

# MSc Mathematics and Finance Module Guide

## Programme Structure

FHEQ Level 7

Students study all compulsory modules.

| Module Code                     | Module Title                                     | Core/ Compulsory | Term       | Credits   |
|---------------------------------|--------------------------------------------------|------------------|------------|-----------|
| <a href="#">MATH70107</a>       | Fundamentals of Option Pricing                   | Compulsory       | 1          | 7.5       |
| <a href="#">MATH70108</a>       | Statistical Methods for Finance                  | Compulsory       | 1          | 7.5       |
| <a href="#">MATH70109</a>       | Stochastic Processes                             | Compulsory       | 1          | 7.5       |
| <a href="#">MATH70110</a>       | Quantitative Risk Management                     | Compulsory       | 1          | 7.5       |
| <a href="#">MATH70111</a>       | Interest Rates Models                            | Compulsory       | 2          | 7.5       |
| <a href="#">MATH70112</a>       | Computing for Finance                            | Compulsory       | 1 & 2      | 5         |
| <a href="#">MATH70113</a>       | Simulation Methods for Finance                   | Compulsory       | 2          | 7.5       |
| Total Compulsory Module Credits |                                                  |                  |            | 50        |
| Module Code                     | Module Title                                     | Elective         | Term       | Credits   |
| <a href="#">MATH70116</a>       | Deep Learning                                    | Elective         | 1.1        | 5         |
| <a href="#">MATH70129</a>       | Portfolio Management                             | Elective         | 1.1        | 5         |
| <a href="#">MATH70118</a>       | Quantum Machine Learning in Finance              | Elective         | 1.2        | 5         |
| <a href="#">MATH70122</a>       | Optimization in Machine Learning                 | Elective         | 1.2        | 5         |
| <a href="#">MATH70119</a>       | Numerical Methods in Finance                     | Elective         | 2.1        | 5         |
| <a href="#">MATH70120</a>       | Reinforcement Learning                           | Elective         | 2.1        | 5         |
| <a href="#">MATH70125</a>       | Market Microstructure                            | Elective         | 2.1        | 5         |
| <a href="#">MATH70126</a>       | Stochastic Control in Finance                    | Elective         | 2.1        | 5         |
| <a href="#">MATH70065</a>       | Generative Modelling in Finance                  | Elective         | 2.2        | 5         |
| <a href="#">MATH70121</a>       | Quantitative Trading of Fixed Income Derivatives | Elective         | 2.2        | 5         |
| <a href="#">MATH70127</a>       | Quantitative Trading and Price Impact            | Elective         | 2.2        | 5         |
| <a href="#">MATH70128</a>       | Volatility Modelling                             | Elective         | 2.2        | 5         |
| Total Elective Modules Credits  |                                                  |                  |            | 25        |
| Module Code                     | Module Title                                     | Elective         | Term       | Credits   |
| MATH70114                       | MSc Mathematics and Finance Research Project     | Core             | 3 & Summer | 15        |
| Total Core Module Credits       |                                                  |                  |            | 15        |
| <b>Total Programme Credits</b>  |                                                  |                  |            | <b>90</b> |

Students should take seven compulsory core modules and choose only five from a menu of elective modules. It is not possible to take modules that in total exceed 25 ECTS.

The information displayed in this guide is correct at the time of publication and is **subject to change**. Updated information will be posted on your Programme Canvas page.

# Module Descriptions

## Compulsory Modules

### MATH70107: Fundamentals of Option Pricing

**Module Leader:** Prof. H Zheng

**Assessment:** 100% exam

#### Description

This module is an introduction to Option Pricing theory, first in finite discrete time (focusing mostly on the binomial and trinomial models, and starting in the one-period setting), and then most importantly in continuous time (focusing on the Black-Scholes model).

