

IMPERIAL

DEPARTMENT OF MATHEMATICS

GUIDE TO MODULES

THIRD YEAR (BSc/MSci) 2026-2027

Notes and syllabus details of modules for Mathematics students in their Third Year.

For degree programmes:

G100, G103	MATHEMATICS (BSc, MSci)
G104	MATHEMATICS WITH A YEAR ABROAD (MSci)
G102	MATHEMATICS WITH MATHEMATICAL COMPUTATION
G125	MATHEMATICS (PURE MATHEMATICS)
G1F3	MATHEMATICS WITH APPLIED MATHEMATICS/MATHEMATICAL PHYSICS
G1G3	MATHEMATICS WITH STATISTICS
G1GH	MATHEMATICS WITH STATISTICS FOR FINANCE

Information on the Mathematics modules available to third-year students on the Joint Mathematics and Computing Programmes can be found in this guide.

These notes should be read in conjunction with your undergraduate student handbook and the programme specifications for your year. Some of the information may be subject to alteration.

Updated information will be posted in [Maths TIPS](#) on Canvas.

Dr Thibault Bertrand, Director of Undergraduate Studies, Mathematics
September 2026

THIRD YEAR PROGRAMME

In the 3rd year of your Mathematics programme, you choose from a wide range of elective modules. There are no compulsory modules, but if you wish to graduate with a specialist degree title, then you need to choose a certain number of modules related to the specialism.

You are advised to read these notes carefully and discuss your choice of options with your Personal Tutor. An 'Option Fair' will take place after exams in the Summer Term; this will include presentations about the modules on offer from the Pure, Statistics and Applied Mathematics and Mathematical Physics sections of the Department.

Each third year Mathematics module has a credit value of 7.5 ECTS and has a module code beginning with MATH6. In this guide, we will refer to these as MATH6 modules.

In your third year, you take modules for a total credit value of 60 or 62.5 ECTS. This will consist of eight 7.5 ECTS modules or seven 7.5 ECTS modules and two 5 ECTS modules. You must take **at least seven MATH6 modules**. The remaining ECTS can come from second year Mathematics electives not taken in year 2 (5 ECTS each), a further MATH6 module (7.5 ECTS), a module from another Department (subject to approval by DUGS) or certain CLCC or Business School BPES modules (5 ECTS). Where a very strong case can be made, the DUGS may allow some 3rd year students to take a small number of fourth year (level 7) Mathematics modules.

Examples

A total credit value of 60 or 62.5 ECTS allows you to take a combination of 7.5 and 5 ECTS modules in year 3. Some example of possible module combinations include:

- Eight MATH6 modules;
- Seven MATH6 modules and two MATH5 electives not taken previously;
- Seven MATH6 modules, one MATH5 elective not taken previously, one approved CLCC/ Business School module;
- Seven MATH6 modules, one MATH5 elective not taken previously, one Department of Computing module (5 ECTS);
- Seven MATH6 modules, one Department of Computing module, one approved CLCC/ Business School module.

At most one of your modules can be from CLCC or BPES; not one of each. You may take two computing modules.

CLCC / BPES

Registration for CLCC (Horizons) and Business School (BPES) modules opens in May and the Mathematics UG Office sends round an email detailing the registration process. At most one

of your modules can be from CLCC or BPES; not one of each. The other 5 ECTS should come from a year 2 maths option or a computing module.

CLCC (Horizons) Information

In Mathematics, any one Horizons module may be taken for credit in Year 3 with the following exceptions: Language modules and the Change Makers Independent Project may only be taken for extra-credit. Horizons options are available here:

<https://www.imperial.ac.uk/horizons/taking-a-module-for-credit/module-options-by-department/>

Enrolment will take place on the Horizons Enrolment system:

<https://www.imperial.ac.uk/horizons/enrolment/>

Module descriptions can be found here

<https://www.imperial.ac.uk/horizons/module-options/>

Some module descriptions are not currently available as they are being updated, but they will be ready before enrolment opens.

BPES information

You are only allowed to take one of the following

- Managing Innovation - BUSI60045 (Autumn)
- Finance & Financial Management - BUSI60043 (Spring)
- Corporate Finance Online – BUSI60040 (Spring)

for credit in Year 3. If you took one of these modules for i-Explore in year 2 you may not of course take it again for credit in year 3.

In addition, if you took Corporate Finance Online in year 2 for i-Explore, you may not register for Finance & Financial Management for credit next year, because of shared content between these modules. More information on BPES is available here:

<https://www.imperial.ac.uk/business-school/programmes/undergraduate-study/bpes-programme/>

Further remarks on non-MATH6 modules

Each year the Department of Computing makes a small number of its modules available to 'external students' (such as Mathematics students). These modules are usually advertised towards the start of the new academic year and the Mathematics UG Office will email you with details when available. Note that assessment for these modules is often at the end of term, when you will also have other coursework deadlines, so you will need to manage your workload carefully.

Students wishing to take a 4th year (level 7) module in their 3rd year require permission from the DUGS. Such requests should not normally be made until close to the start of the academic year and are only granted exceptionally and where there is a strong academic rationale.

Very occasionally the Department allows a student to take a module (other than those indicated above) from another Department. This requires discussion with, and approval from, the Mathematics DUGS and the agreement of the other Department. Such requests should not normally be made until close to the start of the academic year.

Computation of your year total

If you take 60 ECTS, consisting of 8 modules all of which are worth 7.5 ECTS, then these modules are weighted equally in the computation of your year 3 total.

If you take 62.5 ECTS, consisting of seven MATH6 modules and two 5 ECTS modules, then the required 52.5 ECTS of MATH6 modules will still count for 87.5 percent of the year total. The remaining 12.5 percent of the year total will come from the average of your marks for the two 5 ECTS modules. This ensures that the weighting on MATH6 modules in the year total is consistent across all students regardless of whether they are taking 60 or 62.5 ECTS.

Specialist BSc degree programmes

Choice of electives is dependent on Degree programme. Students on specialist programmes are required to take a certain number of electives related to their programme, as set out below. Students not meeting the requirements may be graduated with an alternative degree programme (such as BSc Mathematics (G100)).

For any of the specialist degree programmes, the BSc independent research project module, MATH60050, may be taken in place of one of the specialist modules, provided that the project is related to the specialism.

ADVICE ON THE CHOICE OF OPTIONS AND MODULE REGISTRATION

Students are advised to read these notes carefully and to discuss their option selections with their Personal Tutor.

It is anticipated that lecturers will give advice on suitable books at the start of each module. Students should contact the proposed lecturers if they desire any further details about module content in order to make their choice of course options.

Students are strongly advised to maintain a balanced teaching load across the Autumn and Spring Terms. They are encouraged, during the first two weeks of each term, to explore their elective options by attending introductory lectures and considering whether the content, teaching pattern, and style of delivery of a module are well suited to their interests and background. In both the Autumn and Spring Terms, module registrations must be completed by **Monday of Week 4 at 1.00 pm**. In the Autumn Term, students may register for between **three and six modules inclusive**. In addition, in each term students may take **no more than two modules for which 50% or more of the assessment takes place during term time** (see table below). At the point of Spring Term registration, students may amend their choices only by **dropping at most one Autumn Term module**; at this point, no further Autumn term module can be chosen, the only permitted change is to substitute Spring modules for Autumn modules. Any module registered for in the Autumn Term may be dropped by the Spring Term registration deadline, including modules for which 50% or more of the assessment takes place during term time. At the time of Spring term module registration, students will only be permitted to register for the number of modules necessary to complete their ECTS count. For instance, if a student takes five 7.5 ECTS modules in Term 1, they will be able to register for: (1) three 7.5 ECTS modules in Term 2 if they do not drop any Term 1 modules or (2) four 7.5 ECTS modules in Term 2 if they drop one Term 1 module.

Note: For modules outside the department of Mathematics, students must follow the module registration rules of the host department. In particular, you will be committed to the module once you register for the examination or at any earlier point specified by the host department. It is important to note that these modules may have earlier examinations and students are responsible to check that they are aware of these arrangements.

PERSONAL TUTORS AND OTHER ACADEMIC SUPPORT

In Year 3 your Personal Tutor will continue to offer general support for your studies including discussing your module choices and future options. You will meet your Personal Tutor at least twice a term in terms 1 and 2 during Meet Your Personal Tutor Weeks. It is your responsibility to arrange and to attend these meetings. You are welcome to also contact your Personal Tutor at different times of the term with queries. You should discuss with your Personal Tutor your academic progress and any concerns and questions. Personal Tutors are also there to support you with any wellbeing concerns you may have and to signpost you to Departmental or College support services if need be.

In Year 3, Academic support is provided through problems classes and office hours/ question and answer sessions/ discussion boards. The frequency of problems classes is generally less than in previous years: around 5 classes per module. Students are strongly encouraged to set up their own small study groups to support each other by working together on lecture notes and non-assessed problem sheets.

GRADUATION AND PROGRESSION REQUIREMENTS

Graduation Requirements for three-year degree programmes

In order to graduate, you must have passed all modules in the current level of study at first attempt, at resit or by an allowed (dependent on degree programme) compensated pass. In year 3 of BSc programmes, the Board of Examiners may award compensated passes in up to 15 ECTS of modules where the module mark is below the pass mark but at least 30%. The overall weighted average for the year must be at least 40%, including where a module(s) has been compensated, in order for you to graduate.

MSci Degrees (G103 and G104): The MSci is an undergraduate 'Masters' degree with a final year at the level of a taught postgraduate MSc programme. On successful completion, a degree title on the lines of 'Master in Science (incorporating Bachelor's level study)' is awarded. Very occasionally, circumstances may require the Department to graduate an MSci student with a BSc.

Degree Classification

The total of marks for examinations, assessed coursework, progress tests, assignments and projects, with the appropriate year weightings, is calculated and presented at the Board of Examiners' Meeting (normally held at the end of June) for consideration by the Academic Staff and External Examiners. Degree classifications are determined according to the criteria given in the borderline classification algorithm, which can be found on Maths Central. You may also wish to consult the programme specification:

<https://www.imperial.ac.uk/mathematics/undergraduate/course-structure-and-content/>

Criteria for Degree Classification at Borderlines

The discussion here refers to candidates at the first/upper-second borderline. Similar considerations apply to the other grade boundaries.

For all courses, a first is automatic for a programme total P of 69.5 or more. For candidates with $68 \leq P < 69.5$, the classification is determined by considering the uplift criteria as described below.

BSc courses

Classification is based on two uplift criteria: candidates with $68 \leq P < 69.5$ will be promoted to the first class if either of the criteria below is satisfied.

(a) Year 3 Total ≥ 69.5

(b) 30 ECTS in year 3 Mathematics modules at the higher classification (≥ 70 marks). External modules with a high mathematical content may be included, at the discretion of the board of examiners.

MSci courses

Classification is based on three uplift criteria: candidates with $68 \leq P < 69.5$ will be promoted to first class if any two of the criteria below are satisfied.

(a) Year 4 Total ≥ 69.5

(b) MSci project at the higher classification (≥ 70 marks)

(c) 22.5 ECTS in year 4 Mathematics electives at the higher classification (≥ 70 marks). External modules with a high mathematical content may be included, at the discretion of the board of examiners.

Mitigating Circumstances

Candidates with accepted mitigating circumstances for modules where no mitigation has been applied may be uplifted for $65 \leq P < 69.5$. In this case, the uplift criteria above may be modified to account for the circumstances of individual candidates. The rationale for any uplift should nevertheless make reference to the criteria above, e.g. noting the evidence for performance at the higher class in any components of the programme unaffected by mitigating circumstances. Care should be given to ensuring that mitigating circumstances are fully taken into account, without giving unfair advantage to the student, or subjecting them to more demanding requirements.

Progression Requirements for four-year programmes (G103 and G104)

G103: In order to progress to the next level of study, you must have passed all modules (equivalent to 60 ECTS) in the current level of study at first attempt, at resit or by a compensated pass. In year 3 of MSci programmes, the Board of Examiners may award compensated passes in up to 7.5 ECTS of modules where the module mark is below the pass mark but at least 30%. Additionally, the overall weighted average mark for the year, including where a module(s) has been compensated, must normally be at least 58%.

A student who has been permitted to remain on G103 in year 3 despite not having achieved a year total of 60% in year 2 is normally expected to obtain a year total higher than 60% in year 3.

G104: A student who is not permitted to remain on G104 for year 3 will be transferred to a BSc or MSci Mathematics degree.

Full progression information is available in the Programme Specifications:

<http://www.imperial.ac.uk/staff/tools-and-reference/quality-assurance-enhancement/programme-information/programme-specifications/>

MSci (G103, G104) - Overview of Year 4

In year 4 of the MSci you will normally take six fourth year Mathematics modules (MATH7 modules) and complete a year-long research project

All MATH6 modules are also available to 4th year and MSc students in an extended MATH7 format. Typically this will involve additional material ('mastery material') for self-study and an additional question on the main exam paper. There are also some modules which are only normally available to fourth year and MSc students only. The MATH7 taught modules are 7.5 ECTS and have a pass mark of 50%. **No student may take both the 3rd year and 4th year forms of a module.**

The MSci project is worth 15 ECTS. It is a supervised project in some area of Mathematics. The main aim is to develop a deep understanding of a particular area or topic by means of a supervised project in some area of mathematics. The project may be theoretical and/or computational and the topic for each student is chosen in consultation with the Department. A list of project supervisors will be released shortly before the start of the Autumn term of year 4. Students will then be asked to provide preferred research areas/supervisors at the start of the term; allocations will be finalized shortly afterwards. Work on the project should continue throughout all three terms of year 4 and the project is submitted shortly after the year 4 summer examinations.

MODULE ASSESSMENT AND EXAMINATIONS

Most MATH6 modules are examined by one written examination of 1.5 hours in length.

Most modules have some in-term assessment (coursework), which may take the form of an in-class test, problem sheet or a longer project. Typically, coursework makes up 10% of the overall module mark. Some modules have a more substantial coursework component (for example, 25%) and others are assessed entirely within the term. Details can be found in the tables below. Precise details of the number and nature of coursework assignments will be provided at the start of each module.

Students should bear in mind that modules assessed entirely within the term in which they are taught require considerable extra time-commitment during that term. You should note that, in

principle, 7.5 ECTS represents 187.5 hours of effort on a module and completing this in a single term is a substantial task. For this reason, students may take at most two such modules in a term.

The BSc individual research project is assessed by a written report as well as an oral examination.

Reassessment

Students who do not obtain a pass or compensated pass in examinations at the first attempt will be expected to attend resit examinations where appropriate. In determining whether a Mathematics module has been passed at a resit attempt, if a resit exam has been passed, then the Board of Examiners may discount any module marks obtained from coursework and award an overall pass mark for the module. This will apply to 1st and 2nd year modules and any other modules with a final exam worth at least 90 percent of the module mark. In cases where a student has not achieved the required amount of credit and no further resit attempts are permitted, the Board will graduate the student with an appropriate exit award, as detailed in the programme specifications and regulations.

Students who fail a module will normally be provided with a reassessment opportunity prior to the next academic year. At the discretion of the Board of Examiners, reassessment may be offered in an alternative format, such as an oral examination.

Students who have not achieved the required Passes by the beginning of the new academic year are required by College to spend a year out of attendance. During this time they are not considered College students. This may create a number of issues and hold visa implications.

Students who are required to take a year out due to failed examinations or who take an interruption are not normally permitted to resubmit any coursework previously submitted during their year out.

Resit examinations are for Pass credit only – a maximum mark of 40 percent will be credited. Once a Pass is achieved, no further attempts are permitted.

DEGREE CHANGES

Students are able to change between three-year mathematics degree programmes (or dropping down from a four-year to a three-year programme) by completing a Degree Change request and (if appropriate) ensuring that they comply with the requirements for any specialist coding module options.

Students wishing to move to the G103 programme (after the second year) must be able to comply with the Year 2 and 3 mark requirements for that programme.

International students who require a visa are advised to consult the International Student Support Office prior to making ANY degree change as you may be required to apply for a new visa (outside of the UK).

All degree transfer requests should normally be made by 31st of March in the year of the transfer, but transfers can in principle be made in any of years 1, 2 or 3.

If you change your degree coding and are registered on a module outside the Department, including BPES or CLCC, please ensure that you let the host Department know, so that you are not removed from their module lists.

THIRD YEAR MODULE LIST

Note that not all of the individual modules listed below are offered every session and the Department reserves the right to cancel a particular module if, for example, the number of students wishing to attend that module does not make it viable. Similarly, some modules are occasionally run as 'Reading Modules'.

