IMPERIAL

DEPARTMENT OF MATHEMATICS

GUIDE TO OPTIONAL MODULES

FOURTH/FINAL YEAR (MSci)
2025-2026

Notes and syllabus details on Fourth Year modules for students in their Fourth/Final Year

For degree codings:

G103	MATHEMATICS (BSc, MSci)
G104	MATHEMATICS WITH A YEAR ABROAD (MSci)

Joint MATHEMATICS AND COMPUTER SCIENCE programmes are administered by the Department of Computing.

These notes should be read in conjunction with your undergraduate student handbook and the programme specifications for your year. Some of the information may be subject to alteration.

Updated information will be posted on the Maths Central Blackboard site.

Dr Christopher Hallsworth, Mathematics DUGS
September 2025.

FOURTH YEAR OVERVIEW

The MSci Fourth Year is available to those on the G103 and G104 codings who perform to a satisfactory standard in their Third Year, here or abroad. There is considerable overlap with the taught postgraduate MSc programmes in Pure and Applied Mathematics, but the MSci is a separate degree.

The MSci programme is designed to provide a breadth and depth in mathematics to a level of attainment broadly equivalent to that of an MSc degree and takes place over three terms – Term 1 (also known as Autumn Term), Term 2 (also known as Spring Term) and Term 3 (also known as Summer Term).

Students choose six lectured modules from those made available to them in the Department and from certain modules elsewhere. Students also take the compulsory MSci project, which is equivalent to two lecture modules.

Most, but not all, of the MATH7 modules are also available in MATH6 form and 4th year students take the MATH7 version. Fourth Year examinations normally consist of 4 questions and are 2 hours long, whereas the corresponding exams for 3rd year students (if any) contain 3 questions in 1.5 hours. Students may not take an MATH7 module if they have already taken the MATH6 version.

Lectures will take place during Term 1 and Term 2. Each module will typically have three hours per week, which usually includes some classes. The normal expectation is that there should be a 'lecture'/'class' balance of about 5/1. The identification of particular class times within the timetabled periods is at the discretion of the lecturer, in consultation with the class and as appropriate for the module material.

ADVICE ON THE CHOICE OF OPTIONS

Students are advised to read these notes carefully and to discuss their option selections with their Personal Tutor.

It is anticipated that lecturers will give advice on suitable books at the start of each module. Students should contact the proposed lecturers if they desire any further details about module content in order to make their choice of course options.

You will not be committed to your choice of most optional modules until the completion of your examination entry at the beginning of Term 2. The exception to this is that students do become committed to the completion of certain modules examined only by project at some stage during the module, as will be made clear by the lecturer at the start of the module.

MSci PROJECT

Individual Research Project
Compulsory

Supervised by Various Academic Staff
Co-ordinator: Dr T. Bertrand
(Terms 1, 2 & 3)

A fundamental part of the MSci degree is a substantial compulsory project equivalent to two lecture modules (15 ECTS). The main aim of this module is to give a deep understanding of a particular area/topic by means of a supervised project in some area of mathematics. The project may be theoretical and/or computational and the area/topic for each student is chosen in consultation with the Department.

The project provides an excellent *'apprenticeship in research'* and is therefore of particular value to students who are considering postgraduate study leading to a PhD.

The project may be theoretical and/or computational and the topic for each student is chosen in consultation with the Department. A list of project supervisors will be released shortly before the start of the Autumn term of year 4. Students will then be asked to provide preferred research areas/supervisors at the start of the term; allocations will be finalized shortly afterwards. Work on the project should continue throughout all three terms of year 4 and the project is submitted shortly after the year 4 summer examinations.

EXTERNAL MODULES

Subject to the Department's approval, students may take a module given outside the Department, e.g. in the Departments of Physics or Computing. Students must obtain permission from the Director of Undergraduate Studies if they wish to consider such an option. Where this permission is granted, it is always on the understanding that it is at the student's own risk. Students must satisfy themselves that they are comfortable with the methods and timing of assessments, and that external modules have an appropriate ECTS value.

GRADUATION

Students graduating will receive an MSci degree that explicitly incorporates a BSc.

It is normally required that MSci students pass all course components to graduate. However, the Board of Examiners may compensate narrowly failed modules up to 15 ECTS in the Final Year of study.

The total of marks for examinations, assessed coursework, progress tests, assignments and projects, with the appropriate year weightings, is calculated and presented at the Board of Examiners' Meeting (normally held at the end of June) for consideration by the Academic Staff and External Examiners. Degree classifications are determined according to the criteria given in the programme specification and borderline classification algorithm, which can be found on Maths Central. You may also wish to consult the programme specification:

https://www.imperial.ac.uk/mathematics/undergraduate/course-structure-and-content/

G104: For those on a Maths with a year abroad coding, the third year is spent abroad at another university. On return to Imperial, students take the regular Year 4 MSci programme. On the rare occasion that a G104 student performs very poorly in their year away they may, at the discretion of the Senior Tutor, be transferred to the BSc G100 Mathematics Degree and take MATH6 modules in their Final Year.

MARKS, YEAR TOTALS AND YEAR WEIGHTINGS

Within the Department each total module assessment is rescaled so that overall performances in different modules may be compared. The rescaling onto the scale 0 – 100 marks is such that 50 then corresponds to the lowest Pass Honours mark for a Masters level module and 70 corresponds to the lowest First Class performance.

Marks from the modules taken in the fourth year are combined into a year total expressed as a percentage.

The aggregate marks from each year will be combined with the following percentage weightings to produce an overall aggregate mark:

For G103

Year 1: 7.50 percent
Year 2: 20.00 percent
Year 3: 36.25 percent
Year 4: 36.25 percent,

For G104

Year 1: 7.50 percent
Year 2: 25.00 percent
Year 3: 25.00 percent
Year 4: 42.50 percent.

Criteria for Degree Classification at Borderlines

The discussion here refers to candidates at the first/upper-second borderline. Similar considerations apply to the other grade boundaries.

For all courses, a first is automatic for a programme total P of 69.5 or more. For candidates with $68 \leq P < 69.5$, the classification is determined by considering the uplift criteria as described below.

BSc courses

Classification is based on two uplift criteria: candidates with $68 \leq P < 69.5$ will be promoted to the first class if either of the criteria below is satisfied.

(a) Year 3 Total ≥ 69.5

(b) 30 ECTS in year 3 Mathematics modules at the higher classification (≥ 70 marks). External modules with a high mathematical content may be included, at the discretion of the board of examiners.

MSci courses

Classification is based on three uplift criteria: candidates with $68 \leq P < 69.5$ will be promoted to first class if any two of the criteria below are satisfied.

(a) Year 4 Total ≥ 69.5

(b) MSci project at the higher classification (≥ 70 marks)

(c) 22.5 ECTS in year 4 Mathematics electives at the higher classification (≥ 70 marks). External modules with a high mathematical content may be included, at the discretion of the board of examiners.

Mitigating Circumstances

Candidates with accepted mitigating circumstances for modules where no mitigation has been applied may be uplifted for $65 \leq P < 69.5$. In this case, the uplift criteria above may be modified to account for the circumstances of individual candidates. The rationale for any uplift should nevertheless make reference to the criteria above, e.g. noting the evidence for performance at the higher class in any components of the programme unaffected by mitigating circumstances. Care should be given to ensuring that mitigating circumstances are fully taken into account, without giving unfair advantage to the student, or subjecting them to more demanding requirements.

ECTS

To comply with the European ‘Bologna Process’, degree programmes are required to be rated via the ECTS (European Credit Transfer System) – which is based notionally on hour counts for elements within the degree. In principle, **1 ECTS should equate to around 25 hours of study (including examinations and private study).**

As in Third Year, each Fourth Year mathematics module has an ECTS value of 7.5 except for the Individual Research Project which has an ECTS value of 15. You should check the ECTS value of any modules taken from other departments You must complete 60 ECTS over the year.

MODULE ASSESSMENT AND EXAMINATIONS

Most MATH7 modules are examined by one written examination of 2 hours in length. Written examinations for MATH6 modules are typically 1.5 hours in length.

Most modules have some in-term assessment (coursework), which may take the form of an in-class test, problem sheet or a longer project. Typically, coursework makes up 10% of the overall module mark. Some modules have a more substantial coursework component (for example, 25%) and others are assessed entirely within the term. Details can be found in the tables below. Precise details of the number and nature of coursework assignments will be provided at the start of each module.

Students should bear in mind that modules assessed entirely within the term in which they are taught require considerable extra time-commitment during that term. You should note that, in principle, 7.5 ECTS represents 187.5 hours of effort on a module and completing this in a single term is a substantial task. For this reason, students may take at most one such module in a term.

Note: Students who take a module which is wholly assessed within the term will be deemed to be officially registered on the module after the submission of a specified number of pieces of assessed work for that module. Thus, once a certain point is reached in one of these modules, a student will be committed to completing it. In contrast, students only become committed to modules with summer examinations when they enter for the examinations in February. For modules outside Mathematics which have earlier examinations, you will be committed to the module once you register for the examination or at any earlier point specified by the host Department.

The MSci individual research project is assessed by a written report as well as an oral examination.

Further details of the assessment for the module Topics in Advanced Statistics can be found in the module description: the assessment pattern is different for the different component modules. Note that your chosen component modules for this module will appear separately on your transcript.

Students who do not obtain Passes in examinations at the first attempt may be expected to attend resit examinations. The Exam Board may award compensated passes in up to 15 ECTS of modules where the module mark is below the pass mark (50%) but at least 40%.

Resit examinations are for Pass credit only – a maximum mark of the pass mark (50 percent for Masters level modules) will be credited. Once a Pass is achieved, no further attempts are permitted.

FOURTH YEAR MODULE LIST

Note that not all of the individual modules listed below are offered every session and the Department reserves the right to cancel a particular module if, for example, the number of students attending that module does not make it viable. Similarly, some modules are occasionally run as 'Reading/Seminar Courses'.

Modules marked below with a * are also available in MATH6 form for Third Year undergraduates students (who typically take a shorter examination). When a module is offered it is usually, but not always, available in both forms. **No student may take both the MATH6 and MATH7 forms of a module.**

All MATH6 and MATH7 modules except the MSci project are equally weighted and are worth 7.5 ECTS unless otherwise specified. The MSci project is double-weighted and is worth 15 ECTS. The module Topics in Advanced Statistics is weighted the same as a standard MATH7 module but is worth 10 ECTS and students require permission from DUGS to take this module.

In the tables below, the % Exam indicates the weighting attached to the final written exam in Summer, unless otherwise indicated. The % CW indicates the weighting attached to any assessed work that is completed during the module. This may include in-class tests, projects, or problem sets to be turned in.

The groupings of modules below have been organised to indicate some natural affinities and connections.

The indicated lecturers are provisional; TBC indicates 'to be confirmed'.

APPLIED MATHEMATICS/MATHEMATICAL PHYSICS/NUMERICAL ANALYSIS

FLUIDS

College Module Code	Module Titles	Terms	Lecturer	% exam	% CW
MATH70001*	Fluid Dynamics 1	1	Professor X. Wu	90	10
MATH70002*	Fluid Dynamics 2	2	Professor J. Mestel	90	10

MATH70003*	Introduction to Geophysical Fluid Dynamics	2	Dr P. Berloff	90	10
MATH70051	Vortex Dynamics	2	Professor D. Crowdy	90	10
MATH70052	Hydrodynamic Stability	2	Professor X. Wu	90	10

MATHEMATICAL METHODS

MATH70004*	Asymptotic Methods	1	Dr O. Schnitzer	90	10
MATH70005*	Optimisation	1	Dr D. Kalise	90	10
MATH70006*	Applied Complex Analysis	2	Dr X. Guan	90	10
MATH70141*	Introduction to Game Theory	1	Dr S. Brzezicki	90	10

DYNAMICS

MATH70007*	Dynamics of Learning and Iterated Games	1	Professor S. van Strien	10 (Mid Term Test 30 (Oral)	60
MATH70008*	Dynamical Systems	1	Professor J. Lamb	90	10
MATH70009*	Bifurcation Theory	2	Professor D. Turaev	90	10
MATH70053	Random Dynamical Systems and Ergodic Theory: Seminar Course	2	Professor J. Lamb	40 (Oral)	60
MATH70143	Dynamics, Symmetry and Integrability	2	Professor D. Holm	90	10
MATH70023*	Computational Dynamical Systems	1	Dr E. Keaveny	50	50
MATH70146	Advanced Topics in Dynamical Systems	2	Professor S. van Strien	40 (Oral)	60

FINANCE

MATH70012*	Mathematical Finance: An Introduction to Option Pricing	1	Dr P. Siorpaes	90	10
MATH70130*	Stochastic Differential Equations in Financial Modelling	1	Professor D. Brigo	90	10

BIOLOGY

MATH70014*	Mathematical Biology	1	B. Bassols-Cornudella	90	10
MATH70137*	Mathematical Biology 2: Systems Biology	2	Dr O.Karin	90	10

MATHEMATICAL PHYSICS

MATH70015*	Quantum Mechanics I	1	Dr E-M Graefe	90	10
MATH70016*	Special Relativity and Electromagnetism	1	Dr G. Pruessner	90	10
MATH70017*	Tensor Calculus and General Relativity	2	Dr C. Ford	90	10
MATH70018*	Quantum Mechanics 2	2	Dr R. Barnett	90	10
MATH70147*	Statistical Mechanics	1	Dr T. Bertrand	50	50

APPLIED ANALYSIS

MATH70054*	Introduction to Stochastic Differential Equations and Diffusion Processes	1	Professor P. Bressloff	90	10
MATH70019*	Theory of Partial Differential Equations	1	Dr M. Sorella	90	10
MATH70021*	Advanced Partial Differential Equations 2	2	Professor M. Coti Zelati	90	10
MATH70135*	Advanced Partial Differential Equations 1	1	Professor G. Pavliotis	90	10

COMPUTATIONAL MATHEMATICS

MATH70022*	Finite Elements: Numerical Analysis and Implementation	2	Professor C. Cotter & Professor D. Ham	50	50
MATH70024*	Computational Linear Algebra	1	Dr A. Broms	50	50
MATH70025*	Computational Partial Differential Equations	2	Dr S. Mughal	0	100
MATH70026*	Methods for Data Science	2	Dr B. Bravi	0	100
MATH70148	Probabilistic Generative Models	2	Dr F. Tobar	60	40
MATH70134	Mathematical Foundations of Machine Learning	1	Dr N. Boulle	0	100

PURE MATHEMATICS

ANALYSIS

Note that some Analysis modules are taught by members of the Analysis, Probability and PDEs group, which sits within the Applied section of the department for organisation purposes – please see the Applied Analysis table.

College Module Code	Module Titles	Terms	Lecturer	% exam	% CW
MATH70028*	Probability Theory 1	1	Dr Z. Zhang	90	10
MATH70029*	Functional Analysis	1	Professor P. Germain	90	10
MATH70030*	Fourier Analysis and Theory of Distributions	2	Dr I. Krasovsky	90	10
MATH70055	Stochastic Calculus with Applications to non-Linear Filtering	2	Professor D. Crisan	90	10
MATH70031*	Probability Theory 2	2	B. Degallier	90	10

GEOMETRY

College Module Code	Module Titles	Terms	Lecturer	% exam	% CW
MATH70032*	Geometry of Curves and Surfaces	2	Dr P. Cameron	90	10
MATH70033*	Algebraic Curves	1	Dr S. Sivek	90	10
MATH70034*	Algebraic Topology	2	Dr S. Sivek	90	10
MATH70056	Algebraic Geometry	2	Dr. M. Booth	90	10
MATH70057	Riemannian Geometry	2	Dr M. Guaraco	90	10
MATH70058	Manifolds	1	Dr Y. Sun	90	10
MATH70059	Differential Topology	2	Dr M-A Lawn	90	10
MATH70060	Complex Manifolds	2	D. Parise	90	10
MATH70140*	Geometric Complex Analysis	2	Dr D. Cheraghi	90	10

ALGEBRA AND DISCRETE MATHEMATICS

MATH70035*	Algebra 3	1	Professor A. Corti	90	10
MATH70036*	Group Theory	1	Ms K. Kansal	90	10
MATH70037*	Galois Theory	1	Professor A. Corti	90	10
MATH70038*	Graph Theory	1	Dr M. Zordan	90	10
MATH70039*	Group Representation Theory	2	Dr N. Tamam	90	10
MATH70040*	Formalising Mathematics	2	Dr B. Mehta	0	100
MATH70061	Commutative Algebra	1	Prof P. Cascini	90	10

MATH70062	Lie Algebras	1	Professor T. Gee	90	10
MATH70063	Algebra 4	2	Dr B. Briggs	90	10
MATH70132*	Mathematical Logic	2	Professor D. Evans	90	10

NUMBER THEORY

MATH70041*	Number Theory	1	Professor A. Caraiani	90	10
MATH70042*	Algebraic Number Theory	2	Dr M. Pagano	90	10
MATH70064	Elliptic Curves	1	Prof Y. Lekili	90	10

STATISTICS

College Module Code	Module Titles	Terms	Lecturer	% exam	% CW
MATH70043*	Statistical Theory	1	Dr G. Rioux	90	10
MATH70044*	Applied Statistical Inference	1	Dr C. Hallsworth	75	25
MATH70045*	Applied Probability	1	Dr R. Ryder	90	10
MATH70046*	Time Series Analysis	2	Dr E. Cohen	100	
MATH70047*	Stochastic Simulation	2	Dr D. Akyildiz	75	25
MATH70048*	Survival Models	2	Dr R. Nuermaimaiti	90	10
MATH70049*	Introduction to Statistical Learning	2	Professor G. Nason	100	-
MATH70139*	Spatial Statistics	2	Dr A Sykulski	90	10
	Topics in Advanced Statistics (choose any two of the three options below; requires permission from DUGS)	2			
MATH70081	Non-parametric Statistics	2 (wks 7 – 11)	Dr Ray	100	0
MATH70083	Statistical Learning for High Dimensional Data	2 (wks 7 – 11)	Dr M. Evangelou	30 (Time limited class test	70
MATH70013	Advanced Simulation Methods	2 (wks 2 – 6)	Dr N. Kantas		100

PROJECT (Compulsory)

College Module Code	Module Titles	Terms	Lecturer	% exam	% CW

MATH70050	Research Project in Mathematics (MSci Project)	1, 2 + 3	Dr T. Bertrand	0	100

FOURTH YEAR MATHEMATICS SYLLABUSES

MATH70001 Fluid Dynamics 1

Brief Description

Fluid dynamics investigates motions of both liquids and gases. Being a major branch of continuum mechanics, it does not deal with individual molecules, but with an 'averaged' motion of the medium (i.e. collections of molecules). The aim is to predict the velocity, pressure and temperature fields in flows arising in nature and engineering applications. In this module, the equations governing fluid flows are derived by applying fundamental physical laws to the continuum. This is followed by descriptions of various techniques to simplify and solve the equations with the purpose of describing the motion of fluids under different conditions.

