-- derive and apply properties of harmonic functions and conformal mappings.

Module content (syllabus):
An indicative list of sections and topics is:

Term 1; 20 lectures:
Higher dimensional derivatives: Definition of higher dimensional derivative, chain rule. Directional derivatives, partial derivatives, Df(p) in terms of partial derivatives, Higher derivatives, higher dimensional Taylor's theorem, Symmetry of mixed partials (statement of results). Inverse function theorem, implicit function theorem.

Metric spaces: Definition, examples. Topologically equivalent metrics, isometries, Lipschitz maps, Open sets, bounded sets, examples, unions, intersections, Continuity in terms of open sets, Closed sets, closure, limit points, Separable metric spaces, Topological spaces.
Compact spaces: Definition in terms of open covers, Basic features, existence of convergent sub-sequences, Continuous maps and compact sets, Sequential compactness.
Completeness: Definition, examples, Point-wise convergence and uniform convergence in function spaces, Incompleteness of C(X), Continuity of the integration on function spaces, Arzela-Ascoli, Fixed point theorem.
Connectedness: Definition, examples, Continuous image of a connected set.

Term 2: 20 lectures, Complex Analysis:
Holomorphic Functions: Definition using derivative, Cauchy-Riemann equations, Polynomials, Power series,
Rational functions, Moebius transformations,
Cauchy's Integral Formula: Complex integration along curves, Goursat's theorem, Local existence of primitives and Cauchy's theorem in a disc, Evaluation of some integrals, Homotopies and simply connected domains, Cauchy's integral formulas.
Applications of Cauchy's integral formula: Morera's theorem, Sequences of holomorphic functions, Holomorphic functions defined in terms of integrals, Schwarz reflection principle.
Meromorphic Functions: Zeros and poles. Laurent series. The residue formula, Singularities and meromorphic functions, The argument principle and applications, The complex logarithm.
Harmonic functions: Definition, and basic properties, Maximum modulus principle.
Conformal Mappings: Definitions, Preservation of Angles, Statement of the Riemann mapping theorem.

There will be 20 lectures and 7-8 problem classes per term, supplemented by office hours.

Assessment:
Mid-term test, term 1: 10 percent

Mid-term test, term 2: 10 percent
Main exam, 3 hours, term 3: 80 percent

MATH50003 Linear Algebra and Numerical Analysis

Brief description: The first part of this module builds on the Linear Algebra component of the 1st year module Linear Algebra and Groups. We examine how to find matrices for a linear transformation which reflect its important features, culminating in the rational and Jordan canonical forms. We prove the fundamental Cayley-Hamilton Theorem. We also cover the beginnings of the theory of bilinear maps. The second term of this module focuses on numerical analysis and introduces the algorithms underpinning a vast range of applications and numerical methods: from ways to obtain orthonormal basis of finite dimensional vector spaces to numerical approximations of functions of one variable (using polynomials) and solutions of ODEs.

ECTS and term: 10 ECTS (core), terms 1 and 2.

Learning Outcomes:
On successful completion of this module, you will be able to:
– define direct sums, and state and prove their elementary properties;
– analyse linear transformations via their invariant subspaces and describe the form of matrices for these transformations with respect to appropriate bases;
– calculate triangular form for a linear transformation over the complex numbers, and develop arguments using it;
– state, apply, and explain aspects of the proof of the Cayley-Hamilton Theorem;
– understand and calculate the rational and Jordan canonical form of a linear transformation over appropriate fields;
– explain, and develop arguments using, primary and cyclic decomposition;
– understand and apply the elementary theory of bilinear maps, with particular emphasis on inner product spaces;
– use efficient computational methods to perform various computations in linear algebra, including finding an orthonormal basis of a finite dimensional vector space and finding the solution of linear systems of equations;
– approximate functions of one variable by polynomials;
– compute good approximations of one dimensional integrals;
– numerically approximate solutions to simple ODEs.