The following notes relate to the tables on optional modules for Year 3 as below:

- The indicated lecturers are provisional; TBC indicates 'to be confirmed'.
- All MATH6 modules are equally weighted and have a credit value of 7.5 ECTS
- In the tables below, the *Exam-based Assessment %* indicates the weighting attached to the final exam in Summer, unless otherwise indicated. This would normally be a written exam but other forms of examination, such as oral exams are also possible (including for resit examinations). The *In-term Assessment %* indicates the weighting attached to any assessed work that is completed during the module. This may include in-class tests, projects, or problem sets to be turned in.

APPLIED MATHEMATICS AND MATHEMATICAL PHYSICS

College Module Code	Module Title	Term	Lecturer	Exam-based Assessment %	In-term Assessment %
MATH60001	Fluid Dynamics 1	1	Prof X. Wu	90	10

MATH60002	Fluid Dynamics 2	2	Prof J. Mestel	90	10
MATH60003	Introduction to Geophysical Fluid Dynamics	2	Dr P. Berloff	90	10
MATH60004	Asymptotic Methods	1	Dr O. Schnitzer	90	10
MATH60005	Optimisation	1	Dr D. Kalise	90	10
MATH60006	Applied Complex Analysis	2	Dr M. Vona	90	10
MATH60007	Dynamics of Learning and Iterated Games	1	Prof S. van Strien	-	100
MATH60008	Dynamical Systems	1	Prof D. Turaev	90	10
MATH60009	Bifurcation Theory	2	Prof D. Turaev	90	10
MATH60010	Geometric Mechanics	2	Prof D. Holm	90	10
MATH60014	Mathematical Biology	1	Dr B. Bassols-Cornudella	90	10
MATH60015	Quantum Mechanics I	1	Dr E-M Graefe	90	10
MATH60016	Special Relativity and Electromagnetism	1	Dr G. Pruessner	90	10
MATH60017	Tensor Calculus and General Relativity	2	Dr C. Ford	90	10
MATH60018	Quantum Mechanics II	2	Dr R. Barnett	90	10
MATH60019	Theory of Partial Differential Equations	1	Dr M. Bagnara	90	10
MATH60021	Advanced Partial Differential Equations 2	2	Prof R-O Radu	90	10
MATH60022	Finite Elements: Numerical Analysis and Implementation	2	Prof C. Cotter & Prof D. Ham	50	50
MATH60023	Computational Dynamical Systems	1	Dr E. Keaveny	50	50
MATH60024	Computational Linear Algebra	1	Dr A. Broms	50	50

MATH60025	Computational Partial Differential Equations	2	Dr S. Mughal	50	50
MATH60026	Methods for Data Science	2	Dr B. Bravi	-	100
MATH60054	Introduction to Stochastic Differential Equations and Diffusion Processes	1	Prof P. Bressloff	90	10
MATH60135	Advanced Partial Differential Equations 1	1	Prof G. Pavliotis	90	10
MATH60141	Introduction to Game Theory	1	Dr S. Brzezicki	90	10
MATH60146	Advanced Dynamical Systems	2	Prof J. Lamb	-	100
MATH60147	Statistical Mechanics	1	Dr T. Bertrand	50	50

PURE MATHEMATICS

Note that some Analysis modules are taught by members of the Analysis, Probability and PDEs group, which sits within the Applied section of the department for organisation purposes – please see the Applied Mathematics and Mathematical Physics table.

College Module Code	Module Title	Term	Lecturer	Exam-based Assessment %	In-term Assessment %
MATH60028	Probability Theory 1	1	Dr T. Rosati	90	10
MATH60029	Functional Analysis	1	Prof M. Taylor	90	10
MATH60030	Fourier Analysis and Theory of Distributions	2	Dr I. Krasovsky	90	10
MATH60031	Probability Theory 2	2	Dr B. Dagallier	90	10
MATH60032	Geometry of Curves and Surfaces	2	Dr M. Schmalian	90	10
MATH60033	Geometry I: Algebraic Curves	1	Prof A. Caraiani	90	10
MATH60034	Geometry II: Algebraic Topology	2	Dr S. Sivek	90	10

MATH60035	Algebra III	1	Prof A. Corti	90	10
MATH60036	Group Theory	1	Prof M. Liebeck	90	10
MATH60037	Galois Theory	1	Prof A. Skorobogatov	90	10
MATH60038	Graph Theory	1	Dr M. Zordan	90	10
MATH60039	Group Representation Theory	2	Dr A. Bottini	90	10
MATH60040	Formalising Mathematics	1	Dr T. Browning	-	100
MATH60041	Number Theory	1	Prof T. Gee	90	10
MATH60042	Algebraic Number Theory	2	Dr D. Kim	90	10
MATH60132	Mathematical Logic	1	Dr J. Hua	90	10
MATH60140	Geometric Complex Analysis	2	Dr A. Jove	90	10

STATISTICS

College Module Code	Module Title	Term	Lecturer	Exam-based Assessment %	In-term Assessment %
MATH60043*	Foundations of Modelling and Inference	1	Prof H. Battey	100	-
MATH60044*	Applied Statistical Inference	1	Dr C. Hallsworth	75	25
MATH60045*	Applied Probability	1	Dr R. Ryder	90	10
MATH60046*	Time Series Analysis	2	Prof E. Cohen	100	-
MATH60047*	Stochastic Simulation	2	Dr O. Ratmann	100	-
MATH60048*	Survival Models	1	Dr R. Nuermairaiti	90	10
MATH60049*	Introduction to Statistical Learning	2	Prof G. Nason	100	-
MATH60139*	Spatial Statistics	2	Dr A. Sykulski	90	10

MATHEMATICAL FINANCE

College Module Code	Module Title	Term	Lecturer	Exam-based Assessment %	In-term Assessment %
MATH60012	Mathematical Finance: An Introduction to Option Pricing	2	Dr P. Siorpaes	90	10
MATH60130	Stochastic Differential Equations in Financial Modelling	1	Prof D. Brigo	100	-

CROSS-MATHEMATICS

College Module Code	Module Title	Term	Lecturer	Exam-based Assessment %	In-term Assessment %
MATH60150	Communicating Mathematics	2	Dr C. Webb	-	100

PROJECT

College Module Code	Module Title	Term	Lecturer	Exam-based Assessment %	In-term Assessment %
MATH60050	Research Project in Mathematics (BSc Project)	2, 3	Dr C. Ford	-	100

THIRD YEAR MODULE STREAMS

Owing to the large number of electives offered to fourth-year students, it is not possible to guarantee a timetable entirely free from clashes across all available modules. In order to support students in making their module choices, the electives have been organised into **thematic streams**, each comprising modules with natural affinities and academic connections. In drawing up the timetable for fourth-year electives, every effort is made to ensure that modules **within a given stream** are scheduled without overlap. However, it is not possible to provide the same assurance for modules chosen **across different streams**. These streams are intended to serve as an indicative guide to selection rather than as a constraint, and students remain free to choose from the full range of electives on offer, subject to any relevant academic requirements.

Stream 1 – FLUID DYNAMICS

MATH60001 Fluid Dynamics 1, MATH60002 Fluid Dynamics 2, MATH60003 Introduction to Geophysical Fluid Dynamics, MATH60004 Asymptotic Methods, MATH60006 Applied Complex Analysis, MATH60025 Computational PDEs

Stream 2 – MATHEMATICAL PHYSICS

MATH60010 Geometric Mechanics, MATH60015 Quantum Mechanics I, MATH60016 Special Relativity & Electromagnetism, MATH60017 Tensor Calculus & General Relativity, MATH60018 Quantum Mechanics II, MATH60054 An Introduction to Stochastic Differential Equations and Diffusion Processes, MATH60147 Statistical Mechanics

Stream 3 – DYNAMICAL SYSTEMS & CHAOS

MATH60007 Dynamics of Learning and Iterated Games, MATH60008 Dynamical Systems, MATH60009 Bifurcation Theory, MATH60014 Mathematical Biology, MATH60023 Computational Dynamical Systems, MATH60141 Introduction to Game Theory

Stream 4 – DIFFERENTIAL EQUATIONS & ANALYSIS

MATH60004 Asymptotic Methods, MATH60006 Applied Complex Analysis, MATH60014 Mathematical Biology, MATH60019 Theory of Partial Differential Equations, MATH60022 Finite Elements: Numerical Analysis & Implementation, MATH60025 Computational PDEs, MATH60029 Functional Analysis, MATH60030 Fourier Analysis and Theory of Distributions, MATH60054 An Introduction to Stochastic Differential Equations and Diffusion Processes, MATH60021 Advanced PDEs 2, MATH60135 Advanced PDEs 1

Stream 5 – COMPUTATIONAL MATHEMATICS

MATH60005 Optimisation, MATH60022 Finite Elements: Numerical Analysis & Implementation, MATH60023 Computational Dynamical Systems, MATH60024 Computational Linear Algebra, MATH60025 Computational PDEs, MATH60040 Formalising Mathematics, MATH60047 Stochastic Simulation

Stream 6 – DATA SCIENCE, STATISTICS & MACHINE LEARNING

MATH60005 Optimisation, MATH60007 Dynamics of Learning and Iterated Games, MATH60026 Methods for Data Science, MATH60043 Foundations of Modelling and Inference, MATH60044 Applied Statistical Inference, MATH60046 Time Series Analysis, MATH60049 Introduction to Statistical Learning, MATH60139 Spatial Statistics

Stream 7 – PROBABILITY & STOCHASTIC PROCESSES

MATH60028 Probability Theory I, MATH60030 Fourier Analysis and Theory of Distributions, MATH60031 Probability Theory 2, MATH60045 Applied Probability, MATH60047 Stochastic Simulation, MATH60054 An Introduction to Stochastic Differential Equations and Diffusion Processes, MATH60130 Stochastic Differential Equations in Financial Modelling

Stream 8 – FINANCIAL MATHEMATICAL

MATH60005 Optimisation, MATH60012 Mathematical Finance: An Introduction to Option Pricing, MATH60028 Probability Theory I, MATH60031 Probability Theory 2, MATH60043 Foundations of Modelling and Inference, MATH60044 Applied Statistical Inference, MATH60045 Applied Probability, MATH60046 Time Series Analysis, MATH60047 Stochastic Simulation, MATH60048 Survival Models, MATH60054 An Introduction to Stochastic Differential Equations and Diffusion Processes, MATH60130 Stochastic Differential Equations in Financial Modelling

Stream 9 – ALGEBRA & NUMBER THEORY

MATH60035 Algebra III, MATH60036 Group Theory, MATH60037 Galois Theory, MATH60039 Representation Theory, MATH60041 Number Theory, MATH60042 Algebraic Number Theory

Stream 10 – GEOMETRY & TOPOLOGY

MATH60010 Geometric Mechanics, MATH60032 Geometry of Curves and Surfaces, MATH60033 Geometry I: Algebraic Curves, MATH60034 Geometry II: Algebraic Topology, MATH60038 Graph Theory, MATH60140 Geometric Complex Analysis

Stream 11 – DISCRETE MATHEMATICS, LOGIC & FOUNDATIONS

MATH60038 Graph Theory, MATH60040 Formalising Mathematics, MATH60041 Number Theory, MATH60132 Mathematical Logic

PROGRAMME SPECIFIC MODULE REQUIREMENTS

Choice of electives is dependent on Degree programme. Students on specialist programmes are required to take a certain number of electives related to their programme, as set out in this section.

Students not meeting the requirements may be graduated with an alternative degree programme (such as G100 - BSc in Mathematics).

Students on the following specialist degree codings must take:

G102—Mathematics with Mathematical Computation: 3 modules amongst the following: MATH60022, MATH60024, MATH60025, MATH60026.

G125—Mathematics with Pure Mathematics: 5 modules amongst the following: MATH60028, MATH60029, MATH60030, MATH60031, MATH60032, MATH60033, MATH60034, MATH60035, MATH60036, MATH60037, MATH60038, MATH60039, MATH60040, MATH60041, MATH60042, MATH60132, MATH60140.

G1F3—Mathematics with Applied Mathematics/Mathematical Physics: 5 modules amongst the following:

MATH60001, MATH60002, MATH60003, MATH60004, MATH60005, MATH60006, MATH60007, MATH60008, MATH60009, MATH60010, MATH60014, MATH60015, MATH60016, MATH60017, MATH60018, MATH60019, MATH60022, MATH60023, MATH60024, MATH60025, MATH60026, MATH60054, MATH60135, MATH60141, MATH60147.

G1G3—Mathematics with Statistics: 5 modules amongst the following: MATH60026, MATH60043, MATH60044, MATH60045, MATH60046, MATH60047, MATH60048, MATH60049, MATH60139.

G1GH—Mathematics with Statistics: 5 modules amongst the following: MATH60012, MATH60043, MATH60044, MATH60045, MATH60046, MATH60047, MATH60048, MATH60049, MATH60130, MATH60139.

For any of the specialist degree programmes, the BSc independent research project module, MATH60050, may be taken in place of one of the specialist modules, provided that the project is related to the specialism.

THIRD YEAR MATHEMATICS SYLLABUSES

Modules are listed below in the order of increasing College Module Code. Note that the assessment strategies listed below are indicative and subject to changes, you should refer to the pattern of assessment provided at the start of term.

MATH60001 Fluid Dynamics I

Brief description

Fluid dynamics investigates motions of both liquids and gases. Being a major branch of continuum mechanics, it does not deal with individual molecules, but with an 'averaged' motion of the medium (i.e. collections of molecules). The aim is to predict the velocity, pressure and temperature fields in flows arising in nature and engineering applications. In this module, the equations governing fluid flows are derived by applying fundamental physical laws to the continuum. This is followed by descriptions of various techniques to simplify and solve the equations with the purpose of describing the motion of fluids under different conditions.

Learning outcomes

On successful completion of this module you will be able to

- state the underlying assumptions of the continuum hypothesis;
- compare and contrast the different frameworks that can be used to describe fluid motion and to identify the connections between them;
- derive exact solutions of the Navier-Stokes equations and justify the physical and mathematical assumptions made in obtaining them;
- perform simplifications arising under the assumption of inviscid flow which permit the integration of the Euler equations, leading to results such as Bernoulli's equation and Kelvin's circulation theorem;
- demonstrate a sound understanding of the method of conformal mappings and be able to use this method to analyse various two-dimensional inviscid flows;
- choose the appropriate conformal mapping to solve inviscid flow problems in complicated geometries;
- predict the shape of the flow streamlines for such problems.

Indicative module content

The module is composed of the following sections:

- Introduction: The continuum hypothesis. Knudsen number. The notion of fluid particle. Kinematics of the flow field. Lagrangian and Eulerian frameworks. Streamlines and pathlines. Strain rate tensor. Vorticity and circulation. Helmholtz's first theorem. Streamfunction.
- Governing Equations: Continuity equation. Stress tensor and symmetry, Constitutive relation. The Navier-Stokes equations.
- Exact Solutions of the Navier-Stokes Equations: Couette and Poiseuille flows. The flow between two coaxial cylinders. The flow over an impulsively started plate. Diffusion of a potential vortex.
- Inviscid Flow Theory: Integrals of motion. Kelvin's circulation theorem. Potential flows. Bernoulli's equation. Cauchy-Bernoulli integral for unsteady flows. Two-dimensional flows. Complex potential. Vortex, source, dipole and the flow past a circular cylinder. Adjoint mass. Conformal mapping. Joukovskii transformation. Flows

past aerofoils. Lift force. The theory of separated flows. Kirchhoff and Chaplygin models.

Assessment strategy

Final Exam (90%)

Take-home coursework (10%)

MATH60002 Fluid Dynamics II

Brief description

In this module, we deal with a wide class of realistic problems by seeking asymptotic solutions of the governing Navier-Stokes equations in various limits. We shall start with the “slow, small or sticky” case, when the Reynolds number is low and we obtain the linear Stokes equations. Then we consider the lubrication limit, and show how a thin layer of fluid is able to exert enormous pressures and prevent moving solid bodies from touching. Next we shall consider the “fast and vast” limit of high Reynolds number, which is characteristic of most flows we encounter in everyday life. In the final part of the module we consider a mixture of advanced topics, including flight, bio-fluid-dynamics and an introduction to flow stability.