Learning Outcomes

On successful completion of this module you will be able to

- state the underlying assumptions of the continuum hypothesis;

- compare and contrast the different frameworks that can be used to describe fluid motion and to identify the connections between them;

- derive exact solutions of the Navier-Stokes equations and justify the physical and mathematical assumptions made in obtaining them;

- perform simplifications arising under the assumption of inviscid flow which permit the integration of the Euler equations, leading to results such as Bernoulli's equation and Kelvin's circulation theorem;

- demonstrate a sound understanding of the method of conformal mappings and be able to use this method to analyse various two-dimensional inviscid flows;

- choose the appropriate conformal mapping to solve inviscid flow problems in complicated geometries;

- predict the shape of the flow streamlines for such problems.

Module Content

The module is composed of the following sections:

Introduction: The continuum hypothesis. Knudsen number. The notion of fluid particle. Kinematics of the flow field. Lagrangian and Eulerian frameworks. Streamlines and pathlines. Strain rate tensor. Vorticity and circulation. Helmholtz's first theorem. Streamfunction.

Governing Equations: Continuity equation. Stress tensor and symmetry, Constitutive relation. The Navier-Stokes equations.

Exact Solutions of the Navier-Stokes Equations: Couette and Poiseuille flows. The flow between two coaxial cylinders. The flow over an impulsively started plate. Diffusion of a potential vortex.

Inviscid Flow Theory: Integrals of motion. Kelvin's circulation theorem. Potential flows. Bernoulli's equation. Cauchy-Bernoulli integral for unsteady flows. Two-dimensional flows. Complex potential. Vortex, source, dipole and the flow past a circular cylinder. Adjoint mass. Conformal mapping. Joukovskii transformation. Flows past aerofoils. Lift force. The theory of separated flows. Kirchhoff and Chaplygin models.

MATH70002 Fluid Dynamics 2

Brief Description

In this module, we deal with a wide class of realistic problems by seeking asymptotic solutions of the governing Navier-Stokes equations in various limits. We shall start with the "slow, small or sticky" case, when the Reynolds number is low and we obtain the linear Stokes equations. Then we consider the lubrication limit, and show how a thin layer of fluid is able to exert enormous pressures and prevent moving solid bodies from touching. Next we shall consider the "fast and vast" limit of high Reynolds number, which is characteristic of most flows we encounter in everyday life. In the final part of the module we consider a mixture of advanced topics, including flight, bio-fluid-dynamics and an introduction to flow stability.

Learning Outcomes

On successful completion of this module you will be able to:

- simplify and solve the governing Navier-Stokes equations in situations where there is a short lengthscale in one of the coordinate directions;

- apply the general properties of low Reynolds number flows to predict the drag on slow-moving bodies, like a solid sphere or spherical bubble, and appreciate the causes of the 'Stokes paradox';

- analyse lubrication-like flows in thin layers;

- derive the boundary-layer equations and identify self-similar solutions for flows at large Reynolds number;

- determine stability criteria for various fundamental flows;

- model animal locomotion at low and high Reynolds numbers;

- interpret results from advanced fluid mechanics textbooks and research papers;

- independently appraise and evaluate some advanced topics in viscous fluid mechanics.

Module Content

The module is composed of the following sections:

I – Low-Reynolds-number flows

Dynamic Similarity. Properties of the Stokes equations. Uniqueness and minimal dissipation theorems. The analysis of the flow past a solid sphere and spherical bubble. Stokes paradox.

II – Lubrication Theory

Derivation of Reynolds' lubrication equation and examples. Hele-Shaw and thin film flows.

III – High-Reynolds-number flows; Boundary-layer theory

The notion of singular perturbations. Derivation of boundary-layer equations. Blasius flow, Falkner-Skan solutions and applications. Von Mises variables and their application to periodic boundary layers. Prandtl-Batchelor Theorem for flows with closed streamlines.

IV – Introduction to hydrodynamic stability

Importance of stability. Rayleigh-Taylor and Kelvin-Helmholtz instabilities. Circular flow stability criterion.

V – Swimming and Flight; Animal locomotion

Scallop theorem. Resistive Force Theory. Introduction to 3D-aerofoil theory. Flight strategies.

VI - Advanced Topics

Current research in areas such as convection and magnetohydrodynamics.

MATH70003 Introduction to Geophysical Fluid Dynamics

Brief Description

This is an advanced-level fluid-dynamics course with geophysical flavours. The lectures target upper-level undergraduate and graduate students interested in the mathematics of planet Earth, and in the variety of motions and phenomena occurring in planetary atmospheres and oceans. The lectures provide a mix of theory and applications.

Learning Outcomes

On successful completion of this module you will be able to:

- demonstrate a deep understanding of the foundations of geophysical fluid dynamics;

- model a broad range of natural phenomena associated with the atmosphere and ocean;

- appreciate the main concepts and terminology used in the field;

- derive the boundary layer equations for flow in a rotating frame and justify the relative importance of various terms in the equations of motion;

- describe, select appropriately and apply a range of methods and techniques for solving practical problems;

- independently appraise an advanced topic in geophysical fluid dynamics;

- evaluate results from research papers in the field of geophysical fluid dynamics.

Module Content

The module is composed of the following sections:

I - Introduction and basics;

II - Governing equations (continuity of mass, material tracer, momentum equations, equation of state, thermodynamic equation, spherical coordinates, basic approximations);

III - Geostrophic dynamics (shallow-water model, potential vorticity conservation law, Rossby number expansion, geostrophic and hydrostatic balances, ageostrophic continuity, vorticity equation);

IV - Quasigeostrophic theory (two-layer model, potential vorticity conservation, continuous stratification, planetary geostrophy);

V - Ekman layers (boundary-layer analysis, Ekman pumping);

VI - Rossby waves (general properties of waves, physical mechanism, energetics, reflections, mean-flow effect, two-layer and continuously stratified models);

VII - Hydrodynamic instabilities (barotropic and baroclinic instabilities, necessary conditions, physical mechanisms, energy conversions, Eady and Phillips models);

VIII - Ageostrophic motions (linearized shallow-water model, Poincare and Kelvin waves, equatorial waves, ENSO “delayed oscillator”, geostrophic adjustment, deep-water and stratified gravity waves);

IX - Transport phenomena (Stokes drift, turbulent diffusion);

X - Nonlinear dynamics and wave-mean flow interactions (closure problem and eddy parameterization, triad interactions, Reynolds decomposition, integrals of motion, enstrophy equations, classical 3D turbulence, 2D turbulence, transformed Eulerian mean, Eliassen-Palm flux).

MATH70051 Vortex Dynamics

Brief Description

This is an advanced module in applied mathematical methods applied to the subfield of fluid dynamics called vortex dynamics. The module will focus on the mathematical study of the dynamics of vorticity in an ideal fluid in two and three dimensions. The material will be pitched in such a way that it will be of interest to those specializing in fluid dynamics but can also be viewed as an application of various techniques in dynamical systems theory.

Learning Outcomes

On successful completion of this module, you will be able to:

- interpret the role of vorticity within a range of problems in fluid mechanics;

- derive and compare a range of vortex models, from the point vortex models to distributed models, including vortex patches;

- combine your knowledge of different branches of mathematics (e.g. vector calculus, complex analysis and the theories of Hamiltonian dynamical systems and partial differential equations) in order to describe the dynamics of vorticity;

- choose from an array of applied mathematical techniques to explicitly solve for vorticity distributions;

- appraise the role that vortex structures play in modelling physical systems.

Module Content

The module will cover the following topics:

- Eulerian description of fluid flows;

- Incompressible flows and streamfunctions;

- Vorticity, vortex lines and vortex tubes;

- Biot-Savart law;

- Euler's equations and the vorticity equation;

- Kelvin's circulation theorem;

- Bernoulli theorems;

- Point vortex model, complex potentials;

- Point vortex equilibria;

- Dynamics of point vortices;

- Vortex dynamics on a spherical surface;

- Vortex patch models;

- Vortex patch equilibria;

- Vortex patch dynamics and contour dynamics;

- Other distributed vortex models.

MATH70052 Hydrodynamic Stability

Brief Description

Fluid flows may exist in two distinct forms: the simple laminar state which exhibits a high degree of order and the turbulent state characterised by its complex chaotic behaviours in both time and space. The transition from a laminar state to turbulence is due to hydrodynamic instability, which refers to the phenomenon that small disturbances to a simple state amplify significantly thereby destroying the latter. This is of profound scientific and technological importance because of its relevance to mixing and transport in the atmosphere and oceans, drag and aerodynamic heating experienced by air/spacecrafts, jet noise, combustion in engines and even the operation of proposed nuclear fusion devices.

Learning Outcomes

On successful completion of this module you will be able to

- construct the basic concepts underpinning hydrodynamic stability theory;

- predict linear stability properties based on eigenvalue analysis;

- compare and contrast various different instability mechanisms;

- derive various theorems that help us decide whether a flow is stable or unstable;

- model the effects of nonlinearity within an asymptotic framework.

Module Content

Topics covered will be a selection from the following list.

Basic concepts of stability; linear and nonlinear stability, initial-value and eigenvalue problems, normal modes, dispersion relations, temporal/spatial instability.

Buoyancy driven instability: Rayleigh-Benard instability, formulation of the linearised stability problem, Rayleigh number, Rayleigh-Benard convection cells, discussion of the neutral stability properties.

Centrifugal instability: Taylor-Couette flow, formulation of the linear stability problem, Taylor number, Taylor vortices; inviscid approximation, Rayleigh's criterion; viscous theory and solutions, characterization of stability properties; boundary layers over concave walls, Görtler number, Görtler instability, Görtler vortices.

Inviscid/viscous shear instabilities of parallel flows: Inviscid/Rayleigh instability, Rayleigh equation, Rayleigh's inflection point theorem, Fjortoft's theorem, Howard's semi-circle theorem, solutions for special profiles, KelvinHelmholtz instability, general characteristics of instability, critical layer, singularity; Viscous/Tollmien-Schlichting instability, Orr-Sommerfeld (O-S) equation, Squire's theorem, numerical methods for solving the linear stability problem, discussion of instability properties.

Inviscid/viscous shear instabilities of (weakly) non-parallel flows: local-parallel-flow approximation and

application to free shear layers and boundary layers; non-parallel-flow effects, rational explanation of viscous instability mechanism, high-Reynolds-number asymptotic theory, multi-scale approach, parabolised stabilityeq uations; transition process and prediction (correlation); receptivity.

Nonlinear instability: limitations of linear theories, bifurcation and nonlinear evolution; weakly nonlinear theory, derivation of Stuart-Landau and Ginzburg-Landau equations; nonlinear critical-layer theory.

MATH70004 Asymptotic Methods

Brief Description

This advanced course presents a systematical introduction to asymptotic methods, which form one of the cornerstones of modern applied mathematics. The foundation of asymptotic approximations is laid down first. The key ideas and techniques for deriving asymptotic representations of integrals, and for constructing appropriate solutions to differential equations will be explained. The techniques introduced find wide applications in engineering and natural sciences.

Learning Outcomes

On successful completion of this module, you will be able to:

- appreciate the foundation upon which asymptotic approximations are based;

- describe a variety of asymptotic methods and for each method acquire a thorough understanding of the key ideas involved and their mathematical nature;

- demonstrate basic skills in applying each of these methods to solve classical problems;

- combine, modify and extend methods to unfamiliar problems, such as those that emerge from research topics or practical applications;

- outline how asymptotic methods can in principle be applied to a wide variety of problems;

- interpret results from advanced textbooks and research papers on asymptotic methods;

- construct advanced solution techniques by selecting an appropriate combination of different asymptotic methods to solve higher-dimensional problems.

Module Content

I - Asymptotic approximations (fundamentals)

Order notation. Diverging series, asymptotic expansions. Parameter expansions, overlap regions, distinguished limits and uniform approximations.Stokes phenomenon.

II - Introduction to perturbation methods

Asymptotic solution of algebraic equations with a small parameter. Regular vs. singular perturbations. Method of dominant balance. Local analysis of ordinary differential equations.

III - Asymptotic analysis of integrals

Method of integration by parts. Integrals of Laplace type: Laplace's method, Watson's Lemma. Integrals of Fourier type: method of stationary phase. Integral in the complex plane: method of steepest descent. Method of splitting the range of integration.

IV - Matched asymptotic expansion

Inner and outer expansions, matching principles, notions of 'boundary layer' and interior layer. Composite approximation. Application to relaxation oscillations.

V - Methods of multiple scales

WKB approximations including turning-point problems and eigenvalue quantisation. Secular terms and solvability conditions. Poincare-Lindstedt method for periodic solutions. Multiscale method for quasi-periodic solutions. Application to weakly perturbed oscillators, nonlinear resonance, parametric resonance.

VI - A selection of topics from the following: Stokes phenomenon, hyperasymptotics, expansions involving logarithmic terms, homogenisation.

MATH70005 Optimisation

Brief Description

This module is an introduction to the theory and practice of mathematical optimization and its many applications in mathematics, data science, and engineering. The module aims at endowing students with the necessary mathematical background and a thorough methodological toolbox to formulate optimization problems and developing an algorithmic approach to its solution.The module is structured into five parts: (i) formulation and classification of problems; (ii) unconstrained optimization; (iii) stochastic and nature-inspired optimization; (iv) convex optimization; (v) introduction to optimal control and dynamic optimization. The assessed coursework for this module involves a series of computational tasks, to be completed in groups of two or three.

Learning Outcomes

On successful completion of this module you will be able to

- formulate a mathematical optimization problem by identifying a suitable objective and constraints;

- identify the mathematical structure of an optimization problem and, based on this classification, choose an appropriate methodological approach;

- develop a mathematical and computational appreciation of convexity as a fundamental feature in optimization;

- implement different computational optimization algorithms such as gradient descent and related variants;

- analyse the results of a computational optimization method in terms of optimality guarantees, sensitivities, and performance.

- interpret the role played by optimization in its application to computational data science;

- design optimal control approaches relevant to tackling large-scale nonlinear problems.

Module Content

1. Mathematical preliminaries

2. Unconstrained optimization

3. Gradient descent methods

4. Linear and non-linear least squares problems

5. Stochastic gradient descent

6. Nature-inspired optimization

7. Convex sets and functions

8. Convex optimization problems and stationarity

9. KKT conditions

10. Duality

11. Introduction to dynamic optimization and optimal control.

This final topic is linked to the Mastery Material for MSci students which will involve the study of some of the following solution techniques:

- shooting and multiple shooting methods;

- the reduced gradient approach;

- two-point boundary value solvers for optimal control;

- dynamic Programming and the Hamilton-Jacobi PDE;

- the linear-quadratic regulator and the Riccati equation.

This will be examined by way of an extra question on the May examination paper.

MATH70006 Applied Complex Analysis

Brief Description

The aim of this module is to learn tools and techniques from complex analysis and the theory of orthogonal polynomials that can be used in mathematical physics. The course will focus on mathematical techniques, though will also discuss relevant physical applications, such as electrostatic potential theory. The course incorporates computational techniques in the lectures.

Learning Outcomes

On successful completion of this module, you will be able to:

- apply the technique of contour deformation for calculating integrals;

- appreciate the connection that exists between computational tools such as quadrature and orthogonal polynomials and complex analysis;

- evaluate singular integral equations with Cauchy and logarithmic kernels;

- use the Wiener-Hopf method to solve a class of integral equations;

- compute matrix functions using contour inegration;

- interpret results from advanced textbooks and research papers;

- independently appraise and evaluate an advanced topic in complex analysis.

Module Content

This module covers the following topics:

Revision of complex analysis: complex integration, Cauchy's theorem and residue calculus;

Singular integrals: Cauchy, Hilbert, and log kernel transforms;

Potential theory: Laplace's equation, electrostatic potentials, distribution of charges in a well;

Riemann–Hilbert problems: Plemelj formulae, additive and multiplicative Riemann–Hilbert problems;

Orthogonal polynomials: recurrence relationships, solving differential equations, calculating singular integrals;

Integral equations: integral equations on the whole and half line, Fourier transforms, Laplace transforms;

Wiener–Hopf method: direct solution, solution via Riemann–Hilbert methods.