#### Learning Outcome

On successful completion of this module you will be able to display mastery of a complex and specialised area of option pricing, employing advanced skills to conduct professional activity in this field. In particular you will be able

- To demonstrate knowledge of what financial derivatives are, and what they are used for
- To apply the financial and mathematical concepts and ideas underlying the option-pricing theory and use to evaluate options traded on financial markets.
- To use the probabilistic tools and methods needed to develop such theory, especially in the (continuous-time) setting of the Black-Scholes model (these tools will be introduced here in a self-contained manner, and further explored in the Stochastic Processes course)
- To price simple derivatives

#### Module Content

Option pricing, which in 1987 was described by the famed economist Ross as "the most successful theory not only in finance, but in all of economics", is a core area of Mathematical Finance. While financial derivatives have existed ever since civilisations developed writing, it was only in 1900 with Bachelier's PhD thesis, and later in the '70s with (mostly notably) the Nobel-prize winners Black, Scholes and Merton, that this theory was given a firm mathematical basis, using a probabilistic representation of market uncertainty and relying on the no-arbitrage principle. This module is an introduction to such theory, first in finite discrete time (focusing mostly on the binomial and trinomial models, and starting in the one-period setting), and then most importantly in continuous time (focusing on the Black-Scholes model).

We will cover both financial and mathematical concepts, such as:

- Financial markets, forwards, options and financial derivatives
- Self-financing portfolios and non-anticipative strategies
- Absence of arbitrage, the domination property and law of one price
- Complete and incomplete markets, pricing by replication
- Change of measure and the Radon-Nikodym theorem
- Linear programming, equivalent martingale measures, the fundamental theorems of asset pricing, the risk-neutral pricing formula
- Change of numeraire
- Conditional expectation: definition, properties, and computation
- Martingales and Markov processes
- Brownian motion and the Black-Scholes model as continuous-time limits of scaled random walks

- Why Riemann integration and ordinary calculus do not apply to Brownian motion
- Stochastic integral, semimartingales and their canonical decomposition, quadratic variation, Ito's formula
- Hedging derivatives in the Black-Scholes model, delta-hedging
- The Black-Scholes PDE (Partial Differential Equation), the heat equation, and the Feynman-Kac formula
- Girsanov's theorem, the martingale representation theorem, Levy's characterisation of Brownian motion
- Black-Scholes pricing formula for call and put options, the Greeks

## **MATH70108: Statistical Methods for Finance**

**Module Leader:** Prof. T. Cass

**Assessment:** 15% CW, 85% Exam

### **Description**

This course is concerned with essential statistical methods for the analysis of financial data.

### **Learning Outcome**

On completion of this module you will be able to display command of a complex and specialised area of knowledge and skills in statistics, which will assist you in advanced professional activity in the area of quantitative finance. In particular you will be able to:

- Select and apply the statistical tools required by the financial industry.
- Apply the mathematical tools of risk measures and use them to monitor risks of portfolios
- Extract useful information for time series of financial indicators
- Implement and test those results in Python on real data

### **Module Content**

The financial industry has changed dramatically over the past few years, and the new regulations imposed to banks require more statistical knowledge. The aim of this compulsory module is to reflect these changes, and to make students up to date with the current needs of the financial sector. This course is concerned with essential statistical methods for the analysis of financial data. Topics covered include regression methods (including ordinary and generalised least squares), Random matrices, statistical tests Bayesian analysis, parametric estimation methods (including maximum likelihood estimation and classical asymptotic theory), and non-parametric estimation methods. The various methods are illustrated by applications in Finance and tests on real data.

## **MATH70109: Stochastic Processes**

**Module Leader:** Dr E. Neuman

**Assessment:** 100% Exam

### **Description**

This course gives an introduction to probability theory and measure theory and introduces stochastic processes and the basic tools from stochastic analysis to provide the mathematical foundations for option pricing theory.

### **Learning Outcome**

On successful completion of this module you will display mastery of a complex and specialised area of knowledge and skills in stochastic process. In particular you will be able to:

- Apply advanced techniques in integration theory, probability theory, the theory of martingales and stochastic integration.

- Derive the key results covered in the module.
- Apply the ideas in the module to understand complex examples and interpret the results.

## Module Content

This course gives an introduction to probability theory and measure theory and introduces stochastic processes and the basic tools from stochastic analysis to provide the mathematical foundations for option pricing theory. It includes an intermediate introduction to axiomatic probability theory and measure theory, explaining notions like probability spaces, measures, measurable functions, integration with respect to measures, convergence concepts for random variables, joint distributions, independence and conditional expectations. It studies stochastic processes including martingale theory in discrete and continuous time. These in turn involve notions like the quadratic variation. We will cover the stochastic Ito integral, the Ito formula, and their mathematical applications.