Learning outcomes

On successful completion of this module you will be able to:

- simplify and solve the governing Navier-Stokes equations in situations where there is a short lengthscale in one of the coordinate directions;
- apply the general properties of low Reynolds number flows to predict the drag on slow-moving bodies, like a solid sphere or spherical bubble, and appreciate the causes of the ‘Stokes paradox’;
- analyse lubrication-like flows in thin layers;
- derive the boundary-layer equations and identify self-similar solutions for flows at large Reynolds number;
- determine stability criteria for various fundamental flows;
- model animal locomotion at low and high Reynolds numbers;
- interpret results from advanced fluid mechanics textbooks and research papers;
- independently appraise and evaluate some advanced topics in viscous fluid mechanics.

Indicative module content

The module is composed of the following sections:

I – Low-Reynolds-number flows

Dynamic Similarity. Properties of the Stokes equations. Uniqueness and minimal dissipation theorems. The analysis of the flow past a solid sphere and spherical bubble. Stokes paradox.

II – Lubrication Theory

Derivation of Reynolds’ lubrication equation and examples. Hele-Shaw and thin film flows.

III – High-Reynolds-number flows; Boundary-layer theory

The notion of singular perturbations. Derivation of boundary-layer equations. Blasius flow, Falkner-Skan solutions and applications. Von Mises variables and their application to periodic boundary layers. Prandtl-Batchelor Theorem for flows with closed streamlines.

IV – Introduction to hydrodynamic stability

Importance of stability. Rayleigh-Taylor and Kelvin-Helmholtz instabilities. Circular flow stability criterion.

V – Swimming and Flight; Animal locomotion

Scallop theorem. Resistive Force Theory. Introduction to 3D-aerofoil theory. Flight strategies.

VI - Advanced Topics

Current research in areas such as convection and magnetohydrodynamics.

Assessment strategy

Final Exam (90%)

Take-home coursework (10%)

MATH60003 Introduction to Geophysical Fluid Dynamics

Brief description

This is an advanced-level fluid-dynamics course with geophysical flavours. The lectures target upper-level undergraduate and graduate students interested in the mathematics of planet Earth, and in the variety of motions and phenomena occurring in planetary atmospheres and oceans. The lectures provide a mix of theory and applications.

Learning outcomes

On successful completion of this module you will be able to:

- demonstrate a deep understanding of the foundations of geophysical fluid dynamics;
- model a broad range of natural phenomena associated with the atmosphere and ocean;
- appreciate the main concepts and terminology used in the field;
- derive the boundary layer equations for flow in a rotating frame and justify the relative importance of various terms in the equations of motion;
- describe, select appropriately and apply a range of methods and techniques for solving practical problems;
- independently appraise an advanced topic in geophysical fluid dynamics;
- evaluate results from research papers in the field of geophysical fluid dynamics.

Indicative module content

The module is composed of the following sections:

- I. Introduction and basics;
- II. Governing equations (continuity of mass, material tracer, momentum equations, equation of state, thermodynamic equation, spherical coordinates, basic approximations);
- III. Geostrophic dynamics (shallow-water model, potential vorticity conservation law, Rossby number expansion, geostrophic and hydrostatic balances, ageostrophic continuity, vorticity equation);
- IV. Quasigeostrophic theory (two-layer model, potential vorticity conservation, continuous stratification, planetary geostrophy);
- V. Ekman layers (boundary-layer analysis, Ekman pumping);
- VI. Rossby waves (general properties of waves, physical mechanism, energetics, reflections, mean-flow effect, two-layer and continuously stratified models);
- VII. Hydrodynamic instabilities (barotropic and baroclinic instabilities, necessary conditions, physical mechanisms, energy conversions, Eady and Phillips models);
- VIII. Ageostrophic motions (linearized shallow-water model, Poincare and Kelvin waves, equatorial waves, ENSO “delayed oscillator”, geostrophic adjustment, deep-water and stratified gravity waves);
- IX. Transport phenomena (Stokes drift, turbulent diffusion);
- X. Nonlinear dynamics and wave-mean flow interactions (closure problem and eddy parameterization, triad interactions, Reynolds decomposition, integrals of motion, enstrophy equations, classical 3D turbulence, 2D turbulence, transformed Eulerian mean, Eliassen-Palm flux).

Assessment strategy

Final Exam (90%)

Take-home coursework (10%)

MATH60004 Asymptotic Methods

Brief description

This advanced course presents a systematical introduction to asymptotic methods, which form one of the cornerstones of modern applied mathematics. The foundation of asymptotic approximations is laid down first. The key ideas and techniques for deriving asymptotic representations of integrals, and for constructing appropriate solutions to differential equations will be explained. The techniques introduced find wide applications in engineering and natural sciences.

Learning outcomes

On successful completion of this module, you will be able to:

- appreciate the foundation upon which asymptotic approximations are based;
- describe a variety of asymptotic methods and for each method acquire a thorough understanding of the key ideas involved and their mathematical nature;
- demonstrate basic skills in applying each of these methods to solve classical problems;
- combine, modify and extend methods to unfamiliar problems, such as those that emerge from research topics or practical applications;
- outline how asymptotic methods can in principle be applied to a wide variety of problems;
- interpret results from advanced textbooks and research papers on asymptotic methods;
- construct advanced solution techniques by selecting an appropriate combination of different asymptotic methods to solve higher-dimensional problems.

Indicative module content

- I. Asymptotic approximations (fundamentals)
Order notation. Diverging series, asymptotic expansions. Parameter expansions, overlap regions, distinguished limits and uniform approximations. Stokes phenomenon.
- II. Introduction to perturbation methods
Asymptotic solution of algebraic equations with a small parameter. Regular vs. singular perturbations. Method of dominant balance. Local analysis of ordinary differential equations.
- III. Asymptotic analysis of integrals
Method of integration by parts. Integrals of Laplace type: Laplace's method, Watson's Lemma. Integrals of Fourier type: method of stationary phase. Integral in the complex plane: method of steepest descent. Method of splitting the range of integration.
- IV. Matched asymptotic expansion
Inner and outer expansions, matching principles, notions of 'boundary layer' and interior layer. Composite approximation. Application to relaxation oscillations.
- V. Methods of multiple scales
WKB approximations including turning-point problems and eigenvalue quantisation. Secular terms and solvability conditions. Poincare-Lindstedt method for periodic solutions. Multiscale method for quasi-periodic solutions. Application to weakly perturbed oscillators, nonlinear resonance, parametric resonance.
- VI. A selection of topics from the following: Stokes phenomenon, hyperasymptotics, expansions involving logarithmic terms, homogenisation.

Assessment strategy

Final Exam (90%)

Take-home coursework (10%)

MATH60005 Optimisation

Brief description

This module is an introduction to the theory and practice of mathematical optimization and its many applications in mathematics, data science, and engineering. The module aims at endowing students with the necessary mathematical background and a thorough methodological toolbox to formulate optimization problems and developing an algorithmic approach to its solution. The module is structured into five parts: (i) formulation and classification of problems; (ii) unconstrained optimization; (iii) stochastic and nature-inspired optimization; (iv) convex optimization; (v) introduction to optimal control and dynamic optimization. The assessed coursework for this module involves a series of computational tasks.

Learning outcomes

On successful completion of this module you will be able to

- formulate a mathematical optimization problem by identifying a suitable objective and constraints;
- identify the mathematical structure of an optimization problem and, based on this classification, choose an appropriate methodological approach;
- develop a mathematical and computational appreciation of convexity as a fundamental feature in optimization;
- implement different computational optimization algorithms such as gradient descent and related variants;
- analyse the results of a computational optimization method in terms of optimality guarantees, sensitivities, and performance;
- interpret the role played by optimization in its application to computational data science;
- design optimal control approaches relevant to tackling large-scale nonlinear problems.

Indicative module content

1. Mathematical preliminaries
2. Unconstrained optimization
3. Gradient descent methods
4. Linear and non-linear least squares problems
5. Stochastic gradient descent
6. Nature-inspired optimization
7. Convex sets and functions
8. Convex optimization problems and stationarity
9. KKT conditions
10. Duality
11. Introduction to dynamic optimization and optimal control.

This final topic is linked to the Mastery Material for MSci students which will involve the study of some of the following solution techniques: shooting and multiple shooting methods; the reduced gradient approach; two-point boundary value solvers for optimal control; dynamic Programming and the Hamilton-Jacobi PDE; the linear-quadratic regulator and the Riccati equation.

This will be examined by way of an extra question on the May examination paper.

Assessment strategy

Final Exam (90%)

Computational group take-home coursework (10%)

MATH60006 Applied Complex Analysis

Brief description

The aim of this module is to learn tools and techniques from complex analysis and the theory of orthogonal polynomials that can be used in mathematical physics. The course will focus on mathematical techniques, though will also discuss relevant physical applications, such as electrostatic potential theory. The course incorporates computational techniques in the lectures.

Learning outcomes

On successful completion of this module, you will be able to:

- apply the technique of contour deformation for calculating integrals;
- appreciate the connection that exists between computational tools such as quadrature and orthogonal polynomials and complex analysis;
- evaluate singular integral equations with Cauchy and logarithmic kernels;
- use the Wiener-Hopf method to solve a class of integral equations;
- use Schwartz-Christoffel maps and apply them in a computational setting;
- interpret results from advanced textbooks and research papers;
- independently appraise and evaluate an advanced topic in complex analysis.

Indicative module content

This module covers the following topics:

- Revision of complex analysis: complex integration, Cauchy's theorem and residue calculus;
- Singular integrals: Cauchy, Hilbert, and log kernel transforms;
- Potential theory: Laplace's equation, electrostatic potentials, distribution of charges in a well;
- Riemann-Hilbert problems: Plemelj formulae, additive and multiplicative Riemann-Hilbert problems;
- Orthogonal polynomials: recurrence relationships, solving differential equations, calculating singular integrals;
- Integral equations: integral equations on the whole and half line, Fourier transforms, Laplace transforms;
- Wiener-Hopf method: direct solution, solution via Riemann-Hilbert methods;
- Singularities of differential equations: analyticity of solutions, regular singular points, hypergeometric functions.

Assessment strategy

Final Exam (90%)

Project (10%)

MATH60007 Dynamics of Learning and Iterated Games

Brief description

Recently there has been considerable interest in modelling learning. The settings to which these models are applied is wide-ranging. Examples include optimization of strategies of populations in ecology and biology, iterated strategies of people in a competitive environment and learning models used by technology companies such as Google.

This module is aimed at discussing a number of such models in which learning evolves over time and which have a game theoretic background. The module will use tools from the theory of dynamical systems and will aim to be rigorous. Topics will include replicator systems, best response dynamics and fictitious games, reinforcement learning and no-regret learning.

Learning outcomes

On successful completion of this module, you will be able to:

- analyse 2D replicator systems for one and two player games;
- work comfortably with the notions of Nash, Correlated Equilibrium, Cournot Equilibrium and Evolutionarily Stable Strategies;
- explain the notion of reciprocity in relation to Iterated Prisoner Dilemma games;
- appreciate the connection between Reinforcement Learning and replicator systems;
- outline the idea behind no regret learning models and the Blackwell approachability theorem;
- derive the proofs behind the methods that are used in the final project;
- appraise and interpret results from advanced textbooks and research papers.

Indicative module content

The module will cover the following topics:

- Replicator systems;
- Rock-paper-scissor games;
- Iterated prisoner dilemma games;
- Best response dynamics;
- Two player games;
- Fictitious games as a learning model;
- Reinforcement learning;
- No regret learning.

Assessment strategy

Take-home coursework (15%)

Project examined by report (45%) and oral examination (40%)

MATH60008 Dynamical Systems

Brief description

The theory of Dynamical Systems is an important area of mathematics which aims at describing objects whose state changes over time. For instance, the solar system comprising the sun and all planets is a dynamical system, and dynamical systems can be found in many other areas such as finance, physics, biology and social sciences. This course provides a rigorous treatment of the foundations of discrete-time dynamical systems.

Learning outcomes

On successful completion of this module, you will be able to:

- demonstrate a familiarity with the basic concepts of topological dynamics;
- provide an outline of the ergodic theory of dynamical systems;
- appreciate the concept of symbolic dynamics through which topological and probabilistic dynamical properties can be understood;
- demonstrate an understanding of precise mathematical characterisations of chaotic dynamics;
- apply the above context in a number of one-dimensional settings, in particular in the context of piecewise affine expanding maps;
- independently appraise and evaluate advanced topological and probabilistic dynamical properties, beyond the foundations;
- independently develop and interpret examples in two and higher dimensions.

Indicative module content

The module covers the following topics:

- Introduction: orbits, periodic orbits and their local stability;
- Topological dynamics: invariant sets and limit sets, coding and sequence spaces, topological conjugacy, transitivity and mixing;
- Chaotic dynamics: sensitive dependence, topological entropy, topological Markov chains;
- Ergodic theory: sigma-algebras and measures, invariant measures, Poincaré recurrence, ergodicity and Birkhoff's Ergodic Theorem, Markov measures and metric entropy;
- Additional reading material in line with M4 objectives.

Assessment strategy

Final Exam (90%)

In-class test (10%)

MATH60031 Bifurcation Theory

Brief description

This module serves as an introduction to bifurcation theory, concerning the study of how the behaviour of dynamical systems such as ODEs and maps changes when parameters are varied. The goal is to acquaint the students with the foundations of the theory, its main discoveries and the universal methods behind this theory that extend beyond its remit.

Learning outcomes

On successful completion of this module, you will be able to:

- exploit basic dimension reduction methods (invariant manifold and invariant foliations);
- apply the method of normal forms;
- demonstrate a sound knowledge of the basics of stability theory;
- appreciate the role of control parameters and to construct bifurcation diagrams;
- describe the mathematical framework associated with classical local and global bifurcations;
- interpret results from advanced textbooks and research papers on bifurcation theory;
- independently appraise and evaluate the transition from periodic to quasiperiodic regimes and to chaos via destruction of quasiperiodicity.

Indicative module content

The following topics will be covered:

- 1) Bifurcations on a line and on a plane;
- 2) Centre manifold theorem; local bifurcations of equilibrium states;
- 3) Local bifurcations of periodic orbits – folds and cusps;
- 4) Homoclinic loops: cases with simple dynamics, Shilnikov chaos, Lorenz attractor;
- 5) Saddle-node bifurcations: destruction of a torus, intermittency, blue-sky catastrophe;
- 6) Routes to chaos and homoclinic tangency.

Assessment strategy

- Final Exam (90%)
- In-class test (10%)

MATH60010 Geometric Mechanics

Brief description

The course surveys a small section of the road to geometric mechanics (GM), by treating several examples in classical mechanics all in the same geometric framework. This framework is based on linear algebra, transformation theory, differential equations, variational calculus, and Lie group invariant variational principles, and their actions on functions and differential forms defined on manifolds.

Learning outcomes

On successful completion of this module, you will be able to:

- describe motion on smooth manifolds using the following definitions: tangent bundle (TM), cotangent bundle (T^*M , phase space), kinetic energy, Lagrangian, variational derivative, Hamilton's variational principle (LagHvp), Euler-Lagrange equations (EL eqns);
- derive and solve Euler-Lagrange equations for a variety of classical problems, including: oscillators, plane pendulum, charged particle motion, motion on a sphere, spherical pendulum, rotating rigid body, geodesic motion on the Lobachevsky half-plane, equations of motion for Newton's law of gravitation;
- prove and apply Noether's theorem about how Lie group symmetries of Hamilton's principle imply conservation laws in the GM framework for transformations of the matrix Lie groups of rotations, translations and scaling (in the context of the example problems);
- link fibre derivative, Legendre transformation, Hamilton's equation, Poisson brackets, and Noether quantities as momentum maps for Lie group actions;
- apply the Geometric Mechanics framework to rigid body motion and Newton's solution for the Kepler problem of planetary motion;
- demonstrate familiarity with the transformation theory for smooth invertible maps and the calculus of differential forms, including exterior derivative, wedge product, contraction with vector fields, and Cartan's formula for Lie derivative through worked exercises;
- relate Lie-Poisson brackets to coadjoint actions of matrix Lie groups on matrix Lie algebras;
- derive the equations for ideal fluid dynamics from Hamilton's principle with symmetry under smooth invertible maps.

Indicative module content

This module explains the principles of mechanics as a unified geometric framework, by using Noether's theorem about Lie-group invariant variational principles to focus on Sophus Lie's momentum map concept as the key for applying Poincaré's geometric approach. The only prerequisites are linear algebra, vector calculus and some familiarity with the Euler-Lagrange variational principles and canonical Poisson brackets in classical mechanics at the beginning undergraduate level. The course presents increasingly advanced ideas as a series of recurring thematic examples and worked exercises for classic problems of oscillation and rotation. For example, it treats the geometry of rigid-body motion from the series of viewpoints of Newton, Lagrange, Hamilton and Poincaré. This unfolding series of treatments of rigid body motion from several viewpoints reveals the framework of geometric mechanics and sets the stage for the introduction of the flows of Hamiltonian vector fields and their Lie-derivative actions on differential forms.