MATH70141 Introduction to Game Theory

Brief Description

This module will give students an insight into the wide variety of mathematics and its many applications within the area of game theory. The module aims to promote an active learning style, involving many classroom games as well as games to be played as homework.

The module will cover the classical theory of games involving concepts of dominance, best response and equilibria, where we will prove Nash's Theorem on the existence of equilibria in games. We will see the concept of when a game is termed zero-sum and prove the related Von Neumann's Minimax Theorem. We will briefly discuss cooperation in games and investigate the interesting Nash bargaining solution which arises beautifully from reasonable bargaining axioms.

Broadening our scope, we will look at the area of combinatorial game theory, building up our intuition through investigating the classical game of Nim in detail. We will also see the concept of a congestion game, often applied to situations involving traffic flow, where we will see the counter intuitive Braess paradox emerge and prove Nash's theorem in another context.

The module will finish with a small tour through some other areas and applications of game theory.

Learning Outcomes

On successful completion of this module you will be able to:

- define the concepts of dominance, best-response and equilibria in a variety of competitive scenarios (games);

- solve (determine all equilibria or find optimal strategies) small games via a variety of techniques: iterated deletion of dominated strategies, finding equaliser strategies, use of subgames;

- determine when a game may be termed zero-sum, and be able to recognise, find and apply minimax and maximin strategies in these games;

- apply game theory to traffic flow or flow of information through networks, appreciating the differences and importance of optimal societal routing as compared with selfish individual routing;

- calculate bargaining solutions in simple co-operative games;

- determine whether communication is beneficial or not in different strategic situations;

- demonstrate an integrated understanding of the concepts of the module by critical, independent study of research articles and books.

Indicative Module Content

1. Recap of some basic notions in probability, calculus and analysis, some recap of induction in a game theoretic context.

2. Motivational/illustrative classroom games.

3. Dominance, best-response and equilibria.

4. Nash's theorem on equilibria in games.

5. Zero-sum games and Von Neumann's minimax theorem.

6. Subgame solutions as extensions to full game solutions.

7. Coopertaive games, the Nash arbitration procedure and bargaining solutions.

8. Congestion games; Braess paradox, selfish routing vs optimal societal routing, existence of equilibria.

9. Combinatorial games; Nim, Nim sums and Nim values, sums of games.

MATH70007 Dynamics of Learning and Iterated Games

Brief Description

Recently there has been considerable interest in modelling learning. The settings to which these models are applied is wide-ranging. Examples include optimization of strategies of populations in ecology and biology, iterated strategies of people in a competitive environment and learning models used by technology companies such as Google.

This module is aimed at discussing a number of such models in which learning evolves over time and which have a game theoretic background. The module will use tools from the theory of dynamical systems and will aim to be rigorous. Topics will include replicator systems, best response dynamics and fictitious games, reinforcement learning and no-regret learning.

Learning Outcomes

On successful completion of this module, you will be able to:

- analyse 2D replicators systems for one and two player games;

- work comfortably with the notions of Nash, Correlated Equilibrium, Course Correlated Equilibrium and Evolutionary Stable Strategies;

- explain the notion of reciprocity in relation to Iterated Prisoner Dilemma games;

- appreciate the connection between Reinforcement Learning and replicator systems;

- outline the idea behind no regret learning models and the Blackwell approachability theorem;

- derive the proofs behind the methods that are used in the final project;

- appraise and interpret results from advanced textbooks and research papers.

Module Content

The module will cover the following topics:

- Replicator systems;

- Rock-paper-scissor games;

- Iterated prisoner dilemma games;

- Best response dynamics;

- Two player games;

- Fictitious games as a learning model;

- Reinforcement learning;

- No regret learning

MATH70008 Dynamical Systems

Brief Description

The theory of Dynamical Systems is an important area of mathematics which aims at describing objects whose state changes over time. For instance, the solar system comprising the sun and all planets is a dynamical system, and dynamical systems can be found in many other areas such as finance, physics, biology and social sciences. This course provides a rigorous treatment of the foundations of discrete-time dynamical systems.

Learning Outcomes

On successful completion of this module, you will be able to:

- demonstrate a familiarity with the basic concepts of topological dynamics;

- provide an outline of the ergodic theory of dynamical systems;

- appreciate the concept of symbolic dynamics through which topological and probabilistic dynamical properties can be understood;

- demonstrate an understanding of precise mathematical characterisations of chaotic dynamics;

- apply the above context in a number of one-dimensional settings, in particular in the context of piecewise affine expanding maps;

- independently appraise and evaluate advanced topological and probabilistic dynamical properties, beyond the foundations;

- independently develop and interpret examples in two and higher dimensions.

Module Content

The module covers the following topics:

- Introduction: orbits, periodic orbits and their local stability;

- Topological dynamics: invariant sets and limit sets, coding and sequence spaces, topological conjugacy, transitivity and mixing;

- Chaotic dynamics: sensitive dependence, topological entropy, topological Markov chains;

- Ergodic theory: sigma-algebras and measures, invariant measures, Poincaré recurrence, ergodicity and Birkhoff's Ergodic Theorem, Markov measures and metric entropy;

- Additional reading material in line with M4 objectives.

MATH70009 Bifurcation Theory

Brief Description

This module serves as an introduction to bifurcation theory, concerning the study of how the behaviour of dynamical systems such as ODEs and maps changes when parameters are varied.The goal is to acquaint the students with the foundations of the theory, its main discoveries and the universal methods behind this theory that extend beyond its remit.

Learning Outcomes

On successful completion of this module, you will be able to:

- exploit basic dimension reduction methods (invariant manifold and invariant foliations);

- apply the method of normal forms;

- demonstrate a sound knowledge of the basics of stability theory;

- appreciate the role of control parameters and to construct bifurcation diagrams;

- describe the mathematical framework associated with classical local and global bifurcations;

- interpret results from advanced textbooks and research papers on bifurcation theory;

- independently appraise and evaluate the transition from periodic to quasiperiodic regimes and to chaos via destruction of quasiperiodicity.

Module Content

The following topics will be covered:

1) Bifurcations on a line and on a plane;

2) Centre manifold theorem; local bifurcations of equilibrium states;

3) Local bifurcations of periodic orbits – folds and cusps;

4) Homoclinic loops: cases with simple dynamics, Shilnikov chaos, Lorenz attractor;

5) Saddle-node bifurcations: destruction of a torus, intermittency, blue-sky catastrophe;

6) Routes to chaos and homoclinic tangency.

MATH70053 Random Dynamical Systems and Ergodic Theory: Seminar Course

Brief Description

This is a course on the theory and applications of random dynamical systems and ergodic theory. Random dynamical systems are deterministic dynamical systems driven by a random input. The goal will be to present a solid introduction to the subject and then to touch upon several more advanced developments in this field.

Learning Outcomes

On successful completion of this module, you will be able to:

- describe the fundamental concepts of random dynamical systems;

- summarize the ergodic theory of random dynamical systems;

- select and critically appraise relevant research papers and chapters of research monographs;

- combine the ideas contained in such papers to provide a written overview of the current state of affairs concerning a particular aspect of random dynamical systems theory;

- thoughtfully engage orally in discussions related to random dynamical systems.

Module Content

Introductory lectures include foundational material on:

- Invariant measures and ergodic theory

- Random (pullback) attractors

- Lyapunov exponents

- Random circle homeomorphisms

Further material is at a more advanced level, touching upon current frontline research. Students select material from research level articles or book chapters.

MATH70143 Dynamics, Symmetry and Integrability

Brief Description

This module on Dynamics, Symmetry and Integrability is a friendly and fast-paced introduction to the geometric approach to proving integrability of classical Hamiltonian systems, at the level suitable for advanced undergraduates and first-year graduate students in mathematics. It fills a gap between traditional classical mechanics texts and advanced mathematical treatments of the geometric approach to integrability. The key idea is to use the momentum maps (e.g. from Noether's theorem) to find enough conservation laws to prove integrability. The main examples of integrable PDEs discussed are those that model shallow water waves, particularly the Korteweg-de Vries and Camassa-Holm equations.

Learning Outcomes

On successful completion of this module, you will be able to:
- describe Hamiltonian motion on a smooth finite-dimensional manifold and demonstrate familiarity with the cotangent bundle (T*M, phase space) and the definition of canonical Poisson brackets, as well as Hamiltonian vector fields, symplectic forms, symplectic transformations and solutions as characteristic flows of Hamiltonian vector fields on T*M;
- define Liouville integrability for finite-dimensional Hamiltonian dynamical systems and appreciate that Liouville integrability requires a sufficient number of functionally independent conservation laws in involution;
- select and use several other methods (introduced via worked examples) for acquiring the conservation laws necessary to prove integrability including: reduction to elliptic curves, isospectral reformulation in Lax pair form using covariant derivatives with zero curvature and transformation of variables to the momentum maps which arise in Noether's theorem;
- determine momaps for Hamiltonian systems with symmetry for a variety of classic finite-dimensional problems including: rigid body motion in Rn, coupled nonlinear oscillators in C2 and C3 and the reduction of the CH equation to finite dimensions which results from a singular momap;

- interpret the Lax pair form of isospectral dynamics as coadjoint motion of a cotangent lift momentum map leading to the Lie-Poisson bracket which features widely in establishing the integrability of Hamiltonian systems;
- derive Magri's theorem for establishing integrability via isospectrality of bi-hamiltonian systems in infinite dimensions through the examples of the KdV and CH water wave equations with soliton solutions;
- appraise and interpret Hamiltonian methods of geometric mechanics for ideal fluid dynamics by applying V.I. Arnold's reformulation of Euler's 2D and 3D fluid solutions as geodesic flow on the manifold of smooth volume preserving flows with respect to the fluid kinetic energy which sets the stage for investigation of nonlinear stability of fluid equilibrium solutions.

Module Content

The module is composed of the following sections:

I - Dynamics
The main ideas of the course are illuminated by considering cases when the solution dynamics on the configuration manifold may be lifted to a (non-Abelian) Lie group symmetry of the Hamiltonian. With an emphasis on applications in ODEs of finite-dimensional mechanical systems, such as the rigid body SO(3) and coupled resonant oscillations U(2), and PDEs of nonlinear waves, such as the KdV and CH equation in infinite dimensions, the properties and results for integrability which are inherited from the geometrical formulation of dynamics induced by Lie group actions are discussed.

II - Symmetry
Symmetries of the Hamiltonian under Lie group transformations and their associated momentum maps are emphasised, both for reducing the number of independent degrees of freedom and in finding conservation laws by Noether's theorem.

III - Integrability
Definition: According to Liouville, a Hamiltonian system on a 2N-dimensional symplectic manifold M2N is completely integrable, if it possesses N functionally independent conservation laws which mutually commute under canonical Poisson brackets. What makes a dynamical system integrable, then? Enough conservation laws!

The course develops a series of geometrical methods for finding mutually Poisson-commuting conservation laws and thereby solving a sequence of integrable Hamiltonian problems ranging from rigid body motion to nonlinear wave PDEs. These methods include isospectral Lax pair formulations and algebraic geometry of elliptic curves for rigid body motion, as well as Lax equations and isospectrality principles due to bi-Hamiltonian structures for the KdV and CH water wave equations. In developing the solvability algorithms for this sequence of problems, the momentum map for the cotangent lift action of a Lie group on a manifold M plays a central role in representing the equations, their solutions and the analysis of their solution behaviour.

MATH70023 Computational Dynamical Systems

Brief Description

Nonlinear differential equations arise across the scientific disciplines and often solutions can only be computed numerically. This module entails developing a foundation in the analysis and implementation of numerical methods to obtain these solutions. Along with generating solutions, the module will examine how these methods can also be used to perform bifurcation and stability analyses of steady and time periodic solutions, analyses often required to understand the qualitative features of the system. The module will have equal emphasis on theory, both in method analysis and background stability analysis, as well as implementation. As such assessment with be 50% project and 50% exam. Students will be expected to use Python in their assessments.

Note: this module replaces the module Numerical Solutions of Ordinary Differential Equations, and so may not be taken by students who have already taken the year 3 module MATH60023.

Learning Outcomes

On the successful completion of the module, you will be able to:

- use classical numerical methods to solve initial value problems;

- analyse different properties of numerical methods (e.g. accuracy and stability);

- compare different methods with respect to accuracy, stability, and computational complexity.

- obtain numerically steady state and time periodic solutions to differential equations

- perform numerically linear stability and Floquet analyses

- conduct numerical bifurcation analyses and understand normal forms for bifurcations

Module Content

This module will cover the following topics:

- Multi-step and multistage methods for solving initial value problems;

- Stability, accuracy and convergence;

- Newton methods for finding steady state and time periodic solutions;

- Algorithms for linear stability and Floquet analyses and associated theory;

- Jacobian-free algorithms for stability analysis

- Normal forms for bifurcations, and weakly nonlinear analysis;

- Applications of the methods to mathematical models arising across science and engineering;

MATH70146 Advanced Topics in Dyanmical Systems

Brief Description

The aim of this course concerns the in-depth study of an advanced topic in the theory of dynamical systems. The precise topic covered will vary from year to year depending on the most exciting recent research directions in the field and on the research directions undertaken in the research of MSc and PhD students in dynamical systems.

Learning Outcomes

On successful completion of this module, you will have:

- developed an understanding of an advanced topic in the theory of dynamical systems;

- worked independently to understand a research paper or a chapter of a research monograph;

- worked independently to produce an essay in which you report on the material that you have mastered;

- the ability to thoughtfully engage in a discussion about an advanced topic in dynamical systems.

Module Content

This course deals with topics in dynamical systems at an advanced level, touching upon current frontline research. Each year a selection will be made of material from the area of local bifurcation theory, global bifurcation theory, ergodic theory of dynamical systems or dynamical systems methods for PDEs/FDEs/networks, etc, in line with current interests of researchers in the dynamical systems group who moderate this course. In recent years, topics that were treated include homoclinic bifurcation theory (2019), circle homeomorphisms (2018) and stochastic approximation (2017). During the academic year 2024-2025 the module will be focussed on numerical methods in dynamical systems and data.

MATH70012 Mathematical Finance: An Introduction to Option Pricing

Brief Description

The mathematical modelling of derivatives securities, initiated by Bachelier in 1900 and developed by Black, Scholes and Merton in the 1970s, focuses on the pricing and hedging of options, futures and other derivatives, using a probabilistic representation of market uncertainty. This module is a mathematical introduction to this theory, in a discrete-time setting. We will mostly focus on the no-arbitrage theory in market models described by

trees; eventually we will take the continuous-time limit of a binomial tree to obtain the celebrated Black-Scholes model and pricing formula.

Learning Outcomes

On successful completion of this module, you will be able to:

- appreciate the fundamental principles involved in pricing derivatives;

- describe and critically analyse simple market models and explore their qualitative properties;

- confidently perform calculations involving pricing and hedging in discrete market models;

- demonstrate a familiarity with some key concepts in modern probability theory and apply them to perform computations;

- outline a mathematical formulation describing the behaviour of a number of financial derivatives;

- construct dynamic programming techniques to solve problems where inter-temporal relations are important;

- appraise and critically evaluate one or more of the advanced topics listed below.

Module Content

The module will cover the following topics:

financial derivatives, arbitrage, no-arbitrage pricing, self-financing portfolios, non-anticipative trading strategies, hedging of derivatives, domination property, complete markets, 'risk-neutral' probabilities, the fundamental theorems of asset pricing, conditional probability and expectation, filtrations, Markov processes, martingales, change of measure.

Extra mastery component will include the following advanced topics: utility, optimal investment.

MATH70130 Stochastic Differential Equations in Financial Modeling

Brief Description

To deal with valuation, hedging and risk management of financial options, we briefly introduce stochastic differential equations using a Riemann-Stiltjes approach to stochastic integration. We introduce no-arbitrage theory in continuous time based on replicating portfolios, self-financing conditions and Ito's formula. We derive prices as risk neutral expectations. We derive the Black Scholes model and introduce volatility smile models. We illustrate valuation of different options and introduce risk measures like Value at Risk and Expected Shortfall, motivating them with the financial crises.

Learning Outcomes

On successful completion of this module you will be able to

- work comfortably with stochastic differential equations commonly encountered in finance

- explain what is meant by no-arbitrage markets and why no-arbitrage is important operationally;

- connect no-arbitrage by replication to the existence of a risk neutral measure;

- price and hedge several types of financial options with several SDE models;

- calculate risk measures such as Value at Risk and Expected Shortfall;

- write code to price options according to SDE models covered in the module.

- independently appraise and evaluate SDE models for financial products.

- adapt a range of numerical methods and apply them in a coherent manner to unfamiliar and open problems in finance.

Module Content

1. Recap of key tools from probability theory

2. Brownian motion

3. Ito and Stratonovich stochastic integrals

4. Ito and Stratonovich stochastic differential equations (SDEs)

5. No-arbitrage through replication

6. No arbitrage though risk neutral measure

7. Derivation of the Black Scholes formula

8. Introduction of a few volatility smile models

9. Pricing of several types of options

10. Introduction to crises and risk measures

11. The Barings collapse and the introduction of value at risk (VaR)

12. Problems of VaR and an alternative: expected shortfall (ES)

13. Numerical examples and problems with risk measures, including software code.

MATH70014 Mathematical Biology

Brief Description

Mathematical biology entails the use of mathematics to model biological phenomena in order to understand these systems, as well as predict their behaviour. It is an incredibly diverse field utilising the complete mathematical toolbox to ascertain insight into many areas of biology and medicine including population dynamics, physiology, epidemiology, cell biology, biochemical reactions, and neurology. This module aims to provide a foundational course in the subject area relying primarily on tools from applied dynamical systems, applied PDEs, asymptotic analysis and stochastic processes.