Module content (syllabus):
This module is composed of two parts:
- Part I - Linear Algebra in Term 1;
- Part II - Numerical Analysis in Term 2;

An indicative list of sections and topics covered in both parts of the module is as follows:

Part I - Linear Algebra (term 1; 20 lectures):
1) Direct sums and quotient spaces in vector spaces; invariance of these under a linear map and related matrices. Triangular form (over complex numbers) and Cayley - Hamilton.
2) Factorization of polynomials (over fields); minimal polynomial of a linear map.
3) Canonical forms: primary decomposition, cyclic decomposition, rational and Jordan canonical form (proofs not examinable).
4) Bilinear maps and forms; inner products. Examples. Gram - Schmidt again. Quadratic forms. Annihilators. Dual spaces.

Part II - Numerical Analysis (term 2):
1) Brief review of Inner product spaces, Gram-Schmidt, Cauchy-Schwarz inequality.
2) Floating point arithmetic and stability of algorithms.
3) Numerical Linear Algebra: orthogonal matrices, positive definite matrices, Cholesky factorization.
4) Orthogonal Polynomials: three term recurrence relationship, Chebyshev polynomials.
5) Polynomial Interpolation: Lagrange and Chebyshev interpolation, existence and uniqueness, divided differences, error analysis.
6) Numerical Integration and Differentiation. Approximation to ODEs.
7) Implementation of algorithms in e.g. Julia, Python or Matlab

There will be 20 lectures and 7-8 problem classes per term, supplemented by office hours.

Assessment:
Mid-term test, term 1: 10 percent
1 hour computer-based test, term 2: 10 percent
Main exam, 3 hours, term 3: 80 percent.

MATH50004 Multivariable Calculus and Differential Equations

Brief description: This module follows the steps of the Calculus and Applications module (first year) and introduces more advanced topics in calculus and ordinary differential equations. It will provide an introduction to multi-dimensional vector calculus and differential operators. Students will be introduced to the calculus of variations and the concept of variational problems. Differential equations play a key role in both pure and applied mathematics. The second part of this module discusses the existence of solutions, and information is obtained about the solutions without necessarily having an explicit expression for these solutions. The importance of these ideas is emphasized by the inclusion of a number of applications in physics, engineering and biology.

ECTS and term: 10 ECTS (core), terms 1 and 2.

Learning Outcomes:
On successful completion of this module, you will be able to:
— perform calculations involving the divergence, gradient and curl and discuss the physical interpretation of these differential operators;
— derive important identities and explain the link between differential operators via integral theorems;
— exhibit geometric understanding of the nature of the differential operators in generalized systems of coordinates in three dimensions by performing line, surface and volume integrals;
— formulate and analyze variational problems;
— determining the existence and uniqueness of ordinary differential equations;
— formulate general solutions of systems of ODEs;
— describe the algebraic structure of the spaces of solutions of linear systems;
— investigate the stability of nonlinear systems.

Module content (syllabus):
This module is composed of two parts:
- Part I - Multivariable Calculus in Term 1;
- Part II - Differential Equations in Term 2;

An indicative list of sections and topics covered in both parts of the module is as follows:

Term 1: Part I - Multivariable Calculus:
1) Introduction to vector calculus: tensor notation and summation convention.
2) Differential operators: gradient, divergence and curl, operations with the gradient, Laplacian, scalar and vector fields;
3) Elements of integration: line, surface and volume integrals, Green's theorem, divergence theorem, Gauss' theorem, Stokes' theorem;
4) Curvilinear coordinates: implicit/inverse function theorems, line and volume elements, gradient, divergence, curl and Laplacian in curvilinear coordinates, changes of variables (jacobian);
5) Calculus of variations: Derivation of Euler-Lagrange Equation; short forms of the equation; extension to constrained problems and to higher dimensions; applications including catenary, brachistochrone, geodesics in various geometries.