After learning a few useful methods of vector calculus in the language of differential forms, the student recognises the geometric structure of ideal fluid dynamics as something which has become familiar and natural; namely, geodesic motion for the kinetic energy metric on

the manifold of smooth invertible maps (L2 norm of the Eulerian fluid velocity). Thus, the course reveals a geometric framework for variational dynamics with symmetry and develops the ideas in the framework from finite to infinite dimensions.

Assessment strategy

Final Exam (90%)

3 x Take-home coursework (10%)

MATH60012 Mathematical Finance: An Introduction to Option Pricing

Brief description

The mathematical modelling of derivatives securities, initiated by Bachelier in 1900 and developed by Black, Scholes and Merton in the 1970s, focuses on the pricing and hedging of options, futures and other derivatives, using a probabilistic representation of market uncertainty. This module is a mathematical introduction to this theory, in a discrete-time setting. We will mostly focus on the no-arbitrage theory in market models described by trees; eventually we will take the continuous-time limit of a binomial tree to obtain the celebrated Black-Scholes model and pricing formula.

Learning outcomes

On successful completion of this module, you will be able to:

- appreciate the fundamental principles involved in pricing derivatives;
- describe and critically analyse simple market models and explore their qualitative properties;
- confidently perform calculations involving pricing and hedging in discrete market models;
- demonstrate a familiarity with some key concepts in modern probability theory and apply them to perform computations;
- outline a mathematical formulation describing the behaviour of a number of financial derivatives;
- construct dynamic programming techniques to solve problems where inter-temporal relations are important;
- appraise and critically evaluate one or more of the advanced topics listed below.

Indicative module content

The module will cover the following topics:

financial derivatives, arbitrage, no-arbitrage pricing, self-financing portfolios, non-anticipative trading strategies, hedging of derivatives, domination property, complete markets, 'risk-neutral' probabilities, the fundamental theorems of asset pricing, conditional probability and expectation, filtrations, Markov processes, martingales, change of measure.

Extra mastery component will include the following advanced topics: utility, optimal investment.

Assessment strategy

Final Exam (90%)

Take-home coursework (10%)

MATH60014 Mathematical Biology

Brief description

Mathematical biology entails the use of mathematics to model biological phenomena in order to understand these systems, as well as predict their behaviour. It is an incredibly diverse field utilising the complete mathematical toolbox to ascertain insight into many areas of biology and medicine including population dynamics, physiology, epidemiology, cell biology, biochemical reactions, and neurology. This module aims to provide a foundational course in the subject area relying primarily on tools from applied dynamical systems, applied PDEs, asymptotic analysis and stochastic processes.

Learning outcomes

On successful completion of this module you will be able to:

- translate biological phenomena into the language of mathematics;
- appreciate canonical problems in epidemiology, ecology, biochemistry and physiology;
- critically analyse sets of ordinary differential equations especially in the non-linear setting;
- critically analyse sets of partial differential equations especially when either travelling-wave solutions or pattern forming phenomena might emerge;
- utilise the concept of stochastic population processes for exact and approximate solutions;
- use the techniques of order-of-magnitude reasoning and dimensional analysis;
- interpret results from the research literature on Mathematical Biology and analyze how the syllabus content relates to this wider body of work;
- appraise and evaluate an advanced topic in Mathematical Biology from a selection of case studies.

Indicative module content

Examples and topics include:

1) One-dimensional systems: existence and uniqueness; fixed points and their stability; bifurcations; logistic growth; SIS epidemic model; spruce budworm model; law of mass action; Michaelis-Menten enzyme dynamics.

2) Multidimensional systems: existence, uniqueness, fixed point stability; two-dimensional systems; SIS model for two populations; genetic control systems; population competition models; predator-prey dynamics and the Lotka-Volterra model.

3) Oscillations and bifurcations: Poincaré-Bendixson Theorem; oscillations in predator-prey models; relaxation oscillators; Fitzhugh-Nagumo model; fixed point bifurcations; Hopf bifurcations and limit cycles.

4) Spatial dynamics: reaction-diffusion equations; Fisher-Kolmogorov equation; travelling waves in predator-prey systems; spatial SIS model; spread of rabies in a fox population; Turing instabilities; pattern formation in one and two dimensions.

5) Stochastic processes: continuous-time Markov chains; simple birth and death processes; stationary probability distributions; logistic growth process; branching processes and drug resistance; multivariate processes; stochastic enzyme dynamics; stochastic predator-prey dynamics.

Assessment strategy

Final Exam (90%)

Take-home coursework (10%)

MATH60015 Quantum Mechanics I

Brief description

Quantum mechanics is one of the most successful theories in modern physics and has an exceptionally beautiful underlying mathematical structure. It provides the basis for many areas of contemporary physics, including atomic and molecular, condensed matter, high-energy particle physics, quantum information theory, and quantum cosmology, and has led to countless technological applications. This module aims to provide an introduction to quantum phenomena and their mathematical description. We will use tools and concepts from various areas of mathematics and physics, such as classical mechanics, linear algebra, probability theory, numerical methods, analysis and geometry.

Learning outcomes

On successful completion of this module, you will be able to:

- appreciate Schrödinger's formulation of quantum mechanics, wave functions and wave equations;
- construct the mathematical framework of quantum mechanics, including the 4 postulates of quantum mechanics and the Dirac notation;
- solve the eigenvalue problem for basic one-dimensional quantum systems;
- exploit the method of stationary states to deduce the time-evolved quantum state from the initial state of a system;
- communicate fluently using the Dirac notation;
- interpret results from advanced quantum mechanics textbooks and research papers;
- independently appraise and evaluate an advanced (more contemporary) topic in quantum mechanics from those listed in the syllabus below.

Indicative module content

The module will cover the following topics:

- Hamiltonian dynamics;
- Schrödinger equation and wave functions;
- stationary states of one-dimensional systems;
- mathematical foundations of quantum mechanics;
- quantum dynamics;
- angular momentum.

A selection of topics among the following additional optional topics will be covered depending on students interests:

- approximation techniques;
- explicitly time-dependent systems;
- geometric phases;
- numerical techniques;
- many-particle systems;
- cold atoms;
- entanglement and quantum information.

Assessment strategy

Final Exam (90%)

Take-home coursework and online quizzes (10%)

MATH60016 Special Relativity and Electromagnetism

Brief description

This module presents a beautiful mathematical description of a physical theory of great historical, theoretical and technological importance. It demonstrates how advances in modern theoretical physics are being made and gives a glimpse of how other theories (say quantum chromodynamics) proceed. This module does not follow the classical presentation of special relativity by following its historical development, but takes the field theoretic route of postulating an action and determining the consequences. The lectures follow closely the famous textbook on the classical theory of fields by Landau and Lifshitz.

Learning outcomes

On successful completion of this module you will be able to

- demonstrate an understanding of the relation between space and time and apply Lorentz transforms;
- appreciate the structure of special relativity as derived from the principle of least action;
- determine relativistic particle trajectories;
- derive Maxwell's equations from first principles and apply them to variety of interactions of charges and fields;
- critically analyse various solutions of the electromagnetic wave equations;
- describe electrostatic interactions and motion using Coulomb's law;
- construct an expansion of electrostatic interactions in terms of multipoles.

Indicative module content

This course follows closely the following book: L.D. Landau and E.M. Lifschitz, Course on Theoretical Physics Volume 2: Classical Theory of Fields.

Special relativity: Einstein's postulates, Lorentz transformation and its consequences, four vectors, dynamics of a particle, mass-energy equivalence, collisions, conserved quantities.

Electromagnetism: Magnetic and electric fields, their transformations and invariants, Maxwell's equations, conserved quantities, wave equation.

Assessment strategy

Final Exam (90%)

2 x Take-home coursework (10%)

MATH60017 Tensor Calculus and General Relativity

Brief description

This module provides an introduction to General Relativity. Starting with the rather simple Mathematics of Special Relativity the goal is to provide you with the mathematical tools to formulate General Relativity. Some examples, including the Schwarzschild space-time are considered in detail.

Learning outcomes

On successful completion of this module, you will be able to:

- appreciate the application of tensors in special relativity;
- demonstrate a working knowledge of tensor calculus;
- explain the concepts of parallel transport and curvature;
- formulate and solve the geodesic equation for a given space-time metric;
- derive Einstein's field equations and analyse Schwarzschild's solution;
- interpret results from advanced general relativity textbooks and research papers;
- appraise and critically evaluate two of the extensions and applications listed below.

Indicative module content

This module will cover the following topics:

1. Special Relativity
2. Tensors in Special Relativity
3. Tensors in General Coordinates Systems
4. Parallel Transport and Curvature
5. General Relativity
6. The Schwarzschild Spacetime
7. Variational Methods
8. Extensions and Applications (selected from gravitational waves, Einstein-Hilbert action, cosmology, Einstein-Cartan theory, differential geometry)

Assessment strategy

Final Exam (90%)

In-class test 1 (5%)

In-class test 2 (5%)

MATH60018 Quantum Mechanics II

Brief description

Quantum mechanics (QM) is one of the most successful theories in modern physics and has an exceptionally beautiful underlying mathematical structure. Assuming some prior exposure to the subject (such as Quantum Mechanics I), this module aims to provide an intermediate/advanced treatment of quantum phenomena and their mathematical description. Quantum theory combines tools and concepts from various areas of mathematics and physics, such as classical mechanics, linear algebra, probability theory, numerical methods, analysis and geometry.

Learning outcomes

On successful completion of this module, you will be able to:

- outline key aspects of quantum mechanics at the intermediate/advanced level;
- harness the power of symmetry in understanding quantum mechanics;
- describe many-particle quantum mechanical systems, and demonstrate familiarity with the formalism of second quantisation;
- solve complex quantum mechanical problems using the machinery introduced in this module;
- use the knowledge gained here as a solid foundation for a research project in quantum mechanics;
- interpret results from advanced quantum mechanics textbooks and research papers;
- appraise and evaluate a topic in quantum mechanics from the syllabus at an advanced level.

Indicative module content

This module will cover the following core topics:

- quantum mechanics in the momentum basis;
- the Heisenberg picture;
- the use of symmetry and general transformations in quantum mechanics;
- Bloch's theorem;
- perturbation theory;
- adiabatic processes;
- second quantisation;
- introduction to many-particle systems;
- Fermi and Bose statistics.

Additional topics include: WKB theory, the Feynman path integral, quantum magnetism.

Assessment strategy

Final Exam (90%)

Take-home coursework (10%)

MATH60019 Theory of Partial Differential Equations

Brief description

In this module, students are exposed to different phenomena which are modelled by partial differential equations. The course emphasizes the mathematical analysis of these models and briefly introduces some numerical methods.

Learning outcomes

On successful completion of this module you will be able to:

- appreciate how to formally differentiate complicated finite dimensional functionals and simple infinite dimensional functionals;
- describe, select and use a variety of methods for solving partial differential equations;
- outline how various partial differential equations respect conservation laws;
- utilize energy methods to critically analyse the stability of solutions to PDEs;
- develop the general method of characteristics and derive the eikonal equation;
- justify the proper use of the calculus of variations in classical settings.

Indicative module content

The module is composed of the following sections:

1. Introduction to PDEs

- 1.1. Basic Concepts
- 1.2. Gauss Theorem

2. Method of Characteristics

- 2.1. Linear and Quasilinear first order PDEs in two independent variables.
- 2.2. Scalar Conservation Laws
- 2.3. Hamilton-Jacobi Equations. General Method of Characteristics.

3. Diffusion

- 3.1. Heat equation. Maximum principle
- 3.2. Separation of variables. Fourier Series.

4. Waves

- 4.1. The 1D wave equation
- 4.2. 2D and 3D waves.

5. Laplace-Poisson equation

- 5.1. Dirichlet and Neumann problems.
- 5.2. Introduction to calculus of variations. The Dirichlet principle.
- 5.3 Finite Element Method.
- 5.4 Lagrangians and the minimum action principle.

Assessment strategy

Final Exam (90%)

Take-home coursework (10%)

MATH60021 Advanced Partial Differential Equations 2

Brief description

The focus of the course is the theory of nonlinear partial differential equations (PDEs) and their modern treatment through analytical techniques. The emphasis is on methods (such as fixed point theorems, Fourier analysis) and how they apply to classical problems involving fluid mechanics and wave propagation.

Learning outcomes

On successful completion of this module you will be able to:

- appreciate the concepts of distribution (differentiation, convergence);
- manipulate the main properties of the Sobolev space H^m for integer m (inbeddings and compactness theorems, Poincaré inequality);
- derive the variational formulation of a specific elliptic boundary value problem and to provide the reasoning leading to the proof of the existence and uniqueness of the solution;
- develop the spectral theory of an elliptic boundary value problem;
- solve a parabolic boundary value problem using the spectral theory of the associated elliptic operator.
- interpret results from advanced textbooks and research papers on the theory of Partial Differential Equations;
- independently appraise and evaluate an advanced topic on Partial Differential Equations, namely the theory of nonlinear elliptic and parabolic equations on the whole space.

Indicative module content

An indicative list of topics is:

1. Fixed point theorems and applications: the contraction mapping principle, Brouwer and Schauder fixed-point theorems, semilinear parabolic equations.
2. Elements of Fourier analysis: the Fourier transform, the method of stationary phases, singular integrals.
3. Energy estimates and compactness: basic notions, spaces involving time, existence of solutions to nonlinear evolution equations.
4. Applications to equations arising in Mathematical Physics: the Euler and Navier-Stokes equations, dispersive equations, nonlinear heat equation, hyperbolic conservation laws.

Assessment strategy

Final Exam (90%)

Take-home coursework (10%)

MATH60022 Finite Elements: Numerical Analysis and Implementation

Brief description

Finite element methods form a flexible class of techniques for numerical solution of PDEs that are both accurate and efficient. The finite element method is a core mathematical technique underpinning much of the development of simulation science. Applications are as diverse as the structural mechanics of buildings, the weather forecast, and pricing financial instruments. Finite element methods have a powerful mathematical abstraction based on the language of function spaces, inner products, norms and operators.

Learning outcomes

On successful completion of this module, you will be able to:

- appreciate the core mathematical principles of the finite element method;
- employ the finite element method to formulate and analyse numerical solutions to linear elliptic PDEs;
- implement the finite element method on a computer;
- compare the application of various software engineering techniques to numerical mathematics;
- generalize the concept of a directional derivative;
- appraise and evaluate techniques for solving nonlinear PDEs using the finite element method.

Indicative module content

This module aims to develop a deep understanding of the finite element method by spanning both its analysis and implementation. In the analysis part of the module, students will employ the mathematical abstractions of the finite element method to analyse the existence, stability and accuracy of numerical solutions to PDEs. At the same time, in the implementation part of the module students will combine these abstractions with modern software engineering tools to create and understand a computer implementation of the finite element method.

This module is composed of the following sections:

- I - Basic concepts: weak formulation of boundary value problems, Ritz-Galerkin approximation, error estimates, piecewise polynomial spaces, local estimates;
- II - Efficient construction of finite element spaces in one dimension: 1D quadrature, global assembly of mass matrix and Laplace matrix;
- III - Construction of a finite element space: Ciarlet's finite element, various element types, finite element interpolants;
- IV - Construction of local bases for finite elements: efficient local assembly;
- V - Sobolev Spaces: generalised derivatives, Sobolev norms and spaces, Sobolev's inequality;
- VI - Numerical quadrature on simplices: employing the pullback to integrate on a reference element;
- VII - Variational formulation of elliptic boundary value problems: Riesz representation theorem, symmetric and nonsymmetric variational problems, Lax-Milgram theorem, finite element approximation estimates;
- VIII - Computational meshes: meshes as graphs of topological entities, discrete function spaces on meshes, local and global numbering;
- IX - Global assembly for Poisson equation: implementation of boundary conditions, general approach for nonlinear elliptic PDEs;

X - Variational problems: Poisson's equation, variational approximation of Poisson's equation, elliptic regularity estimates, general second-order elliptic operators and their variational approximation;
XI - Residual form and the Gâteaux derivative;
XII - Newton solvers and convergence criteria.