Learning Outcomes

On successful completion of this module you will be able to:

- translate biological phenomena into the language of mathematics;

- appreciate canonical problems in epidemiology, ecology, biochemistry and physiology;

- critically analyse sets of ordinary differential equations especially in the non-linear setting;

- critically analyse sets of partial differential equations especially when either travelling-wave solutions or pattern forming phenomena might emerge;

- utilise the concept of stochastic population processes for exact and approximate solutions;

- use the techniques of order-of-magnitude reasoning and dimensional analysis;

- interpret results from the research literature on Mathematical Biology and analyze how the syllabus content relates to this wider body of work;

- appraise and evaluate an advanced topic in Mathematical Biology from a selection of case studies.

Module Content

Examples and topics include:

1) One-dimensional systems: existence and uniqueness; fixed points and their stability; bifurcations; logistic growth; SIS epidemic model; spruce budworm model; law of mass action; Michaelis-Menten enzyme dynamics.

2) Multidimensional systems: existence, uniqueness, fixed point stability; two-dimensional systems; SIS model for two populations; genetic control systems; population competition models; predator-prey dynamics and the Lotka-Volterra model.

3) Oscillations and bifurcations: Poincaré-Bendixson Theorem; oscillations in predator-prey models; relaxation oscillators; Fitzhugh-Nagumo model; fixed point bifurcations; Hopf bifurcations and limit cycles.

4) Spatial dynamics: reaction-diffusion equations; Fisher-Kolmogorov equation; travelling waves in predator-prey systems; spatial SIS model; spread of rabies in a fox population; Turing instabilities; pattern formation in one and two dimensions.

5) Stochastic processes: continuous-time Markov chains; simple birth and death processes; stationary probability distributions; logistic growth process; branching processes and drug resistance; multivariate processes; stochastic enzyme dynamics; stochastic predator-prey dynamics.

Jupyter notebooks containing codes written in Python will be utilised throughout the course and a working knowledge of, or a willingness to learn and use Python, is expected.

MATH70137 Mathematical Biology 2

Brief Description

This module will provide an introduction to the interdisciplinary field of mathematical systems biology. Drawing on analogies between biological and engineered systems, we will learn about mathematical approaches to model functional aspects of biological systems. We will discuss a wide range of topics, including control, memory, and computation in biological systems. Each topic will be discussed in the context of specific experimental systems.

Learning Outcomes

On successful completion of this module you will:

- be able to describe the major concepts and principles of systems biology, including design principles and emergent properties in biological systems

- be able to develop mathematical theories on functional aspects of biological systems, including biochemical and cellular circuits

- appreciate the role of feedback regulation in biological systems, and acquire tools to analyze feedback systems

- critically evaluate the relation between theory and experiment in systems biology

- acquire an understanding of a range of mathematical and computational motifs that play an important functional role in a wide range of biological systems

- develop an appreciation for the complexity and diversity of biological systems, and an understanding of the role of interdisciplinary approaches in advancing our understanding of these systems

- Demonstrate an integrated understanding of the concepts of the module by critical, independent study of research articles and books.

Indicative Module Content

1. Introduction to biological circuits

2. Negative and positive feedback (responses, oscillations, memory, differentiation)

3. Integral feedback (adaptation, scale invariance)

4. Gradients: sampling and optimization

5. Bifurcations and feedback tuning to bifurcations

6. Hopfield networks

7. Optimal control and learning

MATH70015 Quantum Mechanics I

Brief Description

Quantum mechanics is one of the most successful theories in modern physics and has an exceptionally beautiful underlying mathematical structure. It provides the basis for many areas of contemporary physics, including atomic and molecular, condensed matter, high-energy particle physics, quantum information theory, and quantum cosmology, and has led to countless technological applications. This module aims to provide an introduction to quantum phenomena and their mathematical description. We will use tools and concepts from various areas of mathematics and physics, such as classical mechanics, linear algebra, probability theory, numerical methods, analysis and geometry.

Learning Outcomes

On successful completion of this module, you will be able to:

- appreciate Schrödinger's formulation of quantum mechanics, wave functions and wave equations;

- construct the mathematical framework of quantum mechanics, including the 4 postulates of quantum mechanics and the Dirac notation;

- solve the eigenvalue problem for basic one-dimensional quantum systems;

- exploit the method of stationary states to deduce the time-evolved quantum state from the initial state of a system;

- communicate fluently using the Dirac notation;

- interpret results from advanced quantum mechanics textbooks and research papers;

- independently appraise and evaluate an advanced (more contemporary) topic in quantum mechanics from those listed in the syllabus below.

Module Content

The module will cover the following topics:

- Hamiltonian dynamics;

- Schrödinger equation and wave functions;

- stationary states of one-dimensional systems;

- mathematical foundations of quantum mechanics;

- quantum dynamics;

- angular momentum.

A selection of topics among the following additional optional topics will be covered depending on students interests:

- approximation techniques;

- explicitly time-dependent systems;

- geometric phases;

- numerical techniques;

- many-particle systems;

- cold atoms;

- entanglement and quantum information.

MATH70016 Special Relativity and Electromagnetism

Brief Description

This module presents a beautiful mathematical description of a physical theory of great historical, theoretical and technological importance. It demonstrates how advances in modern theoretical physics are being made and gives a glimpse of how other theories (say quantum chromodynamics) proceed. This module does not follow the classical presentation of special relativity by following its historical development, but takes the field theoretic route of postulating an action and determining the consequences. The lectures follow closely the famous textbook on the classical theory of fields by Landau and Lifshitz.

Learning Outcomes

On successful completion of this module you will be able to

- demonstrate an understanding of the relation between space and time and apply Lorentz transforms;

- appreciate the structure of special relativity as derived from the principle of least action;

- determine relativistic particle trajectories;

- derive Maxwell's equations from first principles and apply them to variety of interactions of charges and fields;

- critically analyse various solutions of the electromagnetic wave equations;

- describe electrostatic interactions and motion using Coulomb's law;

- construct an expansion of electrostatic interactions in terms of multipoles.

Module Content

This course follows closely the following book: L.D. Landau and E.M. Lifschitz, Course on Theoretical Physics Volume 2: Classical Theory of Fields.

Special relativity: Einstein's postulates, Lorentz transformation and its consequences, four vectors, dynamics of a particle, mass-energy equivalence, collisions, conserved quantities.

Electromagnetism: Magnetic and electric fields, their transformations and invariants, Maxwell's equations, conserved quantities, wave equation.

MATH70017 Tensor Calculus and General Relativity

Brief Description

This module provides an introduction to General Relativity. Starting with the rather simple Mathematics of Special Relativity the goal is to provide you with the mathematical tools to formulate General Relativity. Some examples, including the Schwarzschild space-time are considered in detail.

Learning Outcomes

On successful completion of this module, you will be able to:

- appreciate the application of tensors in special relativity;

- demonstrate a working knowledge of tensor calculus;

- explain the concepts of parallel transport and curvature;

- formulate and solve the geodesic equation for a given space-time metric;

- derive Einstein's field equations and analyse Schwarzschild's solution;

- interpret results from advanced general relativity textbooks and research papers;

- appraise and critically evaluate two of the extensions and applications listed below.

Module Content

This module will cover the following topics:

1. Special Relativity

2. Tensors in Special Relativity

3. Tensors in General Coordinates Systems 4. Parallel Transport and Curvature

5. General Relativity

6. The Schwarzschild Spacetime

7. Variational Methods

8. Extensions and Applications (selected from gravitational waves, Einstein-Hilbert action, cosmology, Einstein-Cartan theory, differential geometry)

MATH70018 Quantum Mechanics 2

Brief Description

Quantum mechanics (QM) is one of the most successful theories in modern physics and has an exceptionally beautiful underlying mathematical structure. Assuming some prior exposure to the subject (such as Quantum Mechanics I), this module aims to provide an intermediate/advanced treatment of quantum phenomena and their mathematical description. Quantum theory combines tools and concepts from various areas of mathematics and physics, such as classical mechanics, linear algebra, probability theory, numerical methods, analysis and geometry.

Learning Outcomes

On successful completion of this module, you will be able to:

- outline key aspects of quantum mechanics at the intermediate/advanced level;

- harness the power of symmetry in understanding quantum mechanics;

- describe many-particle quantum mechanical systems, and demonstrate familiarity with the formalism of second quantisation;

- solve complex quantum mechanical problems using the machinery introduced in this module;

- use the knowledge gained here as a solid foundation for a research project in quantum mechanics;

- interpret results from advanced quantum mechanics textbooks and research papers;

- appraise and evaluate a topic in quantum mechanics from the syllabus at an advanced level.

Module Content

This module will cover the following core topics:

- quantum mechanics in the momentum basis;

- the Heisenberg picture;

- the use of symmetry and general transformations in quantum mechanics;

- Elements of Quantum Computation;

- perturbation theory;

- adiabatic processes;

- second quantisation;

- introduction to many-particle systems;

- Fermi and Bose statistics.

Additional topics include: WKB theory, the Feynman path integral, quantum magnetism.

MATH70147 Statistical Mechanics

Brief Description

This module is an introduction to the principles and applications of statistical mechanics, with a special emphasis on phase transitions. We will study in particular how macroscopic behaviors emerge from microscopic interactions, providing insights into systems at equilibrium and the critical phenomena associated with phase transitions.

Phase transitions are ubiquitous in nature and fundamental to understanding a wide array of emergent phenomena in the physical sciences (including the magnetization of materials, order-disorder transitions in solids, topological phase transitions, the liquid-liquid phase separation taking place inside biological cells, transition to collective motion in a flock of starlings) and beyond (including percolation on networks, the dynamics of opinion formation, financial market crashes, neural network phase transitions...).

This module aims to equip students with a solid understanding of the mechanisms underlying phase transitions. By the end of the course, you will be able to analyze and predict the behavior of complex systems. Through a blend of theoretical lectures and computational labs, you will gain a solid grasp of key concepts and mathematical tools in statistical mechanics, a deep appreciation of the universality of critical phenomena, and the ability to tackle advanced problems in a variety of fields.

Learning Outcomes

On successful completion of this module students will be able to:

- Understand the fundamental principles of statistical mechanics including the concepts of ensembles, partition functions and phase transitions;

- classify and analyse different types of phase transitions, understand the concept of order parameters and describe symmetry breaking;

- apply mean-field theory and renormalization group theory to explain critical phenomena and phase transitions in various systems;

- connect the theoretical concepts introduced in lecture to real-world applications in physical and social sciences;

- implement numerical simulations to study the emergence properties of complex systems and compare your results to analytical arguments.

Module Content

Topics to be covered in lecture may include:

(1) Principles of equilibrium statistical mechanics (ensembles, fundamental postulate of equilibrium statistical mechanics, elements of thermodynamics);

(2) Introduction to spin models (including the Ising model and the XY model);

(3) Notion of phase transitions, discontinuous and continuous phase transitions;;

(4) Mean-field approaches;

(5) Transfer matrix approach to the Ising model;

(6) Landau theory of phase transitions - Fluctuations and the breakdown of Landau theory;

(7) Critical points, scaling and critical exponents;

(8) Renormalization group (Kadanoff approach);

(9) Kosterlitz-Thouless transition and spontaneous continuous symmetry breaking;

(10) Phase separation and Cahn-Hilliard equation.

Computational labs may include the following topics:

(1) Short introduction to Monte Carlo methods;

(2) Hard disks simulations – computation of partition function, Maxwell-Boltzmann velocity distribution, and macroscopic observables, liquid-solid transition;

(3) Order-disorder transitions in spin systems and Glauber dynamics;

(4) Generalized Ising models and spin glasses;

(5) Dynamic Monte Carlo methods;

(6) Agent-based models (illustrated via order-disorder transition in the Vicsek model).

MATH70054 Introduction to Stochastic Differential Equations and Diffusion Processes

Brief Description

This module provides an introduction to stochastic differential equations (SDEs), together with the necessary background material from stochastic analysis and the link between SDEs and partial differential equations. The course covers the following topics: elements of the theory of stochastic processes in continuous time, Brownian motion, construction of the Ito stochastic integral, existence and uniqueness theory for SDEs, methods for solving SDEs, connection between SDEs and Markov processes, the Fokker-Planck equation, ergodic theory for SDEs.

Learning Outcomes

On successful completion of this module, you will be able to:

- formulate the basics of the theory of stochastic processes in continuous time;

- appreciate the fundamental properties of Brownian motion;

- apply Ito's theory of stochastic integration;

- prove existence and uniqueness of solutions to stochastic differential equations under certain conditions;

- construct the link between stochastic differential equations and Markov processes;

- connect SDEs and the forward and backward (Fokker-Planck) partial differential equations;

- develop techniques for solving the Fokker-Planck equation;

- assemble tools from elementary Hilbert space theory to study the ergodic properties of SDEs.

Module Content

The module is composed of the following sections:

I - Introduction

II - Elements of probability theory and of stochastic processes in continuous time

III - Brownian motion and stochastic calculus

IV - Stochastic integrals

V - Stochastic differential equations

VI - Applications to partial differential equations

VII - Markov processes and invariant measures

MATH70019 Theory of Partial Differential Equations

Brief Description

In this module, students are exposed to different phenomena which are modelled by partial differential equations. The course emphasizes the mathematical analysis of these models and briefly introduces some numerical methods.

Learning Outcomes

On successful completion of this module you will be able to:

- appreciate how to formally differentiate complicated finite dimensional functionals and simple infinite dimensional functionals;

- describe, select and use a variety of methods for solving partial differential equations;

- outline how various partial differential equations respect conservation laws;

- utilize energy methods to critically analyse the stability of solutions to PDEs;

- develop the general method of characteristics and derive the eikonal equation;

- justify the proper use of the calculus of variations in classical settings.

Module Content

The module is composed of the following sections:

1. Introduction to PDEs

   1.1. Basic Concepts

   1.2. Gauss Theorem

2. Method of Characteristics

   2.1. Linear and Quasilinear first order PDEs in two independent variables.

   2.2. Scalar Conservation Laws

   2.3. Hamilton-Jacobi Equations. General Method of Characteristics.

3. Diffusion

   3.1. Heat equation. Maximum principle

   3.2. Separation of variables. Fourier Series.

4. Waves

   4.1. The 1D wave equation

   4.2. 2D and 3D waves.

5. Laplace-Poisson equation

5.1. Dirichlet and Neumann problems.

5.2. Introduction to calculus of variations. The Dirichlet principle.

5.3 Finite Element Method.

5.4 Lagrangians and the minimum action principle.

MATH70021 Advanced Partial Differential Equations 2

Brief Description

The focus of the course is the theory of nonlinear partial differential equations (PDEs) and their modern treatment through analytical techniques. The emphasis is on methods (such as fixed point theorems, Fourier analysis) and how they apply to classical problems involving fluid mechanics and wave propagation.

Learning Outcomes

On successful completion of this module you will be able to:

- appreciate the concepts of distribution (differentiation, convergence);

- manipulate the main properties of the Sobolev space H^m for integer m (inbeddings and compactness theorems, Poincaré inequality);

- derive the variational formulation of a specific elliptic boundary value problem and to provide the reasoning leading to the proof of the existence and uniqueness of the solution;

- develop the spectral theory of an elliptic boundary value problem;

- solve a parabolic boundary value problem using the spectral theory of the associated elliptic operator.

- interpret results from advanced textbooks and research papers on the theory of Partial Differential Equations;

- independently appraise and evaluate an advanced topic on Partial Differential Equations, namely the theory of nonlinear elliptic and parabolic equations on the whole space.

Module Content

An indicative list of topics is:

1. Fixed point theorems and applications: the contraction mapping principle, Brouwer and Schauder

fixed-point theorems, semilinear parabolic equations.

2. Elements of Fourier analysis: the Fourier transform, the method of stationary phases, singular

integrals.

3. Energy estimates and compactness: basic notions, spaces involving time, existence of solutions to nonlinear evolution equations.

4. Applications to equations arising in Mathematical Physics: the Euler and Navier-Stokes equations, dispersive equations, nonlinear heat equation, hyperbolic conservation laws.

MATH70135 Advanced Partial Differential Equations 1

Brief Description

The focus of this course is on the concepts and techniques for solving partial differential equations (PDEs) that permeate various scientific disciplines. It is designed for a diverse audience in pure and applied mathematics, emphasizing rigor and the development of analytical proofs and techniques. The course places a strong emphasis on the theory of weak solutions of elliptic and parabolic equations and their regularity, involving distributions, Sobolev spaces, and the calculus of variations.
Learning Outcomes

On successful completion of this module you should:

-- have developed an intuition for a variety of partial differential equations.
(behaviour of their solutions, techniques to study them.
-- understand some of the deep connections of PDE to physics and geometry.
-- be able to state and prove well-posedness theorems for a variety of PDE and understand their relevance.
-- be familiar with elliptic equations and elliptic regularity theory.
-- be familiar with hyperbolic equations (wave equations).
-- recognise and engage with current research in PDEs
Module content

An indicative list of topics is:

1. Introduction, examples of PDEs that appear in applications. Elliptic, parabolic, hyperbolic PDEs. Course overview.
2. Distributions: definitions and examples, convergence and differentiability, support and convolution.
3. H¨older and Sobolev spaces: definitions and examples, approximation, traces, compactness (Rellich-
Kondrachov Theorem), Sobolev inequalities (Gagliardo-Nirenberg-Sobolev, Morrey, Poincar´e), duality. Spaces involving time.
4. Elliptic PDEs: Basic existence-uniqueness theory: Strong/uniform ellipticity; weak formulation;

Lax-Milgram; energy estimates. Elliptic regularity theory: Difference quotients; interior and boundary regularity. Elements of calculus of variations.
5. Parabolic PDEs: initial boundary value problems, weak formulation, Galerkin method. Parabolic Regularity theory. Maximum principle, Harnack's inequality. Semigroups, Hille-Yosida theorem.