## MATH70110: Quantitative Risk Management

**Module Leader:** Prof. H. Zheng and Dr C. Rossato

**Assessment:** 30% CW, 70% Exam

## Description

This course introduces the key concepts and methods of quantitative risk management, with an emphasis on market risk and volatility.

## Learning Outcome

On successful completion of this module you will be able to:

- Display mastery of complex risk measures used in specialised areas of finance;
- Outline the taxonomy of risks and the current regulatory framework on financial institutions;
- Explain the mapping from risk factors to loss distributions and the definitions of the standard risk measures, expected shortfall and value-at-risk;
- Apply the standard methods for risk measure forecasting and evaluate the quality of the forecasts;
- Characterise the properties of heavy-tailed distributions and the asymptotic behaviour of maxima of random variables and implement these methodologies in risk measurement;
- Develop multivariate dynamic risk models and apply them to forecasting of portfolio risk;
- Design multivariate models of portfolio and credit risk using copulas;
- Explain the key concepts of market microstructure and relate them to market liquidity risk assessment and forecasting of market risk using high-frequency financial data.

## Module Content

After the financial crisis of 2007-2008, in particular, it has been understood how important proper risk management is for the solvency of financial institutions and for the stability of the entire financial system. This course introduces the key concepts and methods of quantitative risk management, with an emphasis on market risk and volatility. We shall cover the following topics:

- Risk management and stylised facts: taxonomy of risks, the regulatory framework, overview of quantitative risk management, stylised facts of asset returns.
- Basic concepts of risk management: risk factors, loss distributions, risk measures (including value-at-risk and expected shortfall), historical simulation, Monte Carlo simulation, back testing.
- Univariate time series modelling: ARMA and GARCH models, estimation and forecasting, applications to risk measures.
- Heavy-tailed distributions and extreme value theory: characterisations of heavy-tailed distributions and examples, the distribution of maxima, modelling of threshold exceedances, applications to risk

measures.

- Multivariate time series and covariance modelling: multivariate time series models, multivariate GARCH models, applications to equity portfolio risk.
- Copulas and dependence modelling: basic properties of copulas, classification of copulas with examples, measuring dependence, estimation of copulas, applications to portfolio and credit risk.
- Market microstructure and high-frequency data: market microstructure primer, market liquidity risk, volatility estimation and forecasting using high-frequency data, applications to risk measures.

## **MATH70111: Interest Rate Models**

**Module Leader:** Prof. D. Brigo and Dr Y. Zhang

**Assessment:** 100% Exam

### **Description**

The course presents a detailed analysis of interest rates markets, from modelling to pricing derivatives. It covers both the classical tools as well as the many changes that have been implemented and regulated since the 2007-2008 crisis.

### **Learning Outcome**

On successful completion of this module, you will be able to:

- Display mastery in the theory and practice of term structure of interest rates when including also credit risk, funding liquidity costs, collateral modeling and multiple curves
- Apply and price non-linear valuation and their approximation via XVA.
- Extend the theory of valuation of interest-rate derivatives to post-2008 finance.

### **Module Content**

The paradigm of derivatives valuation is shifting from complex payouts designed on simple single asset class risks to simple products that are now managed by analysing previously neglected complex and interconnected nonlinear risks. The course starts by briefly putting derivatives valuation into context, in connection also with the onset of the 2007-2008 crisis that prompted many of the changes we are seeing now. The course then moves to classic interest rate models based on a risk free rate, on classical instantaneous forward rates, and on default free LIBOR and SWAP rates, also in presence of volatility smile. Several families of models are introduced and studied in detail, with an eye both to a rigorous theoretical derivation and to practical implementation and calibration. Following the classical part, the increasingly important issues of multiple discount curves, credit risk, credit and debit valuation adjustments, collateral modeling, gap risk and funding liquidity costs are analysed quantitatively. The related notions of CVA, DVA and FVA are analysed and criticized in detail, and their significance for the general derivatives valuation paradigm is discussed. The specific case of trading through central clearing (CCPs) is hinted at. Finally, an analysis of Risk measures for interest rate derivatives products is presented, with a case study highlighting the role of correlation and dependence in Risk measurement.