Assessment strategy

Final Exam (50%)

Term-long implementation assignment (50%)

MATH60023 Computational Dynamical Systems

Brief description

Nonlinear differential equations arise across the scientific disciplines and often solutions can only be computed numerically. This module entails developing a foundation in the analysis and implementation of numerical methods to obtain these solutions. Along with generating solutions, the module will examine how these methods can also be used to perform bifurcation and stability analyses of steady and time periodic solutions, analyses often required to understand the qualitative features of the system. The module will have equal emphasis on theory, both in method analysis and background stability analysis, as well as implementation. As such assessment will be 50% project and 50% exam. Students will be expected to use Python in their assessments.

Learning outcomes

On the successful completion of the module, you will be able to:

- use classical numerical methods to solve initial value problems;
- analyse different properties of numerical methods (e.g. accuracy and stability);
- compare different methods with respect to accuracy, stability, and computational complexity;
- obtain numerically steady state and time periodic solutions to differential equations;
- perform numerically linear stability and Floquet analyses;
- conduct numerical bifurcation analyses and understand normal forms for bifurcations.

Indicative module content

This module will cover the following topics:

- Multi-step and multistage methods for solving initial value problems;
- Stability, accuracy and convergence;
- Newton methods for finding steady state and time periodic solutions;
- Algorithms for linear stability and Floquet analyses and associated theory;
- Jacobian-free algorithms for stability analysis
- Normal forms for bifurcations, and weakly nonlinear analysis;
- Applications of the methods to mathematical models arising across science and engineering;

Assessment strategy

Final Exam (50%)

2 x Take-home coursework (50%)

MATH60024 Computational Linear Algebra

Brief description

Linear systems of equations arise in countless applications and problems in mathematics, science and engineering. Often these systems are large and require a computer to solve. This course provides an overview of the algorithms used to solve linear systems and eigenvalue problems, in terms of their development, stability properties, and application.

Learning outcomes

On successful completion of this module, you will be able to

- describe, select and use algorithms for QR decomposition of matrices;
- solve least-squares problems using QR decomposition;
- find numerical solutions to eigenvalue problems;
- critically analyse various iterative methods for solving linear systems;
- combine the techniques you have mastered in order to assess unseen algorithms;

Indicative module content

The module will cover the following topics:

0) Efficient implementation of numerical algorithms using NumPy.

1) QR factorisation and least squares problems.

2) Conditioning and finite precision arithmetic.

3) Eigenvalue problems:

power method and variants, Householder reduction to tridiagonal form, the QR algorithm.

3) Iterative methods:

Krylov subspace methods: Lanczos method and Arnoldi iteration, conjugate gradient method, GMRES, preconditioning, the conjugate gradient method, symmetric stationary methods as preconditioners.

Assessment strategy

Take-home coursework 1 (10%)

Take-home coursework 2 (10%)

In-class test (30%)

Final Exam (50%)

MATH60025 Computational Partial Differential Equations

Brief description

This module will introduce a variety of computational approaches for solving partial differential equations, focusing mostly on finite difference methods, but also touching on finite volume and spectral methods. Students will gain experience implementing the methods and writing/modifying short programs in Matlab or another programming language of their choice. Applications will be drawn from problems arising in areas such as Mathematical Biology and Fluid Dynamics.

Learning outcomes

On successful completion of this module, you will be able to:

- appreciate the physical and mathematical differences between different types of PDES;
- design suitable finite difference methods to solve each type of PDE;
- outline a theoretical approach to testing the stability of a given algorithm;
- determine the order of convergence of a given algorithm;
- demonstrate familiarity with the implementation and rationale of multigrid methods;
- develop finite-difference based software for use on research level problems;
- communicate your research findings as a poster, in a form suitable for presentation at a scientific conference;
- demonstrate an integrated understanding of the concepts of the module by independent study of related material.

Indicative module content

The module will cover the following topics:

- 1) Introduction to Finite Differences
- 2) Classification of PDEs
- 3) Explicit and Implicit methods for Parabolic PDEs
- 4) Iterative Methods for Elliptic PDEs. Jacobi, Gauss-Seidel, Overrelaxation
- 5) Multigrid Methods
- 6) Hyperbolic PDEs. Nonlinear Advection/Diffusion systems. Waves and PMLs

as well as various advanced practical topics from Fluid Dynamics, which will depend on the final project.

Assessment strategy

- Final Exam (50%)
- Take-home coursework 1 (20%)
- Take-home coursework 2 (30%)

MATH60026 Methods for Data Science

Brief description

This module provides an hands-on introduction to the methods of modern data science. Through interactive lectures, the student will be introduced to data visualisation and analysis as well as the fundamentals of machine learning.

Learning outcomes

On successful completion of this module, you will be able to:

- Visualise and explore data using computational tools;
- Appreciate the fundamental concepts and challenges of learning from data;
- Analyse some commonly used learning methods;
- Compare learning methods and determine suitability for a given problem;
- Describe the principles and differences between supervised and unsupervised learning;
- Clearly and succinctly communicate the results of a data analysis or learning application;
- Appraise and evaluate new algorithms and computational methods presented in scientific and mathematical journals;
- Design and implement newly-developed algorithms and methods.

Indicative module content

The module is composed of the following sections:

- Introduction to computational tools for data analysis and visualisation;
- Introduction to exploratory data analysis;
- Mathematical challenges in learning from data: optimisation;
- Methods in Machine Learning: supervised and unsupervised; neural networks and deep learning; graph-based data learning;
- Machine learning in practice: application of commonly used methods to data science problems. Methods include: regressions, k-nearest neighbours, random forests, support vector machines, neural networks, principal component analysis, k-means, spectral clustering, manifold learning, network statistics, community detection;
- Current research questions in data analysis and machine learning and associated numerical methods.

Assessment strategy

Two projects which include a take-home computational component and a time-limited in-class component:

Project 1 (40%)

Project 2 (60%)

MATH60028 Probability Theory 1

Brief description

This module rigorously develops mathematical probability theory using measure theory and applying many of the tools developed in the Year 2 module Lebesgue Theory and Integration (or an equivalent module). This module will give students a solid foundation for formulating and studying random phenomena using mathematics, laying the ground work for further studies in areas such as stochastic processes, stochastic calculus, and statistical mechanics.

Learning outcomes

On successful completion of this module, you should be able to:

- Demonstrate knowledge of fundamental notions and structures of Probability Theory;
- Use the tools developed in a variety of applications;
- Demonstrate problem solving skills in probability theory;
- Demonstrate additional competence in the subject through self-study of more advanced material;
- Combine material from across the module to solve more advanced problems;
- Communicate your knowledge of the area in a concise, accurate and coherent manner.

Indicative module content

An indicative list of topics is:

1. Probability spaces, measurability and random variables, convergence of random variables.
2. Independence, Weak Law of Large Numbers.
3. Borel-Cantelli Lemmas, Strong Law of Large Numbers.
4. Convergence in distribution, characteristic functions, Central Limit Theorem.
5. Conditional expectation and discrete time martingales.

Assessment strategy

Take-home coursework 1 (5%)
Take-home coursework 2 (5%)
Final Exam (90%)

MATH60029 Functional Analysis

Brief description

This module brings together ideas of continuity and linear algebra. It concerns vector spaces with a distance, and involves linear maps. The vector spaces are often spaces of functions. It is an important requirement for further study of many areas of Mathematical Analysis including PDEs, Stochastic Analysis, Dynamical Systems and Quantum Mechanics.

Learning outcomes

On successful completion of this module, you should be able to:

- Demonstrate knowledge of fundamental notions and structures of Functional Analysis by proving a range of results;
- Use the tools developed in a variety of applications;
- Demonstrate problem solving skills in functional analysis;
- Demonstrate additional competence in the subject through self-study of more advanced material;
- Synthesise topics from across the module to solve problems on more advanced applications;
- Communicate your knowledge of the area in a concise, accurate and coherent manner.

Indicative module content

An indicative list of topics is:

1. Banach spaces: definition, Schauder basis.
2. Linear operators: boundedness, continuity. Dual spaces, Hahn-Banach lemma, Riesz representation theorem in L_p .
3. Baire category and UBP (Banach-Steinhaus), open mapping/closed graph theorem, closable operators
3. Hilbert spaces, the projection theorem and the Riesz representation theorem, orthogonal bases, Lax-Milgram.
4. Compactness: Riesz lemma. Compact operators, examples: Hilbert-Schmidt, Arzela-Ascoli.
5. Spectrum and resolvent: basics. Spectral Theory in Hilbert Spaces, symmetric, self-adjoint, Spectral theorem for compact self-adjoint operators on a Hilbert space (Riesz-Schauder)

Assessment strategy

- Take-home coursework 1 (5%)
- Take-home coursework 2 (5%)
- Final Exam (90%)

MATH60030 Fourier Analysis and the Theory of Distributions

Brief description

Fourier analysis is an important tool used in various branches of mathematics and beyond. The module provides a deeper understanding of it than what is briefly mentioned in general analysis courses. It also connects it to the theory of distributions. As a result of studying the module, students will understand the basics of the Fourier analysis and theory of distributions which will be sufficient for most branches of mathematics.

Learning outcomes

On successful completion of this module, you will be able to:

- understand the issues of convergence for Fourier series;
- apply the Fourier and Laplace transforms;
- understand the motivation behind the notion of distribution;
- be in command of the basics of Fourier analysis and distribution theory sufficient for working in many areas of mathematics;
- demonstrate competence with further advanced material in the area designated for self-study;
- synthesise material from across the module to apply to advanced topics.

Indicative module content

The module will assume familiarity with measure theory and functional analysis, especially L^p spaces and linear functionals.

Indicative content: Orthogonal systems in infinite-dimensional Euclidean spaces, Bessel inequality, Parseval equality, general Fourier series, trigonometric basis in $L_2[-\pi, \pi]$, convergence of trigonometric Fourier series, Fejer's theorem and applications, Fourier transform and its properties, application to solution of differential equations, Plancherel theorem, Laplace transform, linear functionals, distributions, basic properties of distributions and applications, Fourier transform for distributions.

Those students who decide to do a PhD in a closer related area of analysis will be able to use the acquired basic knowledge and skills to relatively easily extend their knowledge to more sophisticated areas of the theory.

This Level 7 version of the module will involve extra self-study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

Assessment strategy

- Take-home coursework 1 (5%)
- Take-home coursework 2 (5%)
- Final Exam (90%)

MATH60031 Probability Theory 2

Brief description

This module builds on Probability Theory 1, building a theory of continuous time stochastic processes (with Brownian Motion as a key example) along with introducing students to rigorous stochastic calculus. This module gives crucial background for students interested in stochastic differential equations, stochastic partial differential equations, and more generally stochastic analysis.

Learning outcomes

On successful completion of this module, you should be able to:

- demonstrate your understanding of the concepts and results associated with the elementary theory of Markov processes, including the proofs of a variety of results
- apply these concepts and results to tackle a range of problems, including previously unseen ones
- apply your understanding to develop proofs of unfamiliar results
- demonstrate additional competence in the subject through the study of more advanced material
- combine ideas from across the module to solve more advanced problems
- communicate your knowledge of the area in a concise, accurate and coherent manner.

Indicative module content

An indicative list of topics is:

1. Definition and construction of Brownian Motion, properties of sample Brownian paths, strong Markov property.
2. Continuous time local martingales.
3. Stochastic integrals with Brownian motion, and general continuous local martingales
3. Itô's formula
4. Stochastic differential equations, martingale problems, applications to PDE.
5. Markov semigroups, functional inequalities, and convergence to equilibrium.

Assessment strategy

Take-home coursework 1 (5%)
Take-home coursework 2 (5%)
Final Exam (90%)

MATH60032 Geometry of Curves and Surfaces

Brief description

This module is an introduction to classical theory of differential geometry, where we discuss geometric features of curves and surfaces in (mostly) three dimensional Euclidean spaces.

Learning outcomes

On successful completion of this module, you will be able to:

- identify regular curves and implement different re-parametrisations of curves in two and three dimensional spaces,
- learn about and calculate the geometric quantities of curvature and torsion of a regular curve,
- identify regular surfaces in 3 dimensional spaces using the notions of charts,
- analyse the regularity of maps from one surface into another surface, and also of functions on surfaces,
- use partitions to calculate the basic topological invariant of Euler characteristic,
- learn about the topological classification of compact surfaces, and identify them,
- calculate the first and second fundamental forms of a surface,
- learn about the existence and uniqueness of geodesics on general surfaces,
- link the Gaussian curvature to the local shape of a surface, and present different kinds of examples,
- analyse the global topological features of a surface by integrating local geometric features (Gauss-Bonnet and winding numbers)
- demonstrate competence with further advanced material in the area designated for self-study
- synthesise material from across the module to apply to advanced topics

Indicative module content

This module is an introduction to classical theory of differential geometry, where we discuss geometric features of curves and surfaces in (mostly) three dimensional Euclidean spaces. A curve, which is the trajectory of a particle moving in a smooth fashion, may twist in two manners described by the values called curvature and torsion. The twists of a surface in three dimensional space is naturally more involved. There are different notions of curvature: the Gaussian curvature and the mean curvature. The Gaussian curvature describes the intrinsic geometry of the surface, and the mean curvature describes how it bends in space. We look at several examples of surfaces, and calculate their curvatures. We study the local shapes of surfaces based on their curvatures. For example, the Gaussian curvature of a sphere is strictly positive, which explains why any planar illustration of the countries distorts shapes. Remarkably, these local geometric notions can be combined to derive global information about the topology of the surface (for example the Gauss-Bonnet formula). This module starts with the basic real analysis taught in years 1 and 2, and leads into the more modern and general theory of manifolds.

An indicative list of sections and topics is:

- Curves in two and three-dimensional spaces: re-parametrizations, curvature and torsion, Frenet-Serret formulae, curves are determined by curvature and torsion, winding number and the total curvature,
- Surfaces: Charts, Tangent vectors, and tangent planes, Smooth maps from one surface into another surface, smooth functions on a surface, Normal vectors,

- Curvature of a surface: the first and second fundamental forms, Christoffel symbols, normal curvature, Gaussian curvature, and mean curvature, Gauss's Theorema Egregium,
- Area of a surface,
- Geodesics on a surface: length-minimising curves, existence, non-existence and examples, geodesic curvature,
- Gauss-Bonnet Theorem and applications
- Topological classification of surfaces
- Vector fields and the Poincare-Hopf Theorem

The module will assume familiarity with material in the second-year module Analysis II

This Level 7 version of the module will involve extra self-study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

Assessment strategy

Take-home coursework (10%)

Final Exam (90%)

MATH60033 Geometry I: Algebraic Curves

Brief description

This module is meant as a first encounter with algebraic geometry, through the study of affine and projective plane curves over the field of complex numbers. We will also discuss some complex-analytic aspects of the theory (Riemann surfaces). Important results include the definition of local intersection multiplicities and Bézout's theorem, inflection points and the classification of plane cubics, linear systems of curves, and the degree-genus formula.

Learning outcomes

On successful completion of this module, you will be able to:

- solve geometric problems about affine and projective plane curves with algebraic techniques;
- determine the projectivizations of affine plane curves and the points at infinity;
- determine the tangent lines of plane curves at smooth and singular points;
- compute projective transformations and find convenient coordinate systems;
- compute intersection multiplicities using resultants and the axiomatic characterization;
- formulate, prove and apply Bézout's theorem;
- find inflection points of projective plane curves and use them to classify cubic curves;
- solve enumerative problems by means of the theory of linear systems;
- work with holomorphic charts to determine local and global properties of Riemann surfaces and morphisms;
- compute ramification degrees of morphisms of Riemann surfaces;
- formulate, prove and apply the degree-genus formula for smooth projective plane curves.
- demonstrate competence with further advanced material in the area designated for self-study
- synthesise material from across the module to apply to advanced topics

Indicative module content

An indicative list of sections and topics is:

- Affine plane curves;
- The geometry of projective spaces;
- Projective plane curves;
- Smooth and singular points, tangent lines;
- Projective transformations and the classification of conics;
- Intersection multiplicities (resultants and axiomatic characterization)
- Bézout's theorem on intersections of projective plane curves;
- the Legendre family of cubics, inflection points and the classification of non-degenerate smooth cubics;
- linear systems of projective plane curves, projective duality and enumerative geometry;
- Riemann surfaces;
- local description of morphisms of Riemann surfaces (ramification);
- classification of topological surfaces and genus (informal introduction);
- Riemann-Hurwitz and the degree-genus formula.

Some related topics will appear in the problem sheets and the coursework (e.g., dual curves, group structure on smooth cubics).