MATH70022 Finite Elements: Numerical Analysis and Implementation

Brief Description

Finite element methods form a flexible class of techniques for numerical solution of PDEs that are both accurate and efficient. The finite element method is a core mathematical technique underpinning much of the development of simulation science. Applications are as diverse as the structural mechanics of buildings, the weather forecast, and pricing financial instruments. Finite element methods have a powerful mathematical abstraction based on the language of function spaces, inner products, norms and operators.

Learning Outcomes

On successful completion of this module, you will be able to:

- appreciate the core mathematical principles of the finite element method;

- employ the finite element method to formulate and analyse numerical solutions to linear elliptic PDEs;

- implement the finite element method on a computer;

- compare the application of various software engineering techniques to numerical mathematics;

- generalize the concept of a directional derivative;

- appraise and evaluate techniques for solving nonlinear PDEs using the finite element method.

Module Content

This module aims to develop a deep understanding of the finite element method by spanning both its analysis and implementation. In the analysis part of the module, students will employ the mathematical abstractions of the finite element method to analyse the existence, stability and accuracy of numerical solutions to PDEs. At the same time, in the implementation part of the module students will combine these abstractions with modern software engineering tools to create and understand a computer implementation of the finite element method.

This module is composed of the following sections:

I - Basic concepts: weak formulation of boundary value problems, Ritz-Galerkin approximation, error estimates, piecewise polynomial spaces, local estimates;

II - Efficient construction of finite element spaces in one dimension: 1D quadrature, global assembly of mass matrix and Laplace matrix;

III - Construction of a finite element space: Ciarlet's finite element, various element types, finite element interpolants;

IV - Construction of local bases for finite elements: efficient local assembly;

V - Sobolev Spaces: generalised derivatives, Sobolev norms and spaces, Sobolev's inequality;

VI - Numerical quadrature on simplices: employing the pullback to integrate on a reference element;

VII - Variational formulation of elliptic boundary value problems: Riesz representation theorem, symmetric and nonsymmetric variational problems, Lax-Milgram theorem, finite element approximation estimates;

VIII - Computational meshes: meshes as graphs of topological entities, discrete function spaces on meshes, local and global numbering;

IX - Global assembly for Poisson equation: implementation of boundary conditions, general approach for nonlinear elliptic PDEs;

X - Variational problems: Poisson's equation, variational approximation of Poisson's equation, elliptic regularity estimates, general second-order elliptic operators and their variational approximation;

XI - Residual form and the Gâteaux derivative;

XII - Newton solvers and convergence criteria.

MATH70024 Computational Linear Algebra

Brief Description

Linear systems of equations arise in countless applications and problems in mathematics, science and engineering. Often these systems are large and require a computer to solve. This course provides an overview of the algorithms used to solve linear systems and eigenvalue problems, in terms of their development, stability properties, and application.

Learning Outcomes

On successful completion of this module, you will be able to

- describe, select and use algorithms for QR decomposition of matrices;

- solve least-squares problems using QR decomposition;

- apply LU decomposition to solve linear systems;

- analyse and modify algorithms that take advantage of matrix structure;

- find numerical solutions to eigenvalue problems;

- critically analyse various iterative methods for solving linear systems.

- combine the techniques you have mastered in order to assess unseen algorithms;

- adapt the techniques to analyse related topics such as functions of matrices.

Module Content

The module will cover the following topics:

1) Direct methods:

Triangular and banded matrices, Gauss elimination, LU-decomposition, conditioning and finite-precision arithmetic, pivoting, Cholesky factorisation, QR factorisation and their numerical implementation.

2) Eigenvalue problems:

power method and variants, Jacobi's method, Householder reduction to tridiagonal form, eigenvalues of tridiagonal matrices, the QR method.

3) Iterative methods:

Krylov subspace methods: Lanczos method and Arnoldi iteration, conjugate gradient method, GMRES, preconditioning.

MATH70025 Computational Partial Differential Equations

Brief Description

This module will introduce a variety of computational approaches for solving partial differential equations, focusing mostly on finite difference methods, but also touching on finite volume and spectral methods. Students will gain experience implementing the methods and writing/modifying short programs in Matlab or another programming language of their choice. Applications will be drawn from problems arising in areas such as Mathematical Biology and Fluid Dynamics.

Learning Outcomes

On successful completion of this module, you will be able to:

- appreciate the physical and mathematical differences between different types of PDES;

- design suitable finite difference methods to solve each type of PDE;

- outline a theoretical approach to testing the stability of a given algorithm;

- determine the order of convergence of a given algorithm;

- demonstrate familiarity with the implementation and rationale of multigrid methods;

- develop finite-difference based software for use on research level problems;

- communicate your research findings as a poster, in a form suitable for presentation at a scientific conference.

Module Content

The module will cover the following topics:

1) Introduction to Finite Differences

2) Classificaton of PDEs

3) Explicit and Implicit methods for Parabolic PDEs

4) Iterative Methods for Elliptic PDEs. Jacobi, Gauss-Seidel, Overrelaxation

5) Multigrid Methods

6) Hyperbolic PDEs. Nonlinear Advection/Diffusion systems. Waves and PMLs

as well as various advanced practical topics from Fluid Dynamics, which will depend on the final project.

MATH70026 Methods for Data Science

Brief Description

This module provides an hands-on introduction to the methods of modern data science. Through interactive lectures, the student will be introduced to data visualisation and analysis as well as the fundamentals of machine learning. The module will be assessed through computational projects and an in-class test.

Learning Outcomes

On successful completion of this module, you will be able to:

- Visualise and explore data using computational tools;

- Appreciate the fundamental concepts and challenges of learning from data;

- Analyse some commonly used learning methods;

- Compare learning methods and determine suitability for a given problem;

- Describe the principles and differences between supervised and unsupervised learning;

- Clearly and succinctly communicate the results of a data analysis or learning application;

- Appraise and evaluate new algorithms and computational methods presented in scientific and mathematical journals;

- Design and implement newly-developed algorithms and methods.

Module Content

The module is composed of the following sections:

- Introduction to computational tools for data analysis and visualisation;

- Introduction to exploratory data analysis;

- Mathematical challenges in learning from data: optimisation;

- Methods in Machine Learning: supervised and unsupervised; neural networks and deep learning; graph-based data learning;

- Machine learning in practice: application of commonly used methods to data science problems. Methods include: regressions, k-nearest neighbours, random forests, support vector machines, neural networks, principal component analysis, k-means, spectral clustering, manifold learning, network statistics, community detection;

- Current research questions in data analysis and machine learning and associated numerical methods.

MATH70148 Probabilistic Generative Models

Brief Description

Probabilistic generative models (PGM) are at the forefront of statistical machine learning research and are central in contemporary AI applications. This module develops a foundation for the design, analysis and implementation of PGMs. The module starts from the fundamentals of training and inference and then studies different PGMs, including parametric, autoregressive, explicit, implicit and (Bayesian) nonparametric models. Dissimilarity metrics are also examined, all to establish a comprehensive setting for the analysis of the models under study. Emphasising theory and practice, the module's assessment features two courseworks (20% each) and an exam (60%). Students are expected to be familiar with classic machine learning models and use Python in their assessments.

Learning Outcomes

On the successful completion of the module, you will be able to:

- select which generative model is appropriate for a given data analysis setup;

- train different types of generative models using dedicated Python toolboxes;

- calculate the similarity between different models;

- obtain samples from a trained model in an efficient and accurate manner;

- determine whether a training procedure has concluded successfully

- demonstrate an integrated understanding of the concepts of this module by independent study of related material.

Module Content

This module will cover the following topics:

- Fundamentals: graphical models, exact and approximante Bayesian inference;

- Dissimilarity measures: optimal transport, maximum mean discrepancy, notions of information theory;

- Explicit models: normalising flows;

- Variational autoencoders & generative adversarial networks;

- Autoregressive models: linear filters, recurrent neural networks, transformers;

- Bayesian nonparametric models: Dirichlet process & Gaussian process;

- Diffusion models & score-based models

MATH70134 Mathematical Foundations of Machine Learning

Brief Description

Machine learning techniques such as deep learning have recently achieved remarkable results in a very wide variety of applications such as image recognition, self-driving vehicles, partial differential equation solvers, trading strategies. However, how and why the recent (deep learning) models work is often still not fully understood. In this course we will begin with a general introduction into machine learning and continue to deep learning. We will focus on better some observed phenomena in deep learning aiming to gain insight into the impact of the optimization algorithms and network architecture through mathematical tools.

Learning Outcomes

On successful completion of this module you will be able to:

1) Demonstrate working familiarity with machine learning principles,

2) Design models using a variety of deep learning architectures

3) Implement neural network models in code

4) Select appropriate optimization algorithms to train deep learning models

5) Evaluate the ability of models to generalize by comparing their training and test performance

6) Independently evaluate new methodologies in deep learning

Module Content

- The preliminaries: pre-processing: data cleaning, dimensionality reduction, clustering

- Regression (linear, Bayesian) and classification (a basic overview)

- Neural networks and a variety of architectures (fully-connected, convolutional)

- Generalisation and overfitting

- Training methods for neural networks and their impact on performance

- The role of noise

- Flat minima and escape times

- Links of neural networks to Gaussian processes

- Explainability in neural networks through reconstruction

MATH70028 Probability Theory 1

Brief Description

This module provides a rigorous approach to the fundamental properties of probability. It teaches fundamental notions and structures as well as tools relevant to modern probability theory and applications. The module is important for further study of probability theory and stochastic processes.

Learning Outcomes

On successful completion of this module, you should be able to:

- Demonstrate knowledge of fundamental notions and structures of Probability Theory;

- Use the tools developped in a variety of applications;

- Demonstrate problem solving skills in probability theory

-- Demonstrate additional competence in the subject through self-study of more advanced material

-- Combine material from across the module to solve more advanced problems

- Communicate your knowledge of the area in a concise, accurate and coherent manner.

Module Content

An indicative list of topics is:

Probability spaces. Random variables: (Bernoulli, Rademacher, Gaussian variables with in- tegration by parts formula). Probability Distributions.

Basic probability inequalities: Jensen, Tshebychev, Poincare* & Log-Sobolev Inequalities *. Tail of Distribution Estimates.

Convergence in probability, in p-th moment, almost everywhere. 0-1 Law.

Mutual Independence of Events/Random Variable and Vieta Formula. Product Probability Spaces. Conditional Expectations and Independence. Borel-Cantelli Lemmas.

Weak and Strong Laws of Large Numbers for Random Sequences and Series of Mutually Independent or Weakly Correlated Random Variables.

Applications : [Probabilistic proof of Weierstrass Theorem, Monte Carlo Method for Large Dimensional Integration, Macmillan's Theorem, Infinitely Often Events: Decadence and Re- currence of Human Civilisations, Normal Numbers...]

Weak Convergence & Characteristic Functions. Central Limit Theorem.

Infinite Product of Bernoulli measures versus Gaussian measure.

Birkhoff Ergodic Theorem.* Elements of Brownian motion.* Martingales.*

Topics denoted by * are more advanced and require self-study through directed reading.

MATH70029 Functional Analysis

Brief Description

This module brings together ideas of continuity and linear algebra. It concerns vector spaces with a distance, and involves linear maps. The vector spaces are often spaces of functions. It is an important requirement for further study of many areas of Mathematical Analysis including PDEs, Stochastic Analysis, Dynamical Systems and Quantum Mechanics.

Learning Outcomes

On successful completion of this module, you should be able to:

- Demonstrate knowledge of fundamental notions and structures of Functional Analysis by proving a range of results;

- Use the tools developped in a variety of applications;

- Demonstrate problem solving skills in functional analysis

-- Demonstrate additional competence in the subject through self-study of more advanced material

- Synthesise topics from across the module to solve problems on more advanced applications

- Communicate your knowledge of the area in a concise, accurate and coherent manner.

Module Content

An indicative list of topics is:

Metric Linear Spaces and basic examples of topological spaces with non metrisable topology.

Minkowski and Hoelder Inequality.

Existence of Hamel basis (axiom of choice 1st time).

Normed vector spaces

& example of not normed Frechet space (Schwartz test functions).

Banach spaces.

Classical Banach Spaces: l_p, c, c_0, L_p(μ), C(Ω), C^(m)(Ω).

Closed Subspaces, Completness, Separability and Compactness in Classical Spaces.

Schauder Basis.

Continuous linear maps.

Banach contraction mapping principle and applications to integral equations (Frdholm+Volterra).
Finite dimensional spaces.

The Hilbert space (orthonormal basis).

The Riesz-Fisher Theorem.

The Hahn-Banach Theorem. (Banach Limit.)

Dual spaces: Dual spaces of classical spaces. Reflexive Non-reflexive spaces.

Baire Cathegory Theorem (axiom of choice again).

Principle of Uniform Boundedness. (Application to Fourier Series).

Open Mapping and Closed Graph Theorems.

Compact operators.

Hermitian operators and the Spectral Theorem.

In addition to the above topics, this level 7 version of the module will also involve study of:

Gauge Norms and Orlicz spaces.

Weak topology (Banach-Alaoglu Thm)

Sobolev Spaces.

The module provides a general orientation in contemporary research problems in Mathematical Analysis includ- ing PDEs , Stochastic Analysis, Dynamical Systems and Quantum Mechanics.

MATH70030 Fourier Analysis and Theory of Distributions

Brief Description

Fourier analysis is an important tool used in various branches of mathematics and beyond. The module provides a deeper understanding of it than what is briefly mentioned in general analysis courses. It also connects it to the theory of distributions. As a result of studying the module, students will understand the basics of the Fourier analysis and theory of distributions which will be sufficient for most branches of mathematics.

Learning Outcomes

On successful completion of this module, you will be able to:

On successful completion of this module, you will be able to:

-understand the issues of convergence for Fourier series,

-apply the Fourier and Laplace transforms,

-understand the motivation behind the notion of distribution,

-be in command of the basics of Fourier analysis and distribution theory sufficient for working in many areas of mathematics

– demonstate competence with further advanced material in the area designated for self-study

-- synthesise material from across the module to apply to advanced topics

Module Content

The module will assume familiarity with measure theory and functional analysis, especially L^p spaces and linear functionals.

Indicative content: Orthogonal systems in infinite-dimensional Euclidean spaces, Bessel inequality, Parseval equality, general Fourier series, trigonometric basis in L_2[-Pi,Pi], convergence of trigonometric Fourier series, Fejer's theorem and applications, Fourier transform and its properties, application to solution of differential equations, Plancherel theorem, Laplace transform, linear functionals, distributions, basic properties of distributions and applications, Fourier transform for distributions.

Those students who decide to do a PhD in a closer related area of analysis will be able to use the acquired basic knowledge and skills to relatively easily extend their knowledge to more sophisticated areas of the theory.

This Level 7 version of the module will involve extra self-study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

MATH70055 Stochastic Calculus with Applications to non-Linear Filtering

Brief Description

The module offers a bespoke introduction to stochastic calculus required to cover the classical theoretical results of nonlinear filtering. The first part of the module will equip the students with the necessary knowledge (e.g., Ito Calculus, Stochastic Integration by Parts, Girsanov's theorem) and skills (solving linear stochastic differential equation, analysing continuous martingales, etc) to handle a variety of applications. The focus will be on the use of stochastic calculus to the theory and numerical solution of nonlinear filtering.

Learning Outcomes

On successful completion of this module, you will be able to: a. understand the notion on Brownian motion and able to show that a stochastic process is a Brownian motion, b. Prove that a process is a martingale via Novikov's condition. c. Solve linear SDEs, d. Be able to check whetheran SDE is well-posed. d. understand the mathematical framework of nonlinear filtering e. Deduce the filtering equations. f. Deduce the evolution equation of the mean and variance of the one-dimensional Kalman-Bucy filter, g. Show that the innovation process is a Brownian motion. h. Apply stochastic integration by parts.

Module Content

An indicative list of topics is:

1. Martingales on Continuous Time (Doob Meyer decomposition, L_p bounds, Brownian motion, exponential martingales, semi-martingales, local martingales, Novikov's condition)

2. Stochastic Calculus (Ito's isometry, chain rule, integration by parts)

3. Stochastic Differential Equations (well posedness, linear SDEs, the Ornstein-Uhlenbeck process, Girsanov's Theorem)

4. Stochastic Filtering (definition, mathematical model for the signal process and the observation process)

5. The Filtering Equations (well-posedness, the innovation process, the Kalman-Bucy filter)

Prerequisites: Ordinary differential equations, partial differential equations, real analysis, probability theory.

MATH70031 Probability Theory 2

Brief Description

This module builds on Probability Theory 1, building a theory of continuous time stochastic processes (with Brownian Motion as a key example) along with introducing students to rigorous stochastic calculus.This module gives crucial background for students interested in stochastic differential equations,stochastic partial differential equations, and more generally stochastic analysis.

Learning Outcomes

On successful completion of this module, you should be able to:

-- demonstrate your understanding of the concepts and results associated with the elementary theory of Markov processes, including the proofs of a variety of results

-- apply these concepts and results to tackle a range of problems, including previously unseen ones apply your understanding to develop proofs of unfamiliar results

- demonstrate additional competence in the subject through the study of more advanced material

- combine ideas from across the module to solve more advanced problems

- communicate your knowledge of the area in a concise, accurate and coherent manner.