Term 2: Part II - Ordinary Differential Equations:
1) Ordinary differential equations and initial value problems;
2) Existence and uniqueness of solutions of ODEs: Picard iterates, metric spaces and normed vector spaces, Banach fixed point theorem, Picard-Lindelof theorem, maximal solution, general solutions and flows;

3) Linear systems: algebraic structure of the space of solutions, matrix exponential function, planar linear systems, Jordan normal form, exponential growth behavior, variation of constants formula;
4) Nonlinear systems: stability, invariant sets and limits sets, Lyapunov functions, Poincaré-Bendixson theorem.

There will be 20 lectures and 7-8 problem classes per term, supplemented by office hours.

Assessment:
Mid-term test, term 1: 10 percent
Mid-term test, term 2: 10 percent
Main exam, 3 hours, term 3: 80 percent.

MATH50002 Group Research Project

Brief description: The aim of the Group Research Project is to provide an opportunity for students to further their mathematical research and communication skills while developing transferable team work and presentation skills.

ECTS and term: 5 ECTS (Core, Pass/Fail), term 3.

Learning Outcomes:
On successful completion of this module, you will be able to:
– conduct independent and group research into a select area of mathematics
- work collaboratively in a team to develop a plan, organise time and delegate responsibilities and tasks
- work with group members to share and consolidate research
- discuss independent and group research and findings with both peers and academic supervisor
– present your collaborative research in a cohesive written project using mathematical typesetting and providing appropriate referencing
– communicate your findings orally and expand on the information in your project to peer and staff markers

Module content:
Students are asked to submit their areas of mathematical interest (via options provided to them based on the topics available) which are used to help determine the teams for the project.
Groups meet with their assigned supervisor regularly to identify specific topics and gain support and understanding on independent reading. Groups are expected to meet regularly independent of the supervisor meetings to share information and peer teach topics to each other.

The time allowed for the project is short and groups need to quickly organise themselves to develop a good working relationship. In addition to importance placed on the mathematical content in the final project, effective group work is also a learning outcome of the module. Students are supported with teamwork, research and presentation skills talks and optional workshops on oral group presentations.

Specialty topics may change annually and are based around the lecturers' expertise or research areas. Students are able to give their initial preferences on project areas to help determine groups.

Assessment:
This module is Pass/Fail; it does not count towards the year total. The module will be assessed by means of a group-written report, for which the Pass/Fail outcome will be the same for all members of the group, and a group oral presentation, in which the Pass/Fail outcome will be determined for each individual student separately. In order to pass the module, both components must be passed.

Module Descriptions: Elective Modules

MATH50005 Groups and Rings

Brief description: This module builds on the Group Theory from the 1st year module Linear Algebra & Groups. We see further examples of groups, and we study homomorphisms: maps between groups which preserve aspects of their structure. We look at applications of Group Theory via the idea of a group action, in which the elements of a group are associated with permutations of a set.

The module also introduces rings, another class of algebraic object, which is equipped with both addition and multiplication. Rings play a fundamental role in many parts of mathematics; in particular they underlie much of Number Theory, from which many of our examples are drawn.

ECTS and term: 5 ECTS (Elective), term 1.

Learning Outcomes:
On successful completion of this module, you will be able to:
– identify features of, and develop arguments about, certain groups and classes of group;
– state, apply, and explain aspects of the proof of the categorization theorem for finitely generated abelian groups;
– explain the principles of group actions, and work with elementary examples;
– define rings, subrings, units, zero-divisors, and related terms, and state and prove their basic properties;

– apply the definitions of integral domains, Euclidean domains and unique factorisation domains in examples and results showing the logical relationships between them;
– understand and apply Euclidean methods in certain classes of ring;
– explain the quotient construction in groups and rings, and its connection with homomorphisms and with normal subgroups/ideals;
– develop arguments and obtain information about groups using the 1st isomorphism theorem;
– explain the construction of finite fields, and their basic properties;
– work independently and with peers to understand abstract concepts in algebra.

Module content (syllabus):
An indicative list of sections and topics is:

Groups: Further examples. Normal subgroups, quotient groups and the 1st isomorphism theorem. Finitely generated abelian groups (via Smith normal form). Group actions, orbit-stabiliser and simple applications.