This Level 7 version of the module will involve extra self-study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

Assessment strategy

In-class test (10%)

Final Exam (90%)

MATH60034 Geometry II: Algebraic Topology

Brief description

This module gives a first introduction to algebraic topology. After some preliminary results on quotient spaces and CW-complexes, we discuss fundamental groups and the Galois correspondence for covering spaces. We then move on to homology theory and study simplicial and singular homology, as well as some applications like the Jordan curve theorem and invariance of domain. Throughout the module, we pay special attention to algebraic and categorical aspects.

Learning outcomes

On successful completion of this module, you will be able to:

- define the basic invariants in algebraic topology and prove their main properties;
- use algebraic techniques to distinguish different homotopy types and classify topological objects;
- compute fundamental groups, simplicial homology groups and singular homology groups;
- apply the Galois correspondence to classify covering spaces of topological spaces;
- apply fundamental groups and homology groups to prove fundamental topological properties (Brouwer's fixed point theorem, Jordan's curve theorem, invariance of domain);
- formulate topological and algebraic constructions in a categorical language (universal properties);
- analyze the structure of quotients of topological spaces by covering space actions;
- represent groups geometrically by means of Cayley complexes

Indicative module content

An indicative list of sections and topics is:

- Preliminaries:
 - Homotopy and homotopy type
 - Cell complexes
 - Operations on spaces
- The Fundamental Group:
 - Paths and Homotopy
 - Presentations of groups, amalgamated products and Van Kampen's Theorem
 - Covering Spaces
 - The Galois correspondence -Deck Transformations and Group Actions -Cayley complexes
- Homology
 - Δ -complexes and simplicial homology
 - Singular homology
 - Homotopy invariance
 - Relative homology, exact sequences and excision
 - The equivalence of simplicial and singular homology
 - Mayer-Vietoris Sequences
 - Applications

The main reference for this course is "Algebraic topology" by Hatcher.

This Level 7 version of the module will involve extra self-study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

The module will assume familiarity with the material in the second-year modules: Groups and Rings, Analysis II

Assessment strategy

3 x Take-home coursework (10%)

Final Exam (90%)

MATH60035 Algebra III

Brief description

This course continues the study of commutative rings and introduces the notion of R -module, which is an analogue over rings of the notion of a vector space over a field. Using these ideals we prove fundamental results about various classes of rings, particularly polynomial rings in several variables.

Learning outcomes

On successful completion of this module, you will be able to:

- Understand the detailed theory of finite fields, their classification, and factorization of polynomials over finite fields
- Understand the theory of R -modules and their presentations
- Understand the classification of modules over Euclidean Domains, and how to use Smith Normal Form to determine the isomorphism class of such a module given a presentation
- Use this classification, in the case of $K[T]$ -modules, to prove fundamental results in linear algebra
- Apply several different criteria for irreducibility of polynomials over various base rings
- demonstrate competence with further advanced material in the area designated for self-study
- synthesise material from across the module to apply to advanced topics

Indicative module content

An indicative list of sections and topics is:

- Chinese Remainder Theorem
- Field Extensions and Finite Fields
- R -modules
- Free modules and presentations
- Modules over Euclidean Domains
- Noetherian rings
- Gauss's Lemma and Factorization in polynomial rings
- If R is a UFD, so is $R[X]$
- Irreducible Polynomials and factorization of polynomials

This Level 7 version of the module will involve extra self-study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

Assessment strategy

Take-home coursework (10%)

Final Exam (90%)

MATH60036 Group Theory

Brief description

This module builds on the Group Theory from the 1st year module Linear Algebra & Groups and the 2nd year module Groups and Rings. We start with a discussion of isomorphism theorems, and proceed to further example of groups and operations on them, including automorphism groups and semidirect products. Special attention is given to group actions and permutation groups: primitivity, multiple transitivity etc. Further we discuss solvable and nilpotent groups and their characterizations.

Learning outcomes

On successful completion of this module, you will be able to:

- identify features of, and develop arguments about, certain groups and classes of group;
- explain the principles of group actions, and work with elementary examples;
- construct and work with direct and semidirect products of groups;
- state the definition of and extraspecial group and construct small examples of them;
- construct certain classical series of doubly and triply transitive groups;
- state and prove the structure theorem for nilpotent groups;
- explain the principles of group actions, and work with elementary examples;
- determine the normal structure and calculate the automorphism groups of symmetric groups;
- work independently and with peers to articulate understanding of abstract concepts in algebra.
- demonstrate competence with further advanced material in the area designated for self-study
- synthesise material from across the module to apply to advanced topics

Indicative module content

An indicative list of sections and topics is:

- Definition and basic properties of groups.
- Isomorphism Theorems.
- Sylow subgroups.
- Group actions, primitivity and multiple transitivity.
- Composition series.
- Nilpotent groups.
- Solvable groups.
- Symmetric groups.
- Automorphism group of a group and semidirect products.
- Linear groups: centres and commutator subgroups, with small examples.

Further advanced material on these topics will be set for self-study.

Assessment strategy

Take-home coursework 1 (5%)

Take-home coursework 2 (5%)

Final Exam (90%)

MATH60037 Galois Theory

Brief description

The formula for the solution to a quadratic equation is well-known. There are similar formulae for cubic and quartic equations but no formula is possible for quintics. The module explains why this happens.

Learning outcomes

On successful completion of this module you should be able to:

- state, prove, and apply the fundamental theorem of Galois theory (aka the "Galois correspondence").
- work with simple examples such as cubic polynomials, cyclotomic polynomials, and polynomials over finite fields.
- compute Galois groups of splitting fields of cubic and bi-quadratic polynomials in arbitrary characteristic.
- state and apply the formulas for solving cubic and quartic equations, and to prove that there are no such formulas for equations of degree 5 or larger.
- compute Galois groups over the rationals by the method of Frobenius elements.
- Demonstrate additional competence in the subject through the self-study of designated advanced material
- Combine topics from across the module to obtain more advanced results

Indicative module content

- Irreducible polynomials.
- Field extensions, degrees and the tower law.
- Extending embeddings.
- Normal field extensions, splitting fields, separable extensions.
- Groups of automorphisms, fixed fields.
- The fundamental theorem of Galois theory.
- Finite fields, cyclotomic extensions.
- Extensions of the rationals and Frobenius elements.
- The solubility of polynomials of degree at most 4 and the insolubility of quintic equations.

Material for self-study (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

Assessment strategy

In-class test 1 (5%)
In-class test 2 (5%)
Final Exam (90%)

MATH60038 Graph Theory

Brief description

A graph is a structure consisting of vertices and edges. Graphs are used in many areas of Mathematics, and in other fields, to model sets with binary relations. In this module we study the elementary theory of graphs; we discuss matters such as connectivity, and criteria for the existence of Hamilton cycles. We treat Ramsey's Theorem in the context of graphs, with some of its consequences. We then discuss probabilistic methods in Graph Theory, and properties of random graphs.

Learning outcomes

On successful completion of this module, you will be able to:

- Demonstrate facility with the terminology of graphs and simple graph constructions to analyse examples and prove results
- Explain the proofs of the theorems of König and Menger, and certain other related results. Apply these results to appropriate problems.
- State, prove and apply Turán's Theorem. Describe and apply certain results in the theory of Hamilton cycles, including Dirac's Theorem.
- Explain and reason about Ramsey's Theorem and related results in the context of graph colourings.
- Describe various models of random graphs and apply probabilistic arguments to situations in graph theory.
- Demonstrate competence with further advanced material in the area designated for self-study
- Synthesise material from across the module to apply to advanced topics

Indicative module content

An indicative list of sections and topics is:

- Standard definitions and basic results about graphs. Common graph constructions: complete graphs, complete bipartite graphs, cycle graphs.
- Matchings and König's Theorem. Connectivity and Menger's Theorem.
- Extremal graph theory. The theorems of Mantel and Turán. Hamilton cycles, and conditions for their existence.
- Ramsey Theory for graphs, with applications.
- The Probabilistic Method and random graphs. Evolution of random graphs.

This Level 7 version of the module will involve extra self-study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

Assessment strategy

Take-home coursework (10%)

Final Exam (90%)

MATH60039 Group Representation Theory

Brief description

This module defines and begins the study of representations of groups, focusing on finite-dimensional complex representations of finite groups. These structures encode ways that groups can act as symmetries, which appear throughout mathematics (notably in algebra, number theory, and geometry, but also in analysis) as well as in physics and chemistry, among other places. We explain how to understand and classify these representations through characters, or traces. In the final unit we generalise the theory to finite-dimensional modules over rings, particularly semisimple algebras, whose theory retains many of the features of that of representations of groups.

Learning outcomes

On successful completion of this module you will be able to:

- Recall and use basic definitions in group representations, their character theory, and modules over algebras, particularly finite-dimensional semisimple algebras;
- Explain and work with the features of complex representations of finite groups that allow one to simplify the theory (e.g., semisimplicity, character tables, etc.);
- Apply these results to classify representations of finite groups and semisimple algebras and compute the character tables of finite groups;
- Perform basic constructions of representations of groups and to apply them to obtain all finite-dimensional representations of certain basic groups up to isomorphism;
- Explain the relationship between finite-dimensional irreducible representations of algebras and of finite-dimensional semisimple algebras, and the basic properties of their characters;
- Relate endomorphisms of representations to central elements in groups and semisimple algebras;
- Work independently and with peers to formulate and solve problems in algebra and geometry using tools of representation theory;
- Demonstrate ability to engage with more advanced material via self-study.
- Combine material from across the module to address more challenging problems

Indicative module content

- Basic theory: definitions, Maschke's theorem, Schur's Lemma, classification and construction of representations of finite abelian groups, dihedral groups, and small symmetric and alternating groups;
- Tensor products of representations and homomorphism spaces, the regular representation;
- Character theory: behaviour under direct sums and tensor products, orthogonality relations, computation of character tables of certain groups;
- Finite-dimensional modules over algebras: definition of finite-dimensional semisimple algebras and relationship of finite-dimensional modules of general algebras to those of finite-dimensional semisimple algebras; constructions of projections via the center;
- Other important examples of modules over algebras.

Further material, related to the above but of a more advanced character (consisting of a book chapter or research paper), will be designated for self-study.

Assessment strategy

In-class test 1 (5%)

In-class test 2 (5%)

Final Exam (90%)

MATH60040 Formalising Mathematics

Brief description

Computer theorem provers are mature enough now to tackle most undergraduate level mathematics, and also some much harder level mathematics. As these systems evolve, they will inevitably become useful as tools for researchers, and some believe that one day they will start proving interesting theorems by themselves. This project-assessed course is an introduction to the Lean theorem prover and during it we will learn how to formalise proofs of undergraduate and masters level theorems from across pure mathematics.

Learning outcomes

On successful completion of this module you will be able to

- understand the basics of how type theory can be used as a foundation for pure mathematics;
- understand how to "modularise" mathematical arguments, breaking them up into simple chunks, thus leading to clarity of understanding;
- understand how to "abstract" mathematical arguments, finding the correct generality in which statements should be made, thus again leading to clarity of understanding;
- state and prove many results from undergraduate and masters level pure mathematics modules in the Lean theorem prover;
- develop mathematical theories of your own in the Lean theorem prover;
- write formal proofs of theorems which other mathematicians can understand and follow
- understand how to turn more advanced mathematics into statements of dependent type theory.

Indicative module content

Note that the aim is to both learn the mathematics and to learn how to teach it to a computer. No experience in programming will be assumed. Lean is a functional programming language, so we will be picking up functional programming along the way. If you want to get a feeling for the kind of coding which will be involved, try playing the natural number game.

The following is an indicative list of areas where the mathematics could be drawn from:

- Logic, functions, sets.
- Lattice theory, complete lattices, Galois insertions and Galois connections. Examples in mathematics.
- Groups and subgroups.
- Closure operators in group theory and topology.
- Filters as generalised subsets. Filters form a complete lattice.
- Applications of filters to topology. New proofs of basic results in topology. Tychonoff's theorem.
- Application of filters to analysis. New proofs of basic results in analysis.
- What is cohomology? Group cohomology in low degree.

This level 7 version of the module will involve extension material and more advanced examples than the level 6 version.

Assessment strategy

Take-home quiz (5%)

In-class quiz (5%)

Midterm in-class test (20%)

Project 1 (20%)

Project 2 and associated oral examination (50%)

MATH60041 Number Theory

Brief description

Brief description of module (at most 600 characters): The module is concerned with properties of natural numbers, and in particular of prime numbers, which can be proved by "elementary" methods (such as basic group theory ring theory).

Learning outcomes

On successful completion of this module, you will be able to:

- form arguments about and solve congruences, particular modulo primes, and apply this to the RSA algorithm;
- compute with quadratic residues;
- solve some particular Diophantine equations, including Pell's equation;
- compute continued fractions;
- construct transcendental numbers;
- explain and demonstrate mastery of such further material as is selected by the module leader for self-study;
- Combine material from across the module to solve more advanced problems.

Indicative module content

An indicative list of topics is:

- Euclid's algorithm, unique factorization, linear congruences, Chinese Remainder Theorem.
- The structure of $(\mathbb{Z}/n\mathbb{Z})^\times$, including the Fermat-Euler theorem, Lagrange's theorem, the existence (and non-existence) of primitive roots.
- Primality testing, factorization, and the RSA algorithm (including the basics of the Miller-Rabin test).
- Quadratic reciprocity, Legendre symbols, Jacobi symbols.
- Sums of 2 and 4 squares, using unique factorization in the Gaussian integers.
- Pell's equation, existence of solutions via Dirichlet's theorem.
- Continued fractions, periodicity for quadratic irrationals, algorithm for solving Pell's equation via continued fractions.
- Irrationality, Liouville's theorem, construction of a transcendental number.
- Elementary results on primes in arithmetic progressions.
- Other topics of the lecturer's choice as time permits, e.g. quadratic forms; Möbius inversion and Dirichlet Convolution; the Selberg sieve; particular examples of Diophantine equations.

Mastery material selected for further self-study, to be based on written material such as a textbook excerpt or research paper.

Assessment strategy

In-class test or take-home coursework (10%)

Final Exam (90%)

MATH60042 Algebraic Number Theory

Brief description

An introduction to algebraic number theory using quadratic fields as the main source of examples. We will study rings of integers in finite extensions of the rational and discuss unique factorization and its failure, the decomposition of primes, the finiteness of the ideal class group, and Dirichlet's unit theorem.

Learning outcomes

On successful completion of this module you will be able to:

- explain how unique factorization domains, principal ideal domains and Euclidean domains are related.
- give examples of rings of integers in quadratic fields that are Euclidean domains and also counter-examples.
- define a Dedekind domain and explain why rings of integers in number fields are Dedekind domains.
- write a basis for the rings of integers in any given quadratic number field.
- explain what it means for a prime to be split, inert or ramified in an extension and, given a quadratic ring of integers and a prime, you will be able to say what happens to that prime.
- explain why the class group in a number field is finite and compute examples of class groups of quadratic number fields.
- state Dirichlet's unit theorem and to describe explicitly the group of units in a real or imaginary quadratic field.
- show mastery of more advanced material on these topics which will be set for self-study.
- synthesise arguments from different parts of the module to solve more advanced problems.

Indicative module content

An indicative list of topics is as follows.

We will review / introduce some background from ring theory, discuss unique factorization domains, principal ideal domains and Euclidean domains. We will study Gaussian and Eisenstein integers in detail and see several other examples of quadratic rings of integers. We will then discuss the structure theorem for finitely generated abelian groups, the notion of integral closure, Dedekind domains and study the ideal class group. We will prove that the ideal class group in a number field is finite and compute many examples. We will study the decomposition of primes in number fields and in quadratic fields in particular. We will end by discussing Dirichlet's unit theorem.

Further more advanced material (such as a book chapter or research paper) will be set by the module leader for independent study.

The module will assume familiarity with some topics in algebra, such as commutative rings and modules.

Assessment strategy

Take-home coursework 1 (5%)

Take-home coursework 2 (5%)

Final Exam (90%)

MATH60043 Foundations of Modelling and Inference

Brief description

This module is concerned with principles of statistical inference, principles of modelling and design, with indicating the uncertainty involved in the conclusions of statistical analyses, and with assessing the relative merits of different methods of analysis. Both mathematical and conceptual aspects are involved.