Module Content

An indicative list of topics is:

1. Definition and construction of Brownian Motion, properties of sample Brownian paths, strong

Markov property.

2. Continuous time local martingales.

3. Stochastic integrals with Brownian motion, and general continuous local martingales

3. Itˆo’s formula

4. Stochastic differential equations, martingale problems, applications to PDE.

5. Markov semigroups, functional inequalities, and convergence to equilibrium.

MATH70032 Geometry of Curves and Surfaces

Brief Description

This module is an introduction to classical theory of differential geometry, where we disucss geometric features of curves and surfaces in (mostly) three dimensional Euclidean spaces.

Learning Outcomes

On successful completion of this module, you will be able to:

- identify regular curves and implement different re-parametrisations of curves in two and three dimensional spaces,

- learn about and calculate the geometric quantities of curvature and torsion of a regular curve,

- identify regular surfaces in 3 dimensional spaces using the notions of charts,

- analyse the regularity of maps from one surface into another surface, and also of functions on surfaces,

- use partitions to calculate the basic topological invariant of Euler characteristic,

- learn about the topological classification of compact surfaces, and identify them,

- calculate the first and second fundamental forms of a surface,

- learn about the existence and uniqueness of geodesics on general surfaces,

- link the Gaussian curvature to the local shape of a surface, and present different kinds of examples,

- analyse the global topological features of a surface by integrating local geometric features (Gauss-Bonnet and winding numbers)

– demonstate competence with further advanced material in the area designated for self-study

-- synthesise material from across the module to apply to advanced topics

Module Content

This module is an introduction to classical theory of differential geometry, where we disucss geometric features of curves and surfaces in (mostly) three dimensional Euclidean spaces. A curve, which is the trajectory of a particle moving in a smooth fashion, may twist in two manners described by the values called curvature and torsion. The twists of a surface in three dimensional space is naturally more involved. There are different notions of curvature: the Gaussian curvature and the mean curvature. The Guassian curvature describes the intrinsic geometry of the surface, and the mean curvature describes how it bends in space. We look at several examples of surfaces, and calculate their curvatures. We study the local shapes of surfaces based on their curvatures. For example, the Gaussian curvature of a sphere is strictly positive, which explains why any planar illustration of the countries distorts shapes. Remarkably, these local gometric notions can be combined to derive global information about the topology of the surface (for example the Gauss-Bonnet formula). This module starts with the basic real analysis taught in years 1 and 2, and leads into the more modern and general theory of manifolds.

An indicative list of sections and topics is:

▪ Curves in two and three-dimensional spaces: re-parametrizations, curvature and torsion, Frenet-Serret formulae, curves are determined by curvature and torsion, winding number and the total curvature,

▪ Surfaces: Charts, Tangent vectors, and tangent planes, Smooth maps from one surface into another surface, smooth functions on a surface, Normal vectors,

▪ Curvature of a surface: the first and second fundamental forms, Christoffel symbols, normal curvature, Gaussian curvature, and mean curvature, Gauss's Theorema Egregium,

▪ Area of a surface,

▪ Geodesics on a surface: length-minimising curves, existence, non-existence and examples, geodesic curvature,

▪ Gauss-Bonnet Theorem and applications

▪ Topological classification of surfaces

▪ Vector fields and the Poincare-Hopf Theorem

The module will assume familiarity with material in the second-year module Analysis II

This Level 7 version of the module will involve extra self-study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

MATH70033 Algebraic Curves

Brief Description

This module is meant as a first encounter with algebraic geometry, through the study of affine and projective plane curves over the field of complex numbers. We will also discuss some complex-analytic aspects of the theory (Riemann surfaces). Important results include the definition of local intersection multiplicities and Bézout's theorem, inflection points and the classification of plane cubics, linear systems of curves, and the degree-genus formula.

Learning Outcomes

On successful completion of this module, you will be able to:

- solve geometric problems about affine and projective plane curves with algebraic techniques;

- determine the projectivizations of affine plane curves and the points at infinity;

- determine the tangent lines of plane curves at smooth and singular points;

- compute projective transformations and find convenient coordinate systems;

- compute intersection multiplicities using resultants and the axiomatic characterization;

- formulate, prove and apply Bézout's theorem;

- find inflection points of projective plane curves and use them to classify cubic curves;

- solve enumerative problems by means of the theory of linear systems;

- work with holomorphic charts to determine local and global properties of Riemann surfaces and morphisms;

- compute ramification degrees of morphisms of Riemann surfaces;

- formulate, prove and apply the degree-genus formula for smooth projective plane curves.

– demonstate competence with further advanced material in the area designated for self-study

-- synthesise material from across the module to apply to advanced topics

Module Content

An indicative list of sections and topics is:

- Affine plane curves;

- The geometry of projective spaces;

- Projective plane curves;

- Smooth and singular points, tangent lines;

- Projective transformations and the classification of conics;

- Intersection multiplicities (resultants and axiomatic characterization)

- Bézout's theorem on intersections of projective plane curves;

- the Legendre family of cubics, inflection points and the classification of non-degenerate smooth cubics;

- linear systems of projective plane curves, projective duality and enumerative geometry;

- Riemann surfaces;

- local description of morphisms of Riemann surfaces (ramification);

- classification of topological surfaces and genus (informal introduction);

- Riemann-Hurwitz and the degree-genus formula.

Some related topics will appear in the problem sheets and the coursework (e.g., dual curves, group structure on smooth cubics).

This Level 7 version of the module will involve extra self-study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

MATH70034 Algebraic Topology

Brief Description

This module gives a first introduction to algebraic topology. After some preliminary results on quotient spaces and CW-complexes, we discuss fundamental groups and the Galois correspondence for covering spaces. We then move on to homology theory and study simplicial and singular homology, as well as some applications like the Jordan curve theorem and invariance of domain. Throughout the module, we pay special attention to algebraic and categorical aspects.

Learning Outcomes

On successful completion of this module, you will be able to:

- define the basic invariants in algebraic topology and prove their main properties;

- use algebraic techniques to distinguish different homotopy types and classify topological objects;

- compute fundamental groups, simplicial homology groups and singular homology groups;

- apply the Galois correspondence to classify covering spaces of topological spaces;

- apply fundamental groups and homology groups to prove fundamental topological properties (Brouwer's fixed point theorem, Jordan's curve theorem, invariance of domain);

- formulate topological and algebraic constructions in a categorical language (universal properties); analyze the structure of quotients of topological spaces by covering space actions;

- represent groups geometrically by means of Cayley complexes

Module Content

An indicative list of sections and topics is:

Preliminaries:

- Homotopy and homotopy type

- Cell complexes

- Operations on spaces

The Fundamental Group:

- Paths and Homotopy

- Presentations of groups, amalgamated products and Van Kampen's Theorem

- Covering Spaces

- The Galois correspondence -Deck Transformations and Group Actions -Cayley complexes

Homology

- Δ-complexes and simplicial homology

- Singular homology

- Homotopy invariance

- Relative homology, exact sequences and excision

- The equivalence of simplicial and singular homology

- Mayer-Vietoris Sequences

- Applications

The main reference for this course is "Algebraic topology" by Hatcher.

This Level 7 version of the module will involve extra self-study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

The module will assume familiarity with the material in the second-year modules: Groups and Rings, Analysis II

MATH70056 Algebraic Geometry

Brief Description

Algebraic geometry is the study of the space of solutions to polynomial equations in several variables. In this module you will learn to use algebraic and geometric ideas together, studying some of the basic concepts from both perspectives and applying them to numerous examples.

Learning Outcomes

On successful completion of this module, you will be able to:

1. Understand the dictionary between algebra and geometry that arises from zero loci of polynomials in n-dimensional space;

2. Understand and compute irreducible and connected components of such zero loci;

3. Understand the concepts of regular and rational maps and their algebraic and geometric meaning;

4. Understand the projective space and the role it plays in compactifying zero loci of polynomials;

5. Understand the notion of dimension of zero loci of polynomials and their behaviour under regular maps;

6. Apply dimension theory and the Zariski topology in examples such as those coming from parameter spaces;

7. Understand how to generalise these ideas to the setting of more general commutative rings via (maximal) spectra (Mastery)

Module Content

An indicative list of topics is:

Affine varieties, projective varieties. The Nullstellensatz.

Regular and rational maps between varieties. Completeness of projective varieties.

Dimension. Parameter spaces.

Examples of algebraic varieties.

Spectrum and maximum spectrum (mastery)

Prerequisites: Commutative Algebra

MATH70057 Riemannian Geometry

Brief Description

The main aim of this module is to understand geodesics and curvature and the relationship between them. Using these ideas we will show how local geometric conditions can lead to global topological constraints.

Learning Outcomes

On successful completion of this module, you will be able to:

- Understand the relevant structures required to make sense of differential topological notions, such as derivatives of smooth functions, and geometric notions, such as lengths and angles, on an abstract manifold.

- Define the Lie derivative and covariant derivative of a tensor field.

- Define geodesics and understand their length minimising properties.

- Define and interpret various measures of the curvature of a Riemannian manifold.

- Understand the effect of curvature on neighbouring geodesics.

- Prove the celebrated classical theorems of Bonnet--Myers and Cartan--Hadamard.

Module Content

An indicative list of topics is:

Topological and smooth manifolds, tangent and cotangent spaces, vector bundles, tensor bundles, Lie bracket, Lie derivative, Riemannian metrics, affine connections, the Levi-Civita connection, parallel transport, geodesics, Riemannian distance, the exponential map, completeness and the Hopf--Rinow Theorem, Riemann and Ricci curvature tensors, scalar curvature, sectional curvatures, submanifolds, the second fundamental form and the Gauss equation, Jacobi fields and the second variation of geodesics, the Bonnet--Myers and Cartan--Hadamard Theorems.

Prerequisites: Geometry of Curves and Surfaces and Manifolds

MATH70058 Manifolds

Brief Description

The goal of this course is to introduce the theory of smooth manifolds. The class starts by defining smooth manifolds, submanifolds and tangent spaces. It will then develop more advanced topics like the theory of vector bundles, which will be used to introduce the notion of the tangent bundle, the cotangent bundle, vector fields and differential forms on a smooth manifold. This allows to define integration on an orientable manifold and then to prove Stokes' Theorem on a manifold with boundary.

Learning Outcomes

On successful completion of this module, you will be able to:

- Define smooth manifolds in an intrinsic way, by using the notion of charts, transition functions and smooth atlases.

- Determine sufficient conditions under which the level set of a smooth function is a submanifold.

- Study vector bundles on a manifold and determine necessary and sufficient condition for a vector bundle to be trivial.

- Study vector fields on a manifold and describing them locally, through the use of charts.

- Define the integration of differential forms on an orientable manifold.

- Prove Stokes' Theorem, which is one of the main tools used in differential topology.

Module Content

This module focuses on foundations as well as examples.

An indicative list of topics is:

Smooth manifolds, quotients, smooth maps, submanifolds, rank of a smooth map, tangent spaces, vector fields, vector bundles, differential forms, the exterior derivative, orientations, integration on manifolds (with boundary) and Stokes' Theorem.

MATH70059 Differential Topology

Brief Description

In this module you will understand how geometry and topology interact on smooth manifolds. You will investigate different (co)homology theories, see how to relate them, and study how to use them

to analyse the topology of a manifold.

Learning Outcomes

On sccessful completeion of this module you will be able to:

- Apply the concepts of homology and cohomology, as well as central results such as Poincaré Duality and the De Rham theorem, to investigate manifolds.

- Use a Mayer-Vietoris argument to compute (co)homology groups.

- Describe the topology of a manifold by analysing the critical points of a Morse function and the gradient

flow lines between them.

- Use and explain the equivalence between the different (co)homology theories introduced (De Rham, singular, Morse), for example how the CW complex of a manifold relates to the homology groups generated by the critical points of Morse functions on the manifold.

- Work independently and with peers to formulate and solve problems in geometry using tools of algebraic and differential topology.

Module Content

An indicative list of contents is:

-De Rham Cohomology: Definition, Poincaré's Lemma, Mayer-Vietoris sequences, compactly supported de Rham cohomology, pairings and Poincaré Duality with applications, degree of a map, mapping degree theorem and examples.

-Morse Theory: Introduction and basics, Fundamental Theorems of Morse Theory, the CW-structure associated to Morse-functions, stable and unstable manifolds, Morse-Smale functions, orientations, Morse homology and Morse Homology Theorem. Examples.

-Singular Homology: Basic definitions, properties and examples, De Rham Theorem.

The module will assume familiarity with topics in Algebraic Topology and smooth manifolds. In particular students should be familiar with: Vector fields, differential forms (k-forms, exterior differential, closed and exact forms), integration on manifolds and Stokes' Theorem, basics of homological algebra (exact sequences, Snake Lemma).

MATH70060 Complex Manifolds

Brief Description

The goal of this course is to introduce the theory of almost complex manifolds and complex manifolds. Many important examples will be provided, such as Kähler manifolds and complex projective manifolds. After introducing some of the main tools, as the Hermitian metrics, the Chern connection and the co-homology of a complex manifold, the theory of Hodge decomposition for Kähler manifolds will be presented, together with many of its applications. The class will culminate with the Kodaira embedding theorem and with the main notions of Kodaira-Spencer deformation theory.

Learning Outcomes

On successful completion of this module, you will be able to:

- Study many examples of complex and almost complex manifolds, such as Hopf manifolds, projective spaces, Kähler manifolds, and projective varieties.

- Introduce tools like Hermitian metrics, holomorphic vector bundles and Chern connections on a complex manifold.

- Study harmonic forms on a complex manifolds and then the Dalbout and the de Rham co-homology of a Kähler manifold, culminating with the Hodge decomposition theorem and several of its applications.

- Use holomorphic line bundles to study the Kodaira embedding theorem, which provides a characterisation of complex projective manifolds.

- Introduce the basic notions of the Kodaira-Spener deformation theory.

Module Content

Prerequisite: Manifolds

An indicative list of topics is:

Complex and almost complex manifolds, integrability. Examples such as the Hopf manifold, projective space, projective varieties. Hermitian metrics, Chern connection. Various equivalent formulations of the Kaehler condition. Hodge decomposition for Kaehler manifolds. Line bundles and Kodaira embedding. Statement of GAGA. Basic Kodaira-Spencer deformation theory.

MATH70140 Geometric Complex Analysis

Brief Description

In this module we look at the subject of complex analysis from a more geometric point of view. We shall look at geometric notions associated with domains in the plane and their boundaries, and how they are transformed under holomorphic mappings. In turn, the behaviour of conformal maps is highly dependent on the shape of their domain of definition.

Learning Outcomes

On successful completion of this module, you will be able to:

– identify features of, and develop arguments about, certain holomorphic maps;

– state, apply, and explain aspects of the Riemann Mapping Theorem for arbitrary simply connected plane domains;

– explain the automorphisms of the disk and the upper half plane;

– Explain hyperbolic geometry, and basic notions of length, geodesics, isometries;

– apply area theorem, and derive distortion estimates for arbitrary conformal mappings;

– acquire deeper understanding of holomorphic mappings through generalisation to quasi-conformal mappings;

– appreciate significance of universal bounds in geometric function theory;

– explain the statement of the Beltrami-equation, and generalisation of the Riemann mapping theorem;

-- demonstrate additional competence in the subject through the study of more advanced material.

-- combine results from across the module to solve advanced problems

– work independently and with peers to understand abstract concepts in complex analysis.

Indicative Module Content

Complex analysis is the study of the functions of complex numbers. It is employed in a wide range of topics, including dynamical systems, algebraic geometry, number theory, and quantum field theory, to name a few. On the other hand, as the separate real and imaginary parts of any analytic function satisfy the Laplace equation, complex analysis is widely employed in the study of two-dimensional problems in physics such as hydrodynamics, thermodynamics, Ferromagnetism, and percolations.

In this module we look at the subject of complex analysis from a more geometric point of view. We shall look at geometric notions associated with domains in the plane and their boundaries, and how they are transformed under holomorphic mappings. In turn, the behaviour of conformal maps is highly dependent on the shape of their domain of definition.

An indicative list of topics is:

-Schwarz lemma and automorphisms of the disk,

-Riemann sphere and rational maps,

-Conformal geometry on the disk, Poincare metric, Isometries, Hyperbolic contractions,

-Conformal Mappings, Conformal mappings of special domains, Normal families, Montel's theorem, General form of Cauchy integral formula, Riemann mapping theorem,

-Growth and Distortion estimates, Area theorem,

-Quasi-conformal maps and Beltrami equation, Linear distortion, Dilatation quotient, Absolute continuity on lines, Quasi-conformal mappings, Beltrami equation, application of MRMT,

There will also be extra self-study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

Prerequisites: It will be helpful if you have taken (or are taking) one or more of the following courses:

Functional Analysis, Measure and Integration (or Lebesgue Measure and Integration), Fourier Analysis and Distributions.

MATH70035 Algebra 3

Brief Description

This course continues the study of commutative rings and introduces the notion of R-module, which is an analogue over rings of the notion of a vector space over a field. Using these ideals we prove fundamental results about various classes of rings, particularly polynomial rings in several variables.