## **MATH70112: Computing in Finance**

This course gives an introduction to programming in C++ and in Python for Quantitative Finance problems.

The Python part represents 15 hours of lectures and the C++ part counts for 25 hours of lectures.

### **Part 1: Python**

**Module Leader:** Dr T. Hadfield and Prof J. Jacquier

**Assessment:** 30% CW

### **Description**

Python has become a key programming language today, due to its versatility, open-source nature and wide

range of capabilities, from big data management to numerical analysis and visualisation tools. In and in banks. Python is now a fundamental tool to quantitative analysis, and a pre-requisite for job applications. The goal of this module is to introduce the key concepts of the Python programming language and its main characteristics, in particular:

- Programming features: Functional Programming to Object-oriented programming
- Big data analysis: Pandas, Dask,
- Statistical tools and libraries
- Web-scraping
- Visualisation tools

All these concepts will be illustrated with real data and in the context of Quantitative Finance.

### **Learning Outcome**

- Write Functional Programming and Object-Oriented Programming code in Python
- Import and analyse data in an IPython notebook
- Write optimised code in Python for financial and data-oriented applications
- Develop basic neural networks in Python

### **Module Content**

Python has become a ubiquitous programming language in the financial world. We shall be covering a wide spectrum, from core Functional and Object-oriented Programming to data import, cleaning and analysis.

We shall keep an eye on new Python libraries developed for data applications.

### **Part 2: C++: Object oriented programming**

**Module Leader:** Dr P. Bilokon

**Assessment:** 30% CW, 40% Exam

### **Description**

The module gives an introduction to object-oriented programming in C++. In contrast to structured programming, where a programming task is simply split into smaller parts, which are then coded separately, the essence of object-oriented programming is to decompose a problem into related subgroups, where each subgroup is self-contained and contains its own instructions as well as the data that relates to it. Starting from the simple concept of a class that contains both data and methods relating to that data, the module will cover all the major features of object-oriented programming, e.g. encapsulation, inheritance and polymorphism. To this end, the module will address operator overloading, virtual functions and templates.

### **Learning Outcome**

- Build classes in C++
- Create C++ objects and constructors
- Encapsulate C++ objects
- Write C++ programs making use of pointers
- Write C++ programs to solve financial problems

### **Module Content**

When it comes to speed, nothing really beats C++ and object-oriented programming in C++. In contrast to structured programming, where a programming task is simply split into smaller parts, which are then coded separately, the essence of object-oriented programming is to decompose a problem into related subgroups, where each subgroup is self-contained and contains its own instructions as well as the data that relates to it. Starting from the simple concept of a class that contains both data and methods relating to that data, the course

will cover all the major features of object-oriented programming, e.g. encapsulation, inheritance and polymorphism. To this end, the course will address operator overloading, virtual functions and templates.

We will always use financial applications, with real data, to illustrate the many topics covered in this module.

### **MATH70113: Simulation Methods for Finance**

**Module Leader:** Dr Yufei Zhang

**Assessment:** 20% CW & 80% Exam

#### **Description**

This course is an introduction to simulation methods in finance and more generally to probabilistic numerical methods for PDEs and SDEs.

#### **Learning Outcome**

On successful completion of this module you will be able to display mastery of a complex and specialised area of simulation, employing advanced skills to conduct professional activity in derivative pricing. In particular you will be able to:

- Evaluate most derivatives used in today's financial markets.
- Generate random variables, compute expectations with simulations, and solve SDEs numerically with error analysis.
- Generate risk scenarios and analyse them
- Critically consider quantitatively the new regulations (Basel) imposed on banks, and check them efficiently.

#### **Module Content**

This course is an introduction to simulation methods in finance and more generally to probabilistic numerical methods for PDEs. It starts with discussion of random number generators, statistical tests and moves on to cover numerical schemes for solving stochastic differential equations: the Euler, Milstein and certain higher-order schemes. Properties of weak and strong convergence, consistency and numerical stability are established. It then discusses variance reduction techniques and estimation of sensitivities. The course will be concluded by studying a numerical method for American options and non-linear PDEs.