Learning outcomes

On successful completion of this module, you will be able to:

- Demonstrate an awareness of the assumptions and limitations of statistical models by critically evaluating scientific claims in published work
- Identify the general statistical principles underpinning individual statistical methods;
- Select and apply statistical methods appropriate to commonly encountered modelling settings.
- Demonstrate an integrated understanding of the concepts of this module by independent study of related material

Indicative module content

An indicative list of sections and topics is:

- Principles of modelling
- Principles of inference
- Asymptotic techniques for use in statistics

Assessment strategy

Final Exam (100%)

MATH60044 Applied Statistical Inference

Brief description

This module builds on earlier statistical modelling work, and we begin with a review of the linear model in generality. We extend the linear model in two different directions: firstly, using generalized linear models (GLMs) to handle non-normally distributed errors, and secondly, using random effects to handle correlated observations. The module has an applied focus, relying on statistical computation in R to analyze a variety of data. We will discuss practical techniques for comparing models and for identifying when models fit that fit poorly.

Learning outcomes

On successful completion of this module, you will be able to:

- Use R to fit linear models.
- Select appropriate generalized linear models for modelling data with non-normal error distributions, e.g. poisson, binomial, gamma.
- Use models to give predictions, together with an associated confidence interval.
- Identify models that fit poorly, and suggest improved models.
- Model data with correlated observations using linear mixed models.
- Explain the properties of different estimators of random effects variances, such as maximum likelihood and REML.
- Interpret output from R using plain language.
- Demonstrate an integrated understanding of the concepts of this module by independent study of related material.

Indicative module content

An indicative list of sections and topics is:

- Linear models: Least squares, normal equations, Gauss-Markov theorem, goodness of fit, diagnostics. Studentized residuals. Confidence intervals and prediction intervals.
- Generalized linear models: Specification, estimation (iterated reweighted least squares), inference, diagnostics.
- Mixed models: Specification, estimation (ANOVA, maximum likelihood, REML) and inference (parametric bootstrap) for variance components.
- Independent study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

Assessment strategy

Data analysis task and report (25%)

Final exam (75%)

MATH60045 Applied Probability

Brief description

This module introduces stochastic processes and their applications. The theory of different kinds of processes will be described and illustrated with applications in several areas. The groundwork will be laid for further deep work, especially in such areas as genetics, engineering and finance.

Learning outcomes

On successful completion of this module, you will be able to:

- Use the Poisson process to model random arrivals.
- Extend the Poisson process to accommodate common departures from the Poisson assumptions.
- Work with Markov chains in continuous time.
- Determine the long-term behaviour of a continuous-time Markov chain.
- Determine whether states are recurrent or transient.
- Solve differential and difference equations to determine quantities of interest for stochastic processes.
- Explain basic properties of Brownian motion.
- Demonstrate an integrated understanding of the concepts of this module by independent study of related material.

Indicative module content

An indicative list of sections and topics is:

- Review of probability and discrete time Markov Chains
- Random walks
- Poisson processes and their properties: superposition, thinning of Poisson processes; Non-homogeneous, compound, and doubly stochastic Poisson processes.
- Autocorrelation functions.
- Probability generating functions.
- General continuous-time Markov chains: generator, forward and backward equations, holding times, stationarity, long-term behaviour, jump chain, explosion; birth and death processes, reversibility, recurrence/transience.
- Differential and difference equations and pgfs. Embedded processes. Time to extinction.
- Queues.
- Brownian motion and its properties.
- Independent study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

Assessment strategy

Final Exam (90%)

In-class test 1 (5%)

In-class test 2 (5%)

MATH60046 Time Series Analysis

Brief description

A time series is a series of data points indexed and evolving in time. They are prevalent in many areas of modern life, including science, engineering, business, economics, and finance. This module is a self-contained introduction to the analysis of time series. Weight is given to both the time domain and frequency domain viewpoints, and important structural features (e.g. stationarity, reversibility) are treated rigorously. Attention is given to estimation and prediction (forecasting), and useful computational algorithms and approaches are introduced.

Learning outcomes

On successful completion of this module, you will be able to:

- Appreciate that time series should be considered observations from an underlying stochastic process.
- Define what it means for a time series to be stationary.
- Identify autocorrelation within time series data.
- Work with standard models of time series.
- Appreciate that time series can exhibit trend and seasonality and know how to adjust for these.
- Determine the spectral representation of stationary time series and use the spectral density function to provide an alternative viewpoint of second-order structure.
- Derive and implement estimators of mean, correlation and spectral properties.
- Extend time series models, the notion of stationarity, and frequency domain representations to multivariate time series.
- Derive forecasts from standard time series models and quantify their uncertainty.
- Demonstrate an integrated understanding of the concepts of this module by independent study of related material.

Indicative module content

An indicative list of sections and topics is:

- Discrete time stochastic processes and examples.
- Autocovariance, autocorrelation, stationarity.
- Trend removal and seasonal adjustment.
- AR, MA and ARMA processes, characteristic polynomials, general linear process, invertibility, directionality and reversibility.
- Spectral representation, aliasing, linear filtering.
- Estimation of mean and autocovariance sequence, the periodogram, tapering for bias reduction.
- Parametric model fitting.
- Forecasting.
- Independent study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

Assessment strategy

Final Exam (100%)

MATH60047 Stochastic Simulation

Brief description

Computational techniques have become an important element of modern statistics. Computation particularly underpins simulation methods, which are widely applied when studying models of complex systems, e.g. in biology and in finance. This module provides an up-to-date view of such simulation methods, covering areas from basic random variate generation to advanced MCMC (Markov Chain Monte Carlo) methods. The implementation of stochastic simulation algorithms will be carried out in python, a language that is widely used for statistical computing and well suited to scientific programming more generally.

Learning outcomes

On successful completion of this module, you will be able to:

- Use algorithms for efficient generation of pseudo-random numbers.
- Evaluate intractable definite integrals using Monte Carlo methods.
- Implement diverse Monte Carlo algorithms to draw samples from intractable distributions.
- Interpret and assess the performance of Monte Carlo algorithms, for example by using suitable diagnostic procedures.
- Appropriately design and deploy MC algorithms on substantive data problems, and interpret outputs on substantive data problems.
- Demonstrate an integrated understanding of the concepts of this module by independent study of related material.

Indicative module content

An indicative list of sections and topics is:

- Pseudo-random number generators.
- Generalized methods for random variate generation.
- Monte Carlo integration.
- Variance reduction techniques.
- Monte Carlo methods (including eg Importance samplers, Metropolis-Hastings and Gibbs samplers, Langevin adjusted samplers).
- Monitoring and optimisation of MCMC methods.
- Introduction to sequential Monte Carlo methods.
- Independent study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

Assessment strategy

Computational take-home coursework (0%, must pass)

Final exam (100%) – will include theoretical questions and a reflective question on the coursework

MATH60048 Survival Models

Brief description

Survival models are fundamental to actuarial work, as well as being a key concept in medical statistics.

This module will introduce the ideas, placing particular emphasis on actuarial applications.

Learning outcomes

On successful completion of this module, you will be able to:

- Appreciate survival analysis models as a framework that deals with lifetimes and censored observations;
- Use several methods to define event time distributions and the relation between these definitions;
- Describe, select and use methods for fitting parametric, semi-parametric and non-parametric survival analysis models, including regression models and multi state models.
- Explain the counting process approach to survival analysis and its benefits;
- Use and critically analyse methods occurring in actuarial applications, such as methods for the construction and use of life tables.
- Demonstrate an integrated understanding of the concepts of this module by independent study of related material.

Indicative module content

An indicative list of sections and topics is:

- Concepts of survival models,
- Right and left censored data and randomly censored data.
- Estimation procedures for lifetime distributions:
- Empirical survival functions,
- Kaplan-Meier estimates,
- Cox model.
- Statistical models of transfers between multiple states,
- Maximum likelihood estimators.
- Counting process models.
- Actuarial Applications:
- Life table data and expectation of life,
- Binomial model of mortality,
- The Poisson model,
- Estimation of transition intensities that depend on age,
- Graduation and testing of crude and smoothed estimates for consistency.
- Independent study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

Assessment strategy

Final Exam (90%)

Take-home coursework including a data analysis task (10%)

MATH60049 Introduction to Statistical Learning

Brief description

This module provides an introduction to methods of statistical learning. That is, using statistical and artificial intelligence (AI) methods to learn from data, often when the data set is large, complex or of high dimension. We will consider both supervised and unsupervised learning. For the former we use a training set of data to learn patterns within data and then use our knowledge of those patterns to devise methods for predicting the outcomes of those patterns for new data. For the latter, there is not (usually) an outcome measure, but we seek to learn about the patterns within the data set itself. The methods in this module are of immense interest in academic and business circles and underpins much of the modern data tech industry to achieve tasks such as suggesting movies you might like to watch given information on those that you have watched, and from people with similar viewing patterns, developing machine methods for identifying cancer and seeking new ways to understand the economy. It is strongly recommended that students will have already passed the module Statistical Modelling I, or similar.

Learning outcomes

On successful completion of this module, you will be able to:

- Appreciate the structure and likely distribution(s) of various forms of data and which kinds of methods might be suitable for their analysis.
- Understand the concept of multivariate data and situations where supervised and unsupervised learning might be employed.
- Know and deploy a variety of important statistical and AI methods, using the R language to solve data related problems.
- Understand the limitations of methods and how their application can be checked and or corrected.
- Demonstrate an integrated understanding of the concepts of this module by independent study of related material.

Indicative module content

An indicative list of sections and topics is:

- Linear Models for Regression (variable selection, Lasso, Ridge Regression);
- Linear methods for classification (Logistic Regression);
- Basis expansions (piecewise polynomials and splines; smoothing splines, wavelets);
- Kernel regression (local linear and local polynomial);
- Additive models and Trees (boosting);
- Projection pursuit regression and Neural Networks;
- Support Vector Machines;
- Cluster Analysis and Multidimensional Scaling;
- Random Forests;
- Data Ethics.

Assessment strategy

Final Exam (100%)

MATH60050 Research Project in Mathematics

Brief description

The main aim of this module is to give a deep understanding of a particular area/topic by means of a supervised project in some area of mathematics. The project may be theoretical and/or computational and the area/topic for each student is chosen in consultation with the Department. The project is designed to be an introduction to research work in mathematics.

Learning outcomes

On successful completion of this module you will:

- have pursued a topic of mathematical research, beyond the scope of lectured modules, and demonstrated critical engagement with ideas at the frontier of present mathematical knowledge.
- have produced a written report, finished to a professional standard, containing an exposition of the topic of your project together with an account of any new results or investigations.
- have demonstrated your ability to evaluate and communicate mathematics effectively in both written and spoken formats.

Indicative module content

The project will be on a topic agreed between the student and the supervisor and (where appropriate) the Director of Undergraduate Studies in Mathematics. It will offer scope for the student to master material well beyond that offered in lectured modules, and to reach the frontiers of research.

Assessment strategy

Project work (independence, initiative, originality), written report and oral examination (100%)

Interim oral presentation (0%, must pass)

MATH60054 Introduction to Stochastic Differential Equations and Diffusion Processes

Brief description

This module provides an introduction to stochastic differential equations (SDEs) and diffusion processes, with an emphasis on solution methods and applications. The course covers the following topics: elements of probability theory for stochastic processes in continuous time, random walks and Brownian motion, Ito stochastic calculus from an applied perspective, the connection between SDEs and Fokker-Planck (FP) PDEs, methods for solving SDEs and the FP equation, first passage time problems, and approximation methods. Applications will include noise-induced escape problems, random search processes, molecular motors, stochastic resetting, diffusion approximations of chemical master equations, and reaction rate theory.

Learning outcomes

On successful completion of this module, you will be able to:

- formulate the basics of the theory of stochastic processes in continuous time;
- understand the connection between random walks and Brownian motion
- apply Ito's theory of stochastic integration;
- connect SDEs and the forward and backward Fokker-Planck (FP) equation
- construct stationary solutions and solve boundary value problems of the FP equation
- calculate splitting probabilities and mean first passage times for simple random search processes
- analyze noise-induced escape in the weak-noise limit
- carry out a system-size expansion of a Markov chain master equation

Indicative module content

This module will cover the following topics:

- Elements of probability theory and continuous stochastic processes;
- Random walks and Brownian motion;
- Stochastic differential equations and Ito calculus;
- The Fokker-Planck equation: connections between SDEs and PDEs; equilibrium and non-equilibrium stationary states; boundary value problems; multiplicative noise
- First passage time problems: bistable potentials; weak noise limits; random search processes; extreme statistics;
- Approximation schemes: system-size expansion, method of multiple scales

Assessment strategy

Take-home coursework (10%)

Final Exam (90%)

MATH60130 Stochastic Differential Equations in Financial Modeling

Brief description

To deal with valuation, hedging and risk management of financial options, we briefly introduce stochastic differential equations using a Riemann-Stieltjes approach to stochastic integration. We introduce no-arbitrage theory in continuous time based on replicating portfolios, self-financing conditions and Ito's formula. We derive prices as risk neutral expectations. We derive the Black Scholes model and introduce volatility smile models. We illustrate valuation of different options and introduce risk measures like Value at Risk and Expected Shortfall, motivating them with the financial crises.

Learning outcomes

On successful completion of this module you will be able to

- work comfortably with stochastic differential equations commonly encountered in finance;
- explain what is meant by no-arbitrage markets and why no-arbitrage is important operationally;
- connect no-arbitrage by replication to the existence of a risk neutral measure;
- price and hedge several types of financial options with several SDE models;
- calculate risk measures such as Value at Risk and Expected Shortfall;
- write code to price options according to SDE models covered in the module;
- independently appraise and evaluate SDE models for financial products;
- adapt a range of numerical methods and apply them in a coherent manner to unfamiliar and open problems in finance.

Indicative module content

1. Recap of key tools from probability theory
2. Brownian motion
3. Ito and Stratonovich stochastic integrals
4. Ito and Stratonovich stochastic differential equations (SDEs)
5. No-arbitrage through replication
6. No arbitrage through risk neutral measure
7. Derivation of the Black Scholes formula
8. Introduction of a few volatility smile models
9. Pricing of several types of options
10. Introduction to crises and risk measures
11. The Barings collapse and the introduction of value at risk (VaR)
12. Problems of VaR and an alternative: expected shortfall (ES)
13. Numerical examples and problems with risk measures, including software code.

Assessment strategy

Final Exam (100%)

MATH60132 Mathematical Logic

Brief description

The module is concerned with some of the foundational issues of mathematics: formal logic and set theory. In propositional and predicate logic, we analyse the way in which we reason formally about

mathematical structures. In set theory, we will look at the ZFC axioms and use these to develop the notion of cardinality. Material on model theory will involve all of these topics.

Learning outcomes

On successful completion of this module, you should be able to:

- Understand how the notion of truth is defined precisely in propositional and predicate logic and apply the definitions and accompanying results in a variety of contexts.
- Demonstrate understanding of formal systems for propositional and predicate logic by constructing examples of formal proofs and deductions, and by applying and deriving general results about these.
- Appreciate the expressive power of a first-order language (and its limitations) and compare structures via their first-order theories.
- Relate the semantic and syntactic aspects of formal logic, have an understanding of powerful general results such as the completeness and compactness theorems, and be able to apply these in a variety of ways.
- Use the ZFC axioms to justify constructions in set theory, ranging from elementary applications, to constructions involving transfinite recursion, ordinals, cardinals and applications of these.
- Use general results to compute and compare cardinalities of infinite sets.
- Communicate your knowledge of the area in a concise, accurate and coherent manner.
- Combine your knowledge of predicate logic and set theory in the study of model theory and apply the results to deepen your understanding of theories of first-order structures.

Indicative module content

The module is concerned with some of the foundational issues of mathematics. In propositional and predicate logic, we analyse the way in which we reason formally about mathematical structures. In set theory, we will look at the ZFC axioms and use these to develop the notion of cardinality. These topics have applications to other areas of mathematics: formal logic has applications via model theory and ZFC provides an essential toolkit for handling infinite objects.

An indicative list of sections and topics is:

- Propositional logic: Formulas and logical validity; a formal system; soundness and completeness.
- Predicate logic: First-order languages and structures; satisfaction and truth of formulas; the formal system; Goedel's completeness theorem; the compactness theorem; the Loewenheim- Skolem theorem.
- Set theory: The axioms of ZF set theory; ordinals; cardinality; the Axiom of Choice.
- Model theory: Elementary substructures; the method of diagrams, the Tarski-Vaught test; the general Loewenheim- Skolem Theorems; Reduced products and ultraproducts; Los' theorem.

There are no formal prerequisites for the module although a level of mathematical understanding such as would be provided by a second-year algebra or analysis module,

together with an appetite for abstraction and proofs, will be assumed. We will use basic notions from algebra (groups, rings and vector spaces) in examples.