Rings: Definitions and examples (mainly commutative). Units and zero-divisors. Integral domains, Euclidean domains and unique factorisation. Ideals and quotient rings, first isomorphism theorem. Characteristic of an ID; construction of finite fields.

There will be 20 lectures and 7-8 problem classes, supplemented by office hours.

Assessment:
Mid-term test: 20 percent
Main exam, 1.5 hours, term 3: 80 percent.

MATH50010 Probability for Statistics

Brief description: This module introduces probability concepts that are useful in applications, particularly to statistics. It explores random variables in more detail than the first year module, and focuses in particular on the joint behaviour of several random variables. Properties of large, random samples, such as laws of large numbers and the central limit theorem, are treated in detail, and proofs are given. The second part of the course introduces random processes, particularly Markov chains on a finite state space, which are widely used in applications.

ECTS and term: 5 ECTS (Elective), term 1.

Learning Outcomes:
On successful completion of this module, you will be able to:

- Explain how the Borel sigma algebra arises.
- Use countable additivity to determine probabilities for infinite collections of events.
- Transform multivariate random variables.
- Distinguish between different modes of convergence of sequences of random variables.
- Use the laws of large numbers to determine limiting properties of random variables, giving proofs in some cases.
- Prove the central limit theorem and understand its consequences for statistics.
- Use Markov chains to model random processes.
- Solve systems of linear equations to determine properties of a Markov chain.

Module content (syllabus):
An indicative list of sections and topics is:

- Probability spaces
- The Borel sigma algebra
- Countable additivity and the continuity property
- Univariate and multivariate random variables
- Transformations of multivariate random variables
- Modes of convergence of random variables
- Laws of large numbers
- Joint moment generating functions
- Central limit theorem (including proof)
- Random walks (1D)
- Discrete time Markov chains with finite state space
- Transition probabilities and matrices
- Chapman-Kolmogorov equations
- Expected hitting times and probabilities
- Classification of states
- Limiting and stationary distributions

There will be 2 lectures each week, and 7 problem classes. Every other week, there will be a 1 hour computer-based tutorial. The tutorial will look at practical examples and simulation, using R.

There will be 20 lectures and 7-8 problem classes, supplemented by office hours.

Assessment:
In Class Test: 20 percent
Main exam, 1.5 hours, term 3: 80 percent.

MATH50020 Classical Mechanics

Brief description: Classical Mechanics is developed with emphasis on variational principles rather than Newtonian force laws. Lagrangian and Hamiltonian formulations are considered. The methods are applied to a variety of problems including pendulums, the two-body problem, and relativistic particles. The role of conserved quantities is emphasised. An introduction to more advanced ideas including Hamilton-Jacobi theory and integrability is given.

ECTS and term: 5 ECTS (Elective), term 2.

Learning Outcomes:
On successful completion of this module, you will be able to:
- reformulate Newton's laws through variational principles;
- construct Lagrangians or Hamiltonians for mechanics problems in any coordinate system;
- solve the equations of motion for a wide variety of problems in mechanics;
- identify and exploit constants of the motion in solving dynamics problems;

Module content (syllabus):
The module is composed of the following sections:
1. Mechanics: Particles, Energy, Angular Momentum, Polar Coordinates.
2. Lagrangian Mechanics basic theory: Calculus of Variations, Hamilton's Principle, Constrained motion, Other Lagrangians, Generalised Coordinates, The Energy Function.
3. Lagrangian Mechanics further examples: The two body problem, Rigid Bodies (motion about a fixed axis).
4. Hamiltonian Mechanics: The Hamiltonian, Hamilton's Equations, Poisson Brackets.
5. Canonical Transformations: Canonical Transformations, Generating Functions, Liouville's Theorem.
6. Hamilton-Jacobi Theory: The Hamilton-Jacobi equation, Integrability, Non-integrability.

There will be 20 lectures and 7 problem classes

Assessment:
1 hour in-class midterm exercise: 20 percent
Main exam, 1.5 hours, term 3: 80 percent.