### **Elective modules:**

Electives are grouped in three indicative streams:

- **Derivatives Pricing Stream:** Volatility Modelling, Quantitative Trading of Fixed Income Derivatives, Numerical Methods in Finance
- **Market Microstructure Stream:** Optimisation in Machine Learning, Stochastic Control in Finance, Quantitative Trading and Price Impact, Market Microstructure, Portfolio Management,
- **Machine Learning in Finance Stream:** Reinforcement Learning, Deep Learning, Quantum Machine Learning in Finance, Generative Modelling in Finance

### **MATH70065: Generative Modelling in Finance**

**Module Leader:** Dr C. Salvi

**Assessment:** 100% CW

#### **Description**

The goal of this module is to present the most widely used generative AI algorithms: denoising diffusion models, flow matching and related variants such as neural stochastic differential equations. These models are all based on differential equations and are the backbone of the best image, audio, and video generation models (e.g.,

Stable Diffusion 3 and Movie Gen Video). In addition they have most recently become the state-of-the art in scientific applications such as protein structures (e.g., AlphaFold3 is a diffusion model).

### **Learning Outcome**

At the end of the module students will be able to: 1) Understand the mathematical underpinnings behind recent state-of-the art generative models; 2) Understand how classical ODE and SDE theory can be integrated with modern deep learning tools; 3) Implement their own diffusion model from scratch and apply it to real world examples.

### **Module Content**

1. Introduction to Flow and Diffusion Models;
2. Loss functions and training targets;
3. Training and Optimisation;
4. Extension to discrete data;
5. Applications.

## **MATH70116: Deep Learning**

**Module Leader:** Dr T. Hadfield

**Assessment:** 10% CW, 90% Exam

### **Description**

The module will first introduce the multi-layer neural nets and explain their universal approximation property. Subsequently, the module proceeds to the training of neural nets, starting from the derivation of the gradient of a neural net and its evaluation through backpropagation, culminating in the stochastic gradient descent and related modern optimisation methods.

### **Learning Outcome**

On completion of this module you will be able to display mastery of a complex and specialised area of knowledge and skills in deep learning. In particular you will be able to:

- Explain the structure and components of multi-layer neural nets;
- Recognise the universal approximation properties of neural nets;
- Derive the gradient of a neural net and evaluate it using backpropagation;
- Explain the training of neural nets using stochastic gradient descent and its variants;
- Apply procedures to avoid overfitting;
- Implement and train neural nets using Keras and TensorFlow;
- Develop deep neural networks to solve computational and statistical problems in finance
- Recognise the key principles of recurrent neural nets.

### **Module Content**

Deep learning is subfield of Machine Learning that applies deep neural nets to represent and predict complex data. It has recently revolutionised several areas such as image recognition and artificial intelligence and it is currently gaining traction also in the financial industry. The module will first introduce the multi-layer neural nets and explain their universal approximation property. Subsequently, the module proceeds to the training of neural nets, starting from the derivation of the gradient of a neural net and its evaluation through backpropagation, culminating in the stochastic gradient descent and related modern optimisation methods. Techniques to avoid overfitting in training are also elucidated. The remainder of the module focuses on the practical implementation and training of deep neural nets using Keras and TensorFlow, with examples in computational and statistical finance. Time permitting, elements of recurrent neural nets are also sketched.

## **MATH70118: Quantum Machine Learning in Finance**

**Module Leader:** Dr A. Kondratyev and Prof J. Jacquier

**Assessment:** 100% CW

### **Description**

The goal of this Elective (so far not given in any similar MSc programmes around the world) is to introduce this new technology and these new algorithms and show how they can be used to solve financial problems, in particular

- For portfolio optimisation,
- For data generation,
- For Machine learning and neural network.

### **Learning Outcome**

On completion of this module you will be able to display mastery of a complex and specialised area of knowledge and skills in Quantum computing. In particular you will be able to:

- Apply the fundamentals of Quantum circuits
- Develop quantum neural networks and apply them for Quantum Machine Learning and Data generation
- Select and use the right quantum algorithms such as the Variational Quantum Eigensolver, and Quantum Approximate Optimisation Algorithm for practical problems
- Use Quantum computing techniques for optimisation

### **Module Content**

Quantitative Finance is a rapidly changing environment, and the financial industry is always on the lookout for new techniques and new technologies able to harness the rise of big data and the availability of computing power. Quantum computing, though not a recent field, has gained huge popularity in the past few years with the development of small-scale quantum computers and quantum annealers. These have in turn pushed for new algorithms, hybrid between classical and quantum, and tailored for such computers. The financial industry is now looking at such developments and there is a common agreement that this will be one of the leading advances in the coming decade.