Assessment strategy

Take-home coursework 1 (5%)

Take-home coursework 2 (5%)

Final Exam (90%)

MATH60135 Advanced Partial Differential Equations 1

Brief description

The focus of this course is on the concepts and techniques for solving partial differential equations (PDEs) that permeate various scientific disciplines. It is designed for a diverse audience in pure and applied mathematics, emphasizing rigor and the development of analytical proofs and techniques. The course places a strong emphasis on the theory of weak solutions of elliptic and parabolic equations and their regularity, involving distributions, Sobolev spaces, and the calculus of variations.

Learning outcomes

On successful completion of this module you should be able to:

- demonstrate an overview of a variety of partial differential equations, the behaviour of their solutions, and techniques to study them.
- understand and communicate some of the deep connections of PDEs to physics and geometry.
- state and prove well-posedness theorems for a variety of PDEs and explain their relevance.
- apply elliptic regularity theory in the theory of elliptic PDEs.
- apply suitable techniques to hyperbolic equations (wave equations).
- recognise and engage with current research in PDEs

Indicative module content

An indicative list of topics is:

- Introduction, examples of PDEs that appear in applications. Elliptic, parabolic, hyperbolic PDEs. Course overview.
- Distributions: definitions and examples, convergence and differentiability, support and convolution.
- Hölder and Sobolev spaces: definitions and examples, approximation, traces, compactness (Rellich-Kondrachov Theorem), Sobolev inequalities (Gagliardo-Nirenberg-Sobolev, Morrey, Poincaré), duality. Spaces involving time.
- Elliptic PDEs: Basic existence-uniqueness theory: Strong/uniform ellipticity; weak formulation; Lax-Milgram; energy estimates. Elliptic regularity theory: Difference quotients; interior and boundary regularity. Elements of calculus of variations.
- Parabolic PDEs: initial boundary value problems, weak formulation, Galerkin method. Parabolic Regularity theory. Maximum principle, Harnack's inequality. Semigroups, Hille-Yosida theorem.

Assessment strategy

Take-home coursework or in-class test (10%)
Final Exam (90%)

MATH60139 Spatial Statistics

Brief description

Data collected in space are common in many applications including climate science, epidemiology, and economics. This module covers theoretical and methodological statistical foundations for spatial data. The module is structured to cover in detail the three fundamental forms in which spatial data are collected: gridded data, network data, and point pattern data. For each data type, stochastic models will be defined and explored including random fields and point processes. Properties including isotropy, stationarity and homogeneity will be formalised and explored in the context of each model and data type. In addition, techniques for spatial interpolation will be studied.

Learning outcomes

On successful completion of this module students will be able to:

- Recognise the key differences between different types of spatial data (gridded, network and point pattern).
- Apply different classes of spatial covariance models to gridded and network data.
- Formulate different intensity functions for point pattern data.
- Define the concepts of homogeneity, stationarity and isotropy in the context of spatial models.
- Select, derive and apply appropriate methods for spatial interpolation.
- Demonstrate an integrated understanding of the concepts of the module by critical, independent study of research articles and books.

Indicative module content

1. Introduction to spatial data (gridded, network and point pattern data)
2. Random Fields and Covariance Functions
3. Spatial Interpolation and Kriging
4. Network Data and Markov Random Fields
5. Spatial Point Processes

Assessment strategy

Computational take-home coursework (10%)
Final Exam (90%)

MATH60140 Geometric Complex Analysis

Brief description

In this module we look at the subject of complex analysis from a more geometric point of view. We shall look at geometric notions associated with domains in the plane and their boundaries, and how they are transformed under holomorphic mappings. In turn, the behaviour of conformal maps is highly dependent on the shape of their domain of definition.

Learning outcomes

On successful completion of this module, you will be able to:

- identify features of, and develop arguments about, certain holomorphic maps;
- state, apply, and explain aspects of the Riemann Mapping Theorem for arbitrary simply connected plane domains;
- explain the automorphisms of the disk and the upper half plane;
- explain hyperbolic geometry, and basic notions of length, geodesics, isometries;
- apply the area theorem, and derive distortion estimates for arbitrary conformal mappings;
- acquire deeper understanding of holomorphic mappings through generalisation to quasi-conformal mappings;
- appreciate significance of and apply universal bounds in geometric function theory;
- explain the statement of the Beltrami-equation, and generalisation of the Riemann mapping theorem;
- work independently and with peers to understand abstract concepts in complex analysis.

Indicative module content

Complex analysis is the study of the functions of complex numbers. It is employed in a wide range of topics, including dynamical systems, algebraic geometry, number theory, and quantum field theory, to name a few. On the other hand, as the separate real and imaginary parts of any analytic function satisfy the Laplace equation, complex analysis is widely employed in the study of two-dimensional problems in physics such as hydrodynamics, thermodynamics, Ferromagnetism, and percolations.

In this module we look at the subject of complex analysis from a more geometric point of view. We shall look at geometric notions associated with domains in the plane and their boundaries, and how they are transformed under holomorphic mappings. In turn, the behaviour of conformal maps is highly dependent on the shape of their domain of definition.

An indicative list of topics is:

- Schwarz lemma and automorphisms of the disk;
- Riemann sphere and rational maps;
- Conformal geometry on the disk, Poincare metric, Isometries, Hyperbolic contractions;
- Conformal Mappings, Conformal mappings of special domains, Normal families, Montel's theorem;
- General form of Cauchy integral formula, Riemann mapping theorem; Growth and Distortion estimates, Area theorem;
- Quasi-conformal maps and Beltrami equation, Linear distortion, Dilatation quotient, Absolute continuity on lines, Quasi-conformal mappings, Beltrami equation, application of MRMT.

Prerequisites: It will be helpful if you have taken (or are taking) one or more of the following courses: Functional Analysis, Measure and Integration (or Lebesgue Measure and Integration), Fourier Analysis and Distributions.

Assessment strategy

2 x Take-home courseworks (10%)

Final Exam (90%)

MATH60141 Introduction to Game Theory

Brief description

This module will give students an insight into the wide variety of mathematics and its many applications within the area of game theory. The module aims to promote an active learning style, involving many classroom games as well as games to be played as homework.

The module will cover the classical theory of games involving concepts of dominance, best response and equilibria, where we will prove Nash's Theorem on the existence of equilibria in games. We will see the concept of when a game is termed zero-sum and prove the related Von Neumann's Minimax Theorem. We will briefly discuss cooperation in games and investigate the interesting Nash bargaining solution which arises beautifully from reasonable bargaining axioms.

Broadening our scope, we will look at the area of combinatorial game theory, building up our intuition through investigating the classical game of Nim in detail. We will also see the concept of a congestion game, often applied to situations involving traffic flow, where we will see the counter intuitive Braess paradox emerge and prove Nash's theorem in another context.

The module will finish with a small tour through some other areas and applications of game theory.

Learning outcomes

On successful completion of this module you will be able to:

- define the concepts of dominance, best-response and equilibria in a variety of competitive scenarios (games);
- solve (determine all equilibria or find optimal strategies) small games via a variety of techniques: iterated deletion of dominated strategies, finding equaliser strategies, use of subgames;
- determine when a game may be termed zero-sum, and be able to recognise, find and apply minimax and maximin strategies in these games;
- apply game theory to traffic flow or flow of information through networks, appreciating the differences and importance of optimal societal routing as compared with selfish individual routing;
- calculate bargaining solutions in simple co-operative games;
- determine whether communication is beneficial or not in different strategic situations;
- demonstrate an integrated understanding of the concepts of the module by critical, independent study of research articles and books.

Indicative module content

1. Recap of some basic notions in probability, calculus and analysis, some recap of induction in a game theoretic context.
2. Motivational/illustrative classroom games.
3. Dominance, best-response and equilibria.
4. Nash's theorem on equilibria in games.
5. Zero-sum games and Von Neumann's minimax theorem.
6. Subgame solutions as extensions to full game solutions.
7. Cooperative games, the Nash arbitration procedure and bargaining solutions.

8. Congestion games; Braess paradox, selfish routing vs optimal societal routing, existence of equilibria.
9. Combinatorial games; Nim, Nim sums and Nim values, sums of games.

Assessment strategy

Group take-home coursework (10%)

Final Exam (90%)

MATH60143 Dynamics, Symmetry and Integrability

Brief description

This module on Dynamics, Symmetry and Integrability is a friendly and fast-paced introduction to the geometric approach to proving integrability of classical Hamiltonian systems, at the level suitable for advanced undergraduates and first-year graduate students in mathematics. It fills a gap between traditional classical mechanics texts and advanced mathematical treatments of the geometric approach to integrability. The key idea is to use the momentum maps (e.g. from Noether's theorem) to find enough conservation laws to prove integrability. The main examples of integrable PDEs discussed are those that model shallow water waves, particularly the Korteweg-de Vries and Camassa-Holm equations.

Learning outcomes

On successful completion of this module, you will be able to:

- describe Hamiltonian motion on a smooth finite-dimensional manifold and demonstrate familiarity with the cotangent bundle (T^*M , phase space) and the definition of canonical Poisson brackets, as well as Hamiltonian vector fields, symplectic forms, symplectic transformations and solutions as characteristic flows of Hamiltonian vector fields on T^*M ;
- define Liouville integrability for finite-dimensional Hamiltonian dynamical systems and appreciate that Liouville integrability requires a sufficient number of functionally independent conservation laws in involution;
- select and use several other methods (introduced via worked examples) for acquiring the conservation laws necessary to prove integrability including: reduction to elliptic curves, isospectral reformulation in Lax pair form using covariant derivatives with zero curvature and transformation of variables to the momentum maps which arise in Noether's theorem;
- determine momaps for Hamiltonian systems with symmetry for a variety of classic finite-dimensional problems including: rigid body motion in $\mathbb{R}^n$, coupled nonlinear oscillators in C^2 and C^3 and the reduction of the CH equation to finite dimensions which results from a singular momap;
- interpret the Lax pair form of isospectral dynamics as coadjoint motion of a cotangent lift momentum map leading to the Lie-Poisson bracket which features widely in establishing the integrability of Hamiltonian systems;
- derive Magri's theorem for establishing integrability via isospectrality of bi-hamiltonian systems in infinite dimensions through the examples of the KdV and CH water wave equations with soliton solutions;
- appraise and interpret Hamiltonian methods of geometric mechanics for ideal fluid dynamics by applying V.I. Arnold's reformulation of Euler's 2D and 3D fluid solutions as geodesic flow on the manifold of smooth volume preserving flows with respect to the fluid kinetic energy which sets the stage for investigation of nonlinear stability of fluid equilibrium solutions.

Indicative module content

The module is composed of the following sections:

1 - Dynamics

The main ideas of the course are illuminated by considering cases when the solution dynamics on the configuration manifold may be lifted to a (non-Abelian) Lie group symmetry of the Hamiltonian. With an emphasis on applications in ODEs of finite-dimensional

mechanical systems, such as the rigid body $SO(3)$ and coupled resonant oscillations $U(2)$, and PDEs of nonlinear waves, such as the KdV and CH equation in infinite dimensions, the properties and results for integrability which are inherited from the geometrical formulation of dynamics induced by Lie group actions are discussed.

II - Symmetry

Symmetries of the Hamiltonian under Lie group transformations and their associated momentum maps are emphasised, both for reducing the number of independent degrees of freedom and in finding conservation laws by Noether's theorem.

III - Integrability

Definition: According to Liouville, a Hamiltonian system on a $2N$ -dimensional symplectic manifold M^{2N}

is completely integrable, if it possesses N functionally independent conservation laws which mutually commute under canonical Poisson brackets. What makes a dynamical system integrable, then? Enough conservation laws!

The course develops a series of geometrical methods for finding mutually Poisson-commuting conservation laws and thereby solving a sequence of integrable Hamiltonian problems ranging from rigid body motion to nonlinear wave PDEs. These methods include isospectral Lax pair formulations and algebraic geometry of elliptic curves for rigid body motion, as well as Lax equations and isospectrality principles due to bi-Hamiltonian structures for the KdV and CH water wave equations. In developing the solvability algorithms for this sequence of problems, the momentum map for the cotangent lift action of a Lie group on a manifold M plays a central role in representing the equations, their solutions and the analysis of their solution behaviour.

Assessment strategy

3 x Take-home courseworks (10%)

Final Exam (90%)

MATH60147 Statistical Mechanics

Brief description

This module is an introduction to the principles and applications of statistical mechanics, with a special emphasis on phase transitions. We will study in particular how macroscopic behaviors emerge from microscopic interactions, providing insights into systems at equilibrium and the critical phenomena associated with phase transitions.

Phase transitions are ubiquitous in nature and fundamental to understanding a wide array of emergent phenomena in the physical sciences (including the magnetization of materials, order-disorder transitions in solids, topological phase transitions, the liquid-liquid phase separation taking place inside biological cells, transition to collective motion in a flock of starlings) and beyond (including percolation on networks, the dynamics of opinion formation, financial market crashes, neural network phase transitions...).

This module aims to equip students with a solid understanding of the mechanisms underlying phase transitions. By the end of the course, you will be able to analyze and predict the behavior of complex systems. Through a blend of theoretical lectures and computational labs, you will gain a solid grasp of key concepts and mathematical tools in statistical mechanics, a deep appreciation of the universality of critical phenomena, and the ability to tackle advanced problems in a variety of fields.

Learning outcomes

On successful completion of this module students will be able to:

- Understand the fundamental principles of statistical mechanics including the concepts of ensembles, partition functions and phase transitions;
- classify and analyse different types of phase transitions, understand the concept of order parameters and describe symmetry breaking;
- apply mean-field theory and renormalization group theory to explain critical phenomena and phase transitions in various systems;
- connect the theoretical concepts introduced in lecture to real-world applications in physical and social sciences;
- implement numerical simulations to study the emergence properties of complex systems and compare your results to analytical arguments.
- demonstrate a holistic understanding of the concepts introduced in the course and show mastery of further advanced concepts in statistical mechanics through a critical analysis of reference textbooks and research articles in the field.

Indicative module content

Topics to be covered in lecture may include:

- 1) Principles of equilibrium statistical mechanics (ensembles, fundamental postulate of equilibrium statistical mechanics, elements of thermodynamics);
- 2) Introduction to spin models (including the Ising model and the XY model);
- 3) Notion of phase transitions, discontinuous and continuous phase transitions;;
- 4) Mean-field approaches;
- 5) Transfer matrix approach to the Ising model;
- 6) Landau theory of phase transitions - Fluctuations and the breakdown of Landau theory;
- 7) Critical points, scaling and critical exponents;

8) Renormalization group (Kadanoff approach).

Computational labs may include the following topics:

- 1) Short introduction to Monte Carlo methods;
- 2) Hard disks simulations – computation of partition function, Maxwell-Boltzmann velocity distribution, and macroscopic observables, liquid-solid transition;
- 3) Order-disorder transitions in spin systems and Glauber dynamics;
- 4) Generalized Ising models and spin glasses;
- 5) Dynamic Monte Carlo methods;
- 6) Agent-based models (illustrated via order-disorder transition in the Vicsek model).

Assessment strategy

Computational take-home coursework (50%)

Final Exam (50%)

MATH60150 Communicating Mathematics

Brief description

Communicating Mathematics consists of nine 2 hour weekly placements in local secondary schools assisting with the delivery of mathematics teaching during the spring term. Weekly lectures and fortnightly workshops will support student development of the skills and pedagogy they need to make the most of the placements.

Learning outcomes

On the successful completion of the module, you will be able to:

- 1) Devise appropriate ways to communicate mathematical concepts
- 2) Effectively communicate mathematics to students across a range of abilities and contexts
- 3) Evaluate and adapt to the learning needs of individual students
- 4) Critically evaluate and reflect on their own teaching practices and those of others
- 5) Creatively design and develop materials to support own teaching and enrich the practice of others
- 6) Demonstrate an integrated understanding of the concepts of the module by independent study of related material

Indicative module content

- 1) Introduction to working with children, safeguarding and conduct in the school environment.
- 2) Overview of the National Curriculum, its associated terminology and the mathematics within it. Comparison to other international education systems.
- 3) Teaching methodology and practical presentation skills for mathematics.
- 4) Principles of resource development and design including accessibility, modularity and sharability.

The placements will involve teaching observations, working as a teaching assistant and delivering some whole class teaching.

Assessment strategy

Teaching journal (25%)
Special project report (40%)
Oral presentation (25%)
Teacher assessment (10%)