MATH50006 Lebesgue Measure and Integration

Brief description: Lebesgue's theory of measure and integration is a powerful extension of the Riemann integral introduced in first-year analysis. It is an essential tool in all aspects of analysis and its applications, including probability, stochastic processes and PDEs.

ECTS and term: 5 ECTS (Elective), term 2.

Learning Outcomes:
On successful completion of this module, you will be able to:
- explain key features of the construction of Lebesgue measure, and basic features of the measure as a function,
- explain the limitation of a measure, and measurable sets through algebras of open sets,
- define the measurability of functions in terms of measurable sets, and apply simple criteria to verify measurability,
- outline the Lebesgue integral of a function, and how it relates to the Riemann integral,
- work with spaces of functions as infinite dimensional vector spaces, and algebras, and discuss a variety of such spaces,
- use results on convergence of sequences in measurable functions spaces, and criterial to derive properties of limiting functions and their integrals,
- discuss the relationship between integrability and differentiation,
- explain and use the theorems of Fubini and Tonelli,
- discuss abstract measure theory, and the difficulties with construction of measures and integrals on general spaces.

Module content (syllabus):
An indicative list of sections and topics is:

Motivation: Drawbacks of the Riemann integral, Limits of functions, Length and area, The Fundamental Theorem of Calculus, Measures of sets in R^d,
Measure Theory: Abstract measure theory: Motivation, basic definitions of measure spaces and measures.
Lebesgue Measure in R^d: Volume of rectangles and cubes, The exterior measure, properties, Lebesgue measurable sets, countable additivity. Properties of the Lebesgue measure, Regularity, Invariance, σ-algebras and Borel sets, Non-measurable set.
Measurable functions: Definitions and equivalent formulations of measurability, Sums and products, compositions, limits of measurable functions, “Almost everywhere” properties. Approximation by simple functions and step functions, Egoroff’s and Lusin’s theorems.
Lebesgue integration: Definition using bounded functions on sets of finite measure, Riemann integrable functions are Lebesgue integrable, Integrable functions as a normed vector space, L_p(R^d), dense subsets, Completeness, Fatou’s Lemma, monotone convergence theorem, uniform integrability, Vitali’s Theorem, Fubini’s and Tonelli’s Theorems, statements and proofs.
Differentiation and Integration: Differentiation of the Integral, statement of Lebesgue differentiation theorem, Differentiation of functions, Functions of bounded variation, properties, characterisation, Bounded variation implies differentiable a.e.

Absolute continuity of measures, decomposition theorems by Jordan, Hahn and Lebesgue, Radon-Nikodym Theorem.

There will be 20 lectures and 7-8 problem classes, supplemented by office hours.

Assessment:
Mid Term test: 20 percent
Main exam, 1.5 hours, term 3: 80 percent.

MATH50009 Principles of Programming

Brief description: This module provides students with a core of programming skills beyond those encountered in the first-year module. It focusses on the object-oriented programming model, which is very widely used, and is taught in Python. This module will place students in a stronger position to take computational mathematics electives in third and fourth year, and to undertake individual projects with a computational component.

ECTS and term: 5 ECTS (Elective), term 1.

Learning Outcomes:
On successful completion of this module, you will be able to:

– write short programmes to specifications for mathematical problems;
– create and use classes and objects appropriately;
– understand the concepts of encapsulation and abstraction that underpin object-oriented programming;
– understand the concept of an abstract data type and its implementation;
– demonstrate familiarity with common abstract data types and key algorithms using them;
– demonstrate algorithmic reasoning skills and the ability to understand programme execution in an object-oriented context.
– demonstrate a developing capability to debug and test code.

Module content (syllabus):
An indicative list of sections and topics is:
Concepts in object-oriented programming
– Types, classes, and instances.
– Class and instance attributes and methods.
– Inheritance, composition, and encapsulation.
Object-oriented language constructs
– Class definition and initialisation.
– Special methods and overloading.
– Branching based on type, class, and inheritance. – Single dispatch functions.