Learning Outcomes

On successful completion of this module, you will be able to:

- Understand the detailed theory of finite fields, their classification, and factorization of polynomials over finite fields

- Understand the theory of R-modules and their presentations

- Understand the classification of modules over Euclidean Domains, and how to use Smith Normal Form to determine the isomorphism class of such a module given a presentation

- Use this classification, in the case of K[T]-modules, to prove fundamental results in linear algebra

- Apply several different criteria for irreducibility of polynomials over various base rings

– demonstate competence with further advanced material in the area designated for self-study

-- synthesise material from across the module to apply to advanced topics

Module Content

An indicative list of sections and topics is:

- Chinese Remainder Theorem

- Field Extensions and Finite Fields

- R-modules

- Free modules and presentations

- Modules over Euclidean Domains

- Noetherian rings

- Gauss's Lemma and Factorization in polynomial rings

- If R is a UFD, so is R[X]

- Irreducible Polynomials and factorization of polynomials

This Level 7 version of the module will involve extra self-study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

MATH70036 Group Theory

Brief Description

This module builds on the Group Theory from the 1st year module Linear Algebra & Groups and the 2nd year module Groups and Rings. We start with a discussion of isomorphism theorems, and proceed to further example of groups and operations on them, including automorphism groups and semidirect products. Special attention is given to group actions and permutation groups: primitivity, multiple transitivity etc. Further we discuss solvable and nilpotent groups and their characterizations.

Learning Outcomes

On successful completion of this module, you will be able to:

– identify features of, and develop arguments about, certain groups and classes of group;

– explain the principles of group actions, and work with elementary examples;

– construct and work with direct and semidirect products of groups;

– state the definition of and extraspecial group and construct small examples of them;

– construct certain classical series of doubly and triply transitive groups;

– state and prove the structure theorem for nilpotent groups;

– explain the principles of group actions, and work with elementary examples;

– determine the normal structure and calculate the automorphism groups of symmetric groups;

– work independently and with peers to articulate understanding of abstract concepts in algebra.

– demonstate competence with further advanced material in the area designated for self-study

-- synthesise material from across the module to apply to advanced topics

Module Content

An indicative list of sections and topics is:

Definition and basic properties of groups. Isomorphism Theorems. Sylow subgroups. Group actions, primitivity and multiple transitivity. Composition series. Nilpotent groups. Solvable groups. Symmetric groups. Automorphism group of a group and semidirect products. Linear groups: centres and commutator subgroups, with small examples.

Further advanced material on these topics will be set for self-study.

MATH70037 Galois Theory

Brief Description

The formula for the solution to a quadratic equation is well--known. There are similar formulae for cubic and quartic equations but no formula is possible for quintics. The module explains why this happens.

Learning Outcomes

On successful completion of this module you should be able to:

-- state, prove, and apply the fundamental theorem of Galois theory (aka the "Galois correspondence").

-- work with simple examples such as cubic polynomials, cyclotomic polynomials, and polynomials over finite fields.

-- compute Galois groups of splitting fields of cubic and bi-quadratic polynomials in arbitrary characteristic.

-- state and apply the formulas for solving cubic and quartic equations, and to prove that there are no such formulas for equations of degree 5 or larger.

- compute Galois groups over the rationals by the method of Frobenius elements.

-- Demonstrate additional competence in the subject through the self-study of designated advanced material

-- Combine topics from across the module to obtain more advanced results

Module Content

Familiarity with the following topics from Algebra 3

will be assumed: irreducible polynomials and factorization of polynomial;

Gauss's Lemma and factorization in polynomial rings.

An indicative list of topics is:

Field extensions, degrees and the tower law

Splitting fields, normal extensions, separable extensions

Automorphisms, fixed fields and the fundamental theorem

Examples: cubic and biquadratic extensions, finite fields

Extensions of the rationals and Frobenius elements

Cyclotomic extensions

Kummer theory and the insolubility of quintic equations

Material for self-study (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

MATH70038 Graph Theory

Brief Description

A graph is a structure consisting of vertices and edges. Graphs are used in many areas of Mathematics, and in other fields, to model sets with binary relations. In this module we study the elementary theory of graphs; we discuss matters such as connectivity, and criteria for the existence of Hamilton cycles. We treat Ramsey's Theorem in the context of graphs, with some of its consequences. We then discuss probabilistic methods in Graph Theory, and properties of random graphs.

Learning Outcomes

On successful completion of this module, you will be able to:

- Demonstrate facility with the terminology of graphs and simple graph constructions to analyse examples and prove results

- Explain the proofs of the theorems of König and Menger, and certain other related results. Apply these results to appropriate problems.

- State, prove and apply Turán's Theorem. Describe and apply certain results in the theory of Hamilton cycles, including Dirac's Theorem.

- Explain and reason about Ramsey's Theorem and related results in the context of graph colourings.

- Describe various models of random graphs and apply probabilistic arguments to situations in graph theory.

– demonstate competence with further advanced material in the area designated for self-study

-- synthesise material from across the module to apply to advanced topics

Module Content

An indicative list of sections and topics is:

Standard definitions and basic results about graphs. Common graph constructions: complete graphs, complete bipartite graphs, cycle graphs.

Matchings and König's Theorem. Connectivity and Menger's Theorem.

Extremal graph theory. The theorems of Mantel and Turán. Hamilton cycles, and conditions for their existence.

Ramsey Theory for graphs, with applications.

The Probabilistic Method and random graphs. Evolution of random graphs.

This Level 7 version of the module will involve extra self-study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

MATH70039 Group Representation Theory

Brief Description

This module defines and begins the study of representations of groups, focusing on finite-dimensional complex representations of finite groups. These structures encode ways that groups can act as symmetries, which appear throughout mathematics (notably in algebra, number theory, and geometry, but also in analysis) as well as in physics and chemistry, among other places. We explain how to understand and classify these representations through characters, or traces. In the final unit we generalise the theory to finite-dimensional modules over rings, particularly semisimple algebras, whose theory retains many of the features of that of representations of groups.

Learning Outcomes

On successful completion of this module you will be able to:

- Recall and use basic definitions in group representations, their character theory, and modules over algebras, particularly finite-dimensional semisimple algebras;

- Explain and work with the features of complex representations of finite groups that allow one to simplify the theory (e.g., semisimplicity, character tables, etc.);

- Apply these results to classify representations of finite groups and semisimple algebras and compute the character tables of finite groups;

- perform basic constructions of representations of groups and to apply them to obtain all finite-dimensional representations of certain basic groups up to isomorphism;

- Explain the relationship between finite-dimensional irreducible representations of algebras and of finite-dimensional semisimple algebras, and the basic properties of their characters;

- Relate endomorphisms of representations to central elements in groups and semisimple algebras;

- Work independently and with peers to formulate and solve problems in algebra and geometry using tools of representation theory;

- Demonstrate abilility to engage with more advanced material via self-study.

-- Combine material from across the module to address more challenging problems

Module Content

- Basic theory: definitions, Maschke's theorem, Schur's Lemma, classification and construction of representations of finite abelian groups, dihedral groups, and small symmetric and alternating groups;

- Tensor products of representations and homomorphism spaces, the regular representation;

- Character theory: behaviour under direct sums and tensor products, orthogonality relations, computation of character tables of certain groups;

- Finite-dimensional modules over algebras: definition of finite-dimensional semisimple algebras and relationship of finite-dimensional modules of general algebras

to those of finite-dimensional semisimple algebras; constructions of projections via the center;

- Other important examples of modules over algebras.

- Further material, related to the above but of a more advanced character (consisting of a book chapter or research paper), will be designated for self-study.

MATH70040 Formalising Mathematics

Brief Description

Computer theorem provers are mature enough now to tackle most undergraduate level mathematics, and also some much harder level mathematics. As these systems evolve, they will inevitably become useful as tools for researchers, and some believe that one day they will start proving interesting theorems by themselves. This project-assessed course is an introduction to the Lean theorem prover and during it we will learn how to formalise proofs of undergraduate and masters level theorems from across pure mathematics.

Learning Outcomes

On successful completion of this module you will be able to:

- understand the basics of how type theory can be used as a foundation for pure mathematics;

- understand how to "modularise" mathematical arguments, breaking them up into simple chunks, thus leading to clarity of understanding;

- understand how to "abstract" mathematical arguments, finding the correct generality in which statements should be made, thus again leading to clarity of understanding;

- state and prove many results from undergraduate and masters level pure mathematics modules in the Lean theorem prover;

- develop mathematical theories of your own in the Lean theorem prover;

- write formal proofs of theorems which other mathematicians can understand and follow

- understand how to turn more advanced mathematics into statements of dependent type theory.

Module Content

Note that the aim is to both learn the mathematics and to learn how to teach it to a computer. No experience in programming will be assumed. Lean is a functional programming langauge, so we will be picking up functional programming along the way. If you want to get a feeling for the kind of coding which will be involved, try playing the natural number game.

The following is an indicative list of areas where the mathematics could be drawn from:

*) Logic, functions, sets.

*) Lattice theory, complete lattices, Galois insertions and Galois connections. Examples in mathematics.

*) Groups and subgroups.

*) Closure operators in group theory and topology.

*) Filters as generalised subsets. Filters form a complete lattice.

*) Applications of filters to topology. New proofs of basic results in topology. Tychonoff's theorem.

*) Application of filters to analysis. New proofs of basic results in analysis.

*) What is cohomology? Group cohomology in low degree.

This level 7 version of the module will involve extension material and more advanced examples than the level 6 version.

MATH70061 Commutative Algebra

Brief Description

This module is an introduction to commutative algebra which is the modern foundation of algebraic geometry and algebraic number theory. First we will cover such basic notations as prime and maximal ideals, the nilradical and the Jacobson radical. Then we study the very important construction of localisation both for rings and modules over them. We will apply these to a variety of results, for example primary decomposition of ideals and structure theorems for Artinian rings and discrete valuation rings.

Learning Outcomes

On successful completion of this module, you will be able to:

- define basic notions in commutative algebra and prove their main properties;

- use localisation to relate properties of rings, ideals, modules and morphisms between them with properties of their localisations;

- apply various chain conditions to prove properties of rings and modules satisfying these;

- use other standard arguments in commutative algebra;

Module Content

This module is an introduction to commutative algebra which is the modern foundation of algebraic geometry and algebraic number theory. First we will cover such basic notations as prime and maximal ideals, the nilradical and the Jacobson radical. Then we study the very important construction of localisation both for rings and modules over them. We will apply these to a variety of results, for example primary decomposition of ideals and structure theorems for Artinian rings and discrete valuation rings.

An indicative list of topics is:

Prime and maximal ideals, nilradical, Jacobson radical, localization. Modules. Primary decomposition of ideals. Applications to rings of regular functions of affine algebraic varieties. Artinian and Noetherian rings, discrete valuation rings, Dedekind domains. Krull dimension, transcendence degree. Completions and local rings. Graded rings and their Poincaré series.

After this module, you should be equipped to undertake a course in modern algebraic geometry.

MATH70062 Lie Algebras

Brief Description

This module is an introduction to theory of complex Lie algebras and it culminates in the classification of finite dimensional semisimple complex Lie algebras in terms of root systems. It is completely self-contained, and only relies on a good understanding of linear algebra. However the proofs are quite intricate. It is also a good preliminary to the theory of Lie groups and algebraic groups, which study closely related objects, but the latter require much heavier machinery.

Learning Outcomes

On successful completion of this module, you will be able to:

- define basic notions of Lie algebras, such as ideals, derived series and lower central series,

- prove Engel's and Lie's theorem, and apply them in various contexts,

- prove and apply the additive Jordan decomposition theorem,

- define the Killing form and prove Chevalley's criteria,

- define Cartan and Borel subalgebras and prove their main properties,

- apply the above to complete the proof of the classification theorem of semisimple Lie algebras,

- construct explicitly the classical simple Lie algebras,

- work effectively with roots systems.

Module Content

An indicative list of topics is:

The semisimple complex Lie Algebras: root systems, Weyl groups, Dynkin diagrams, classification. Cartan and Borel subalgebras. Classification of irreducible representations.

MATH70063 Algebra 4

Brief Description

This module is a course of homological algebra. The main result is the existence of derived functors in the category of modules over an associative ring. We cover functors Ext and Tor in greater detail, particularly in the category of abelian groups. We define and study some basic properties of group cohomology.

Learning Outcomes

On successful completion of this module, you will be able to:

– identify features of, and develop arguments about, rings, modules over rings and homomorphisms between them;

– understand the definition of the tensor product of module and use it in a variety of settingss;

– define free, injective, projective, flat modules, state and prove their basic properties;

– state, apply, and explain the proof of the theorem about the existence of derived functors in the category of modules over a ring;

– understand and explain the relation between Ext and extensions of modules;

– compute functors Tor and Ext in specific situations, particularly in the category of abelian groups;

– understand and use for computation the explicit construction of the first and second cohomology groups, as well as the cohomology groups of a cyclic group.

– demonstrate capacity for independent study of an advanced topic to be specified by solving a range of problems.

Module Content

An indicative list of sections and topics is:

Modules over rings: free, projective, injective, flat; tensor product

Functors Hom, Ext, Tor. General definition of a derived functor. Long exact sequences. Injective and projective resolutions. Homotopy.

Group cohomology via homogeneous and inhomogeneous cochains. The case of cyclic groups.

Mastery material for self-study, on advanced material relating to the topics above.

MATH70132 Mathematical Logic

Brief Description

The module is concerned with some of the foundational issues of mathematics: formal logic and set theory. In propositional and predicate logic, we analyse the way in which we reason formally about mathematical structures. In set theory, we will look at the ZFC axioms and use these to develop the notion of cardinality. Material on model theory will involve all of these topics.

Learning Outcomes

On successful completion of this module, you should be able to:

- Understand how the notion of truth is defined precisely in propositional and predicate logic and apply the definitions and accompanying results in a variety of contexts.

- Demonstrate understanding of formal systems for propositional and predicate logic by constructing examples of formal proofs and deductions, and by applying and deriving general results about these.

- Appreciate the expressive power of a first-order language (and its limitations) and compare structures via their first-order theories.

- Relate the semantic and syntactic aspects of formal logic, have an understanding of powerful general results such as the completeness and compactness theorems, and be able to apply these in a variety of ways.

- Use the ZFC axioms to justify constructions in set theory, ranging from elementary applications, to constructions involving transfinite recursion, ordinals, cardinals and applications of these.

- Use general results to compute and compare cardinalities of infinite sets.

- Communicate your knowledge of the area in a concise, accurate and coherent manner.

- Combine your knowledge of predicate logic and set theory in the study of model theory and apply the results to deepen your understanding of theories of first-order structures.

Module Content

The module is concerned with some of the foundational issues of mathematics. In propositional and predicate logic, we analyse the way in which we reason formally about mathematical structures. In set theory, we will look at the ZFC axioms and use these to develop the notion of cardinality. These topics have applications to other areas of mathematics: formal logic has applications via model theory and ZFC provides an essential toolkit for handling infinite objects.

An indicative list of sections and topics is:

Propositional logic: Formulas and logical validity; a formal system; soundness and completeness.

Predicate logic: First-order languages and structures; satisfaction and truth of formulas; the formal system; Goedel's completeness theorem; the compactness theorem; the Loewenheim- Skolem theorem.

Set theory: The axioms of ZF set theory; ordinals; cardinality; the Axiom of Choice.

Model theory: Elementary substructures; the method of diagrams, the Tarski-Vaught test; the general Loewenheim- Skolem Theorems; Reduced products and ultraproducts; Los' theorem.

There are no formal prerequisites for the module although a level of mathematical understanding such as would be provided by a second year algebra or analysis module, together with an appetite for abstraction and proofs, will be assumed. We will use basic notions from algebra (groups, rings and vector spaces) in examples.

MATH70041 Number Theory

Brief Description

Brief description of module (at most 600 characters): The module is

concerned with properties of natural numbers, and in particular of prime

numbers, which can be proved by "elementary" methods (such as basic group

theory ring theory).

Learning Outcomes

On successful completion of this module, you will be able to:

– form arguments about and solve congruences, particular modulo primes, and apply

this to the RSA algorithm;

- compute with quadratic residues;

- solve some particular Diophantine equations, including Pell's

equation;

- compute continued fractions;

- construct transcendental numbers.

- explain and demonstrate mastery of such further material as is selected by the module leader for self-study.

-- Combine material from across the module to solve more advanced problems

Module Content

An indicative list of topics is:

Euclid's algorithm, unique factorization, linear congruences, Chinese

Remainder Theorem.

The structure of $(Z/nZ)^\times$, including the Fermat-Euler theorem,

Lagrange's theorem, the existence (and non-existence) of primitive

roots.

Primality testing, factorization, and the RSA algorithm (including the basics of the

Miller-Rabin test).

Quadratic reciprocity, Legendre symbols, Jacobi symbols.

Sums of 2 and 4 squares, using unique factorization in the Gaussian

integers.

Pell's equation, existence of solutions via Dirichlet's theorem.

Continued fractions, periodicity for quadratic irrationals, algorithm

for solving Pell's equation via continued fractions.

Irrationality, Liouville's theorem, construction of a transcendental

number.

Elementary results on primes in arithmetic progressions.

Other topics of the lecturer's choice as time permits, e.g. quadratic forms; Möbius inversion and

Dirichlet Convolution; the Selberg sieve; particular examples of Diophantine equations.

Mastery material selected for further self-study, to be based on written material such as a textbook exerpt or research paper.

MATH70042 Algebraic Number Theory

Brief Description

An introduction to algebraic number theory using quadratic fields as the main source of examples. We will study rings of integers in finite extensions of the rational and discuss unique factorization and its failure, the decomposition of primes, the finiteness of the ideal class group, and Dirichlet's unit theorem.

Learning Outcomes

On successful completion of this module you will be able to:

-- explain how unique factorization domains, principal ideal domains and Euclidean domains are related.

-- give examples of rings of integers in quadratic fields that are Euclidean domains and also counter-examples.