Abstract data types and algorithms
– Concept of an abstract data type, and its implementation. – Stacks and queues; pushing and popping.
– Trees, graphs and traversal; topological sort.
– Computational complexity, big O notation.
Further programming skills
– Exceptions and exception handling. – Creating tests.
– Debugging using a debugger.
– Good programming style and PEP8.

Familiarity with material in the first-year Mathematics module Introduction to Computing will be assumed.

There will be 1 introductory lecture; 20 hours of computer labs and weekly GTA-led drop-in sessions.
Assessment will consist of two controlled assessment exercises (programming exams). One of these will be during the term and worth 20 percent; the other will be during the main exam period and worth 80 percent of the marks for the module. There will be weekly programming exercises which will not contribute marks to the module. However, in order to qualify to sit the first test, students must commit and push solutions that pass the autograder for the exercises for three weeks, by the deadline given for each week's work.

MATH50011 Statistical Modelling

Brief description: This module extends the statistical ideas introduced in the first year to more complex settings. Mathematically, the central concept is the linear model, a framework for statistical modelling that accommodates multiple predictor variables, continuous and categorial, in a unified way. There will be a focus on fitting models to real data from a variety of problem domains, using R to perform computations.

ECTS and term: 5 ECTS (Elective), term 2.

Learning Outcomes:
On successful completion of this module, you will be able to:

- Use the method of maximum likelihood to determine estimates of parameters.
- Determine the asymptotic sampling properties of maximum likelihood estimators.
- Use the likelihood ratio test to compare nested models.
- Evaluate estimators using their sampling properties, e.g. bias, variance and mean square error.
- Fit linear models using R.
- Assess the goodness of fit of statistical models.

- Carry out parametric inference.
- Use the analysis of variance to evaluate models with categorial variables.

Module content (syllabus):
An indicative list of sections and topics is:

- Estimation of parameters by maximum likelihood
- Likelihood ratio tests and their properties
- Properties of Estimators
- Cramer-Rao Lower Bound
- Linear models:
    - Estimation
    - Hypothesis testing / confidence intervals
    - Goodness of fit
    - Diagnostics
    - ANOVA

There will be 20 lectures and 7-8 problem classes, supplemented by office hours.

Assessment:
Computer-based data analysis task: 10 percent
Test: 10 percent
Main exam, 1.5 hours, term 3: 80 percent.

MATH50008 Partial Differential Equations in Action

Brief description: This module will provide an introduction to partial differential equations and the concept of modelling in applied mathematics. This course is intended to focus on the practical applications of partial differential equations. Along the way, students will be introduced to classical partial differential equations and as well as more advanced examples. Throughout the course, the student will learn basic techniques of resolutions of PDEs. Students will be introduced to basic numerical
methods to solve partial differential equations.

ECTS and term: 5 ECTS (Elective), term 2.

Learning Outcomes:
On successful completion of this module, you will be able to demonstrate knowledge and understanding by:
— knowing how to derive simple partial differential equations in several independent variables;

— demonstrating a good knowledge of the concepts and methods behind modelling in applied mathematics;
— using the proper analytical methods to provide solutions to simple PDEs;
— using partial differential equations to model phenomena in physical, life and social sciences;
— getting familiarized with elementary numerical methods for the resolution of PDEs.

Module content (syllabus):
The module is composed of the following sections:
1) Introduction:
models in applied mathematics and basic properties of PDEs;
2) First-order PDEs:
traffic flow equation, method of the characteristics, conservations laws, Burgers equation
3) Second-order PDEs:
classification of second-order PDEs, the classical trinity (Diffusion, Wave and Laplace equations), direct extensions (nonlinear diffusion equation), applications to musical instruments
4) Further advanced topics in physical, life and social sciences:
examples could include reaction-diffusion equation (wave propagation and pattern formation in biology), chemotaxis, swarming, population dynamics, financial markets and Black-Scholes equation, electrodynamics, fluid dynamics.
5) A short introduction to numerical methods.
There will be 20 lectures and 7-8 problem classes, supplemented by office hours.

Assessment:
Mid-term test: 20 percent
Main exam, 1.5 hours, term 3: 80 percent.