- define a Dedekind domain and explain why rings of integers in number fields are Dedekind domains.

-- write a basis for the rings of integers in any given quadratic number field.

-- explain what it means for a prime to be split, inert or ramified in an extension and, given a quadratic ring of integers and a prime, you will be able to say what happens to that prime.

-- explain why the class group in a number field is finite and compute examples of class groups of quadratic number fields.

-- state Dirichlet's unit theorem and to describe explicitly the group of units in a real or imaginary quadratic field.

-- show mastery of more advanced material on these topics which will be set for self-study

-- synthesise arguments from different parts of the module to solve more advanced problems.

Module Content

An indicative list of topics is as follows.

We will review / introduce some background from ring theory, discuss unique factorization domains, principal ideal domains and Euclidean domains. We will study Gaussian and Eisenstein integers in detail and see several other examples of quadratic rings of integers. We will then discuss the structure theorem for finitely generated abelian groups, the notion of integral closure, Dedekind domains and study the ideal class group. We will prove that the ideal class group in a number field is finite and compute many examples. We will study the decomposition of primes in number fields and in quadratic fields in particular. We will end by discussing Dirichlet's unit theorem.

Further more advanced material (such as a book chapter or research paper) will be set by the module leader for independent study.

The module will assume familiarity will some topics in algebra, such as commutative rings and modules.

MATH70064 Elliptic Curves

Brief Description

An elliptic curve is an algebraic curve in two variables defined by an equation of the form $y^2=x^3+ax+b$. Elliptic curves play an important role in Number Theory, and have been central to many recent advances, such as the proof of Fermat's Last Theorem. In this course we study the theory of elliptic curves and their connections with Number Theory, Geometry and Algebra.

Learning Outcomes

On successful completion of this module, you will be able to:

– solve equations in the p-adic numbers;

- find all rational points on plane conics;

- compute with the group law on an elliptic curve;

- compute the torsion subgroup of an elliptic curve over Q;

- compute the rank of an elliptic curve over Q;

- demonstrate mastery of further advanced material selected for self-study by applying it in a variety of problems.

Module Content

An indicative list of topics is:

The p-adic numbers.

Curves of genus 0 over Q .

Cubic curves and curves of genus 1.

The group law on a cubic curve.

Elliptic curves over p-adic fields and over Q.

Torsion points and reduction mod p.

The weak Mordell-Weil theorem.

Heights.

The (full) Mordell-Weil theorem.

MATH70043 Statistical Theory

Brief Description

This module seeks to provide a more unified perspective of the core statistical problems introduced in earlier modules by developing a general mathematical theory for parametric statistical models. We will deal with the criteria and theoretical results necessary to develop and evaluate statistical procedures for point estimation, hypothesis testing and confidence intervals. We will consider several approaches, including maximum likelihood estimation and Bayesian approaches, and study when they are provably optimal.

Learning Outcomes

On sucessful completion of this module, you will be able to-

-Apply key results related to optimal statistical procedures

- Evaluate and compare estimators using their sampling properties

- Explain what it means for statistics to be sufficient and complete

- Explain the Rao-Blackwell theorem and how it can be used to improve an estimator

- Use elementary ideas from decision theory to evaluate statistical procedures

- Explain the Bayesian approch to estimation

- Demonstrate an integrated understanding of the concepts of this module by independent study of related material.

Module Content

An indicative list of sections and topics is:

- Theories of point estimation and hypothesis testing
- Exponential families
- Sufficiency and the Rao-Blackwell theorem
- The Cramer-Rao lower bound
- Maximum likelihood estimation and its asymptotic theory

- Bayesian estimation
- Decision theory
- Completeness and minimum variance unbiased estimators
- The Neyman-Pearson lemma and likelihood ratio tests
- Independent study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

MATH70044 Applied Statistical Inference

Brief Description

In this module, you will use statistical models to explore data from a wide range of problem domains: biology, medicine, engineering, finance. The emphasis throughout will be on developing practical skills for working with data. The module centres around two different extensions of the linear model. First, the idea of Generalized Linear Models, used to handle data where errors cannot be assumed normally distributed with constant variance. Examples include Poisson or binomial counts, proportions and waiting times. Second, we relax the assumption of independence, introducing Linear Mixed Models, to accommodate correlated observations. We will introduce widely applicable methods and principles useful for the applied data scientist, such as diagnostic plots, bootstrapping and regularization.

Learning Outcomes

On successful completion of this module, you will be able to:

- Use R to fit linear models.

- Select appropriate generalized linear models for modelling data with with non-normal error distributions, e.g. poisson, binomial, gamma.

- Use models to give predictions, together with an associated confidence interval.

- Identify models that fit poorly, and suggest improved models.

- Model data with correlated observations using linear mixed models.

- Explain the properties of different estimators of random effects variances, such as maximum likelihood and REML.

- Interpret output from R using plain language.

- Demonstrate an integrated understanding of the concepts of this module by independent study of related material.

Module Content

An indicative list of sections and topics is:

- Linear models: Least squares, normal equations, Gauss-Markov theorem, goodness of fit, diagnostics. Studentized residuals. Confidence intervals and prediction intervals.

- Generalized linear models: Specification, estimation (iterated reweighted least squares), inference, diagnostics.

- Mixed models: Specification, estimation (ANOVA, maximum likelihood, REML) and inference (parametric bootstrap) for variance components.

- Independent study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

MATH70045 Applied Probability

Brief Description

This module introduces stochastic processes and their applications. The theory of different kinds of processes will be described and illustrated with applications in several areas. The groundwork will be laid for further deep work, especially in such areas as genetics, engineering and finance.

Learning Outcomes

"On successful completion of this module, you will be able to:

- Use the Poisson process to model random arrivals.

- Extend the Poisson process to accommodate common depatures from the Poisson assumptions.

- Work with Markov chains in continous time.

- Determine the long-term behaviour of a continuous-time Markov chain.

- Determine whether states are recurrent or transient.

- Solve differential and difference equations to determine quantities of interest for stochastic processes.

- Explain basic properties of Brownian motion."

- Demonstrate an integrated understanding of the concepts of this module by independent study of related material.

Module Content

An indicative list of sections and topics is:

- Review of probability and discrete time Markov Chains

- Random walks

- Poisson processes and their properties: superposition, thinning of Poisson processes; Non-homogeneous, compound, and doubly stochastic Poisson processes.

- Autocorrelation functions.

- Probability generating functions.

- General continuous-time Markov chains: generator, forward and backward equations, holding times, stationarity, long-term behaviour, jump chain, explosion; birth and death processes, reversibility, recurrence/transience.

- Differential and difference equations and pgfs. Embedded processes. Time to extinction.

- Queues.

- Brownian motion and its properties.

- Independent study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

MATH70046 Time Series Analysis

Brief Description

A time series is a series of data points indexed and evolving in time. They are prevalent in many areas of modern life, including science, engineering, business, economics, and finance. This module is a self-contained introduction to the analysis of time series. Weight is given to both the time domain and frequency domain viewpoints, and important structural features (e.g. stationarity, reversibility) are treated rigorously. Attention is given to estimation and prediction (forecasting), and useful computational algorithms and approaches are introduced.

Learning Outcomes

On successful completion of this module, you will be able to:

- Appreciate that time series should be considered observations from an underlying stochastic process.

- Define what it means for a time series to be stationary.

- Identify autocorrelation within time series data.

- Work with standard models of time series.

- Appreciate that time series can exhibit trend and seasonality and know how to adjust for these.

- Determine the spectral representation of stationary time series and use the spectral density function to provide an alternative viewpoint of second-order structure.

- Derive and implement estimators of mean, correlation and spectral properties.

- Extend time series models, the notion of stationarity, and frequency domain representations to multivariate time series.

- Derive forecasts from standard time series models and quantify their uncertainty.

- Demonstrate an integrated understanding of the concepts of this module by independent study of related material.

Module Content

An indicative list of sections and topics is:

- Discrete time stochastic processes and examples.

- Autocovariance, autocorrelation, stationarity.

- Trend removal and seasonal adjustment.

- AR, MA and ARMA processes, characteristic polynomials, general linear process, invertibility, directionality and reversibility.

- Spectral representation, aliasing, linear filtering.

- Estimation of mean and autocovariance sequence, the periodogram, tapering for bias reduction.

- Parametric model fitting.

- Forecasting.

- Independent study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

MATH70047 Stochastic Simulation

Brief Description

Computational techniques have become an important element of modern statistics. Computation particularly underpins simulation methods, which are widely applied when studying models of complex systems, e.g. in biology and in finance. This module provides an up-to-date view of such simulation methods, covering areas from basic random variate generation to advanced MCMC (Markov Chain Monte Carlo) methods. The implementation of stochastic simulation algorithms will be carried out in R, a language that is widely used for statistical computing and well suited to scientific programming more generally.

Learning Outcomes

On successful completion of this module, you will be able to:

- Use algorithms for efficient generation of pseudo-random numbers.

- Evaluate intractable definite integrals using Use Monte Carlo methods.

- Implement MCMC algorithms to draw samples from intractable distributions.

- Assess the performance of MCMC algorithms using suitable diagnostic procedures.

- Explain how sequential Monte Carlo methods can be used to understand time-structured problems.

- Demonstrate an integrated understanding of the concepts of this module by independent study of related material.

Module Content

An indicative list of sections and topics is:

- Pseudo-random number generators.

- Generalized methods for random variate generation.

- Monte Carlo integration.

- Variance reduction techniques.

- Markov chain Monte Carlo methods (including Metropolis-Hastings and Gibbs samplers).

- Monitoring and optimisation of MCMC methods.

- Introduction to sequential Monte Carlo methods.

- Independent study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

MATH70048 Survival Models

Brief Description

Survival models are fundamental to actuarial work, as well as being a key concept in medical statistics.

This module will introduce the ideas, placing particular emphasis on actuarial applications.

Learning Outcomes

On successful completion of this module, you will be able to:

- Appreciate survival analysis models as a framework that deals with lifetimes and censored observations;

- Use several methods to define event time distributions and the relation between these definitions;

- Describe, select and use methods for fitting parametric, semi-parametric and non-parametric survival analysis models, including regression models and multi state models.

- Explain the counting process approach to survival analysis and its benefits;

- Use and critically analyse methods occurring in actuarial applications, such as methods for the construction and use of life tables.

- Demonstrate an integrated understanding of the concepts of this module by independent study of related material.

Module Content

An indicative list of sections and topics is:

- Concepts of survival models,

- Right and left censored data and randomly censored data.

- Estimation procedures for lifetime distributions:

- Empirical survival functions,

- Kaplan-Meier estimates,

- Cox model.

- Statistical models of transfers between multiple states,

- Maximum likelihood estimators.

- Counting process models.

- Actuarial Applications:

- Life table data and expectation of life,

- Binomial model of mortality,

- The Poisson model,

- Estimation of transition intensities that depend on age,

- Graduation and testing of crude and smoothed estimates for consistency.

- Independent study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

MATH70049 Introduction to Statistical Learning

Brief Description

This module provides an introduction to methods of statistical learning. That is, using statistical and artificial intelligence (AI) methods to learn from data, often when the data set is large, complex or of high dimension. We will consider both supervised and unsupervised learning. For the former we use a training set of data to learn patterns within data and then use our knowledge of those patterns to devise methods for predicting the outcomes of those patterns for new data. For the latter, there is not (usually) an outcome measure, but we seek to learn about the patterns within the data set itself. The methods in this module are of immense interest in academic and business circles and underpins much of the modern data tech industry to achieve tasks such as suggesting movies you might like to watch given information on those that you have watched, and from people with similar viewing patterns, developing machine methods for identifying cancer and seeking new ways to understand the economy. It is strongly recommended that students will have already passed the module Statistical Modelling I, or similar.

Learning Outcomes

On successful completion of this module, you will be able to:

Appreciate the structure and likely distribution(s) of various forms of data and which kinds of methods might be suitable for their analysis.

Understand the concept of multivariate data and situations where supervised and unsupervised learning might be employed.

Know and deploy a variety of important statistical and AI methods, using the R language to solve data related problems.

Understand the limitations of methods and how their application can be checked and or corrected.

Module Content

An indicative list of sections and topics is:Linear Models for Regression (variable selection, Lasso, Ridge Regression); Linear methods for classification (Logistic Regression); Basis expansions (piecewise polynomials and splines; smoothing splines, wavelets); Kernel regression (local linear and local polynomial); Additive models and Trees (boosting); Projection pursuit regression and Neural Networks; Support Vector Machines; Cluster Analysis and Multidimensional Scaling; Random Forests; Data Ethics. A Mastery topic for self-study directed by the Module Leader, on advanced material related to the topics above.

MATH70139 Spatial Statistics

Brief Description

Data collected in space are common in many applications including climate science, epidemiology, and economics. This module covers theoretical and methodological statistical foundations for spatial data. The module is structured to cover in detail the three fundamental forms in which spatial data are collected: gridded data, network data, and point pattern data. For each data type, stochastic models will be defined and explored including random fields and point processes. Properties including isotropy, stationarity and homogeneity will be formalised and explored in the context of each model and data type. In addition, techniques for spatial interpolation will be studied.

Learning Outcomes

On successful completion of this module students will be able to:

- Recognise the key differences between different types of spatial data (gridded, network and point pattern)

- Apply different classes of spatial covariance models to gridded and network data

- Formulate different intensity functions for point pattern data

- Define the concepts of homogeneity, stationarity and isotropy in the context of spatial models

- Select, derive and apply appropriate methods for spatial interpolation

- Demonstrate an integrated understanding of the concepts of the module by critical, independent study of research articles and books.

Indicative Module Content

1. Introduction to spatial data (gridded, network and point pattern data)
2. Random Fields and Covariance Functions
3. Spatial Interpolation and Kriging
4. Network Data and Markov Random Fields
5. Spatial Point Processes

Topics in Advanced Statistics

MATH70081 Non-parametric Statistics

Brief Description

Nonparametric methods aim to provide inference under weaker assumptions than conventional parametric methods. In this module, students will apply modern techniques to a variety of problems, such as estimating distribution functions, density estimation and nonparametric regression.

Learning Outcomes

On successful completion of this module students will be able to:

-Evaluate the strengths and weaknesses of nonparametric methods as flexible data-driven alternatives to parametric modelling;

-Describe basic properties of the empirical distribution function;

-Describe the bias-variance trade-off for nonparametric estimation;

-Construct nonparametric density estimators, for example kernel density estimators, and appraise their relative merits using common statistical criteria, such as mean square error, bias and variance;

-Construct nonparametric regression estimators, for example Nadaraya-Watson estimators or cubic splines, and appraise their relative merits using common statistical criteria;

-Appreciate basis functions, for example B-splines or wavelets, as an efficient method of representing functions and understand how they can be used to construct nonparametric estimators.

Module Content

The module will first introduce the most basic nonparametric estimator, the empirical distribution function, and cover its basic properties and how it can be used to construct confidence intervals. We then consider the problem of nonparametric density estimation, illustrating the key concept of the bias-variance trade-off via histograms and kernel density estimators. To evaluate the statistical properties of the latter, we will examine approximation by convolutions.

The module then covers various methods in nonparametric regression, such as kernel methods, local polynomial regression or splines. We will finally turn to the basis function approach, using suitable bases to provide efficient function approximations and hence derive estimators. This will be illustrated via B-splines and wavelets.

MATH70083 Statistical Learning for High Dimensional Data

Brief Description

This module introduces different models and tools used for the analysis of complex and high-dimensional datasets, like the ones observed in the field of genetics.

Learning Outcomes

On successful completion of this module students will be able to:
- Appreciate the importance of handling high dimensional datasets, through the analysis high dimensional datasets from different fields including genetics
-- Define multiple testing approaches and prove related theorems;

- Derive statistical approaches, including Bayesian and Frequentist, for performing variable selection in the for high dimensional data
- Evaluate different model selection approaches

Module Content

In this module we will develop models and tools to analyse complex and high dimensional datasets as these arise in different application fields as for example the genetics field. This will include statistical and machine learning techniques for multiple testing, penalised regression, clustering, dimensionality reduction and visualisation. The module will cover both Frequentist and Bayesian statistical approaches as well as some modern machine learning unsupervised and supervised approaches.

MATH70013 Advanced Simulation Methods

Brief Description

Modern problems in Statistics require sampling from complicated probability distributions defined on a variety of spaces and setups. In this module we will visit popular advanced sampling techniques, such as Importance Sampling, Markov Chain Monte Carlo, Sequential Monte Carlo. We will consider the underlying principles of each method as well as practical aspects related to implementation, computational cost and efficiency. By the end of the module the students will be familiar with these sampling methods and will have applied them to popular models, such as Hidden Markov Models, which appear ubiquitous in many scientific disciplines.

Learning Outcomes

On successful completion of this module students will be able to:
- Create, evaluate, understand and analyse advanced simulation methods;
- Display mastery of Hidden Markov models and what simulation methods are required to fit them to data;
- Understand, apply and evaluate variance and bias assessment and reduction;
- Display mastery of advanced Monte Carlo methods used in Statistics, such as Importance Sampling, Markov Chain Monte Carlo, Sequential Monte Carlo;
- Understand the basics and foundations of the methods mentioned above. Effort will be made to link the principles behind Monte Carlo methods and practical problems.

Module Content

This module will explore simulation algorithms: Importance Sampling, Markov Chain Monte Carlo, Sequential Monte Carlo and Particle filtering. We will discuss Hidden Markov Models, the simulation of the conditional laws and static parameter estimation. Popular examples: Linear Gaussian models, Stochastic Volatility, SIR models in epidemics etc.