The goal of this Elective (so far not given in any similar MSc programmes around the world) is to introduce students to this new technology and these new algorithms and show them how they can be used to solve financial problems, in particular

- For portfolio optimisation,
- For data generation,
- For Machine learning and neural network.

The module will strike a fair balance between theoretical concepts of Quantum Computing, their implementation (in Python using IBM's Qiskit framework) and their application to real financial problems.

## **MATH70119: Numerical Methods in Finance**

**Module Leader:** Dr Steve Campbell

**Assessment:** 30% CW, 70% Exam

### **Description**

The goal of this module is to complement the Core module on Simulation Methods via introducing several other numerical techniques which are widely adopted by the industry. We shall investigate three popular approaches, namely lattice methods, PDE methods and Fourier methods.

## Learning Outcome

On completion of this module you will be able to display mastery of a complex and specialised area of knowledge and skills in numerical analysis. In particular you will be able to:

- Approximate a continuous-time model by a discrete lattice model and implement the associated algorithms to price European/American options;
- Translate an option pricing problem into a partial differential equation and construct numerical schemes to solve the equation;
- Analyse the convergence and practical limitations of these numerical schemes;
- Implement these algorithms on widely used models and derivatives products, and test them on real data;
- Evaluate options using Fourier transform techniques, and implement Fast Fourier transforms;
- Critically assess existing numerical methods for pricing and hedging in the financial industry.

## Module Content

The goal of this module is to complement the Core module on Simulation Methods via introducing several other numerical techniques which are widely adopted by the industry. We shall investigate three popular approaches, namely lattice methods, PDE methods and Fourier methods.

For each approach, we will start with a theoretical framework, explaining how an option pricing problem can be turned into a dynamic programming problem, a PDE or a Fourier integration. We shall then focus on the numerical methods to solve these problems. Practical implementations on real models/data will be emphasised.

Literature:

Pascucci - PDE and martingale methods in option pricing

Achdou, Pironneau - Computational methods for option pricing

Zhu - Applications of Fourier transform to smile modelling

## MATH70120: Reinforcement Learning

**Module Leader:** Dr Y. Zhang

**Assessment:** 25% CW, 75% Exam

## Description

This module provides an introduction to the fundamental concepts and techniques of reinforcement learning, a key area of machine learning centered on decision-making and sequential problem-solving. Students will delve into the theoretical foundations, implement core algorithms, and explore practical applications across domains such as robotics and finance. Through hands-on exercises, they will acquire the skills needed to design, implement, and analyze reinforcement learning systems effectively.

## Learning Outcome

On completion of this module you will have a comprehensive understanding of the fundamental principles and methodologies of reinforcement learning. In particular you will be able to:

- Formulate decision-making problems as Markov Decision Processes.
- Implement key reinforcement learning algorithms such as dynamic programming, Monte Carlo methods, temporal difference learning, and policy gradient techniques.
- Evaluate reinforcement learning algorithm performance and understand exploration-exploitation trade-offs.
- Apply reinforcement learning techniques to practical decision-making problems in various domains such as robotics, finance, and game AI.

## Module Content

The module will cover the foundational concepts and methods of reinforcement learning. Topics will include multi-armed bandits, Markov Decision Processes, value-based methods (such as Q-learning and SARSA), policy-based methods (such as policy gradient and actor-critic algorithms), and advanced techniques like deep reinforcement learning. Practical aspects will be emphasized through coding exercises and case studies in environments like OpenAI Gym, enabling students to apply theoretical concepts to real-world challenges.

## MATH70121: Quantitative Trading of Fixed Income Derivatives

**Module Leader:** Dr V. Piterbarg

**Assessment:** 100% CW

### Description

The goal of this module is to introduce the technical tools needed to understand the specificities of fixed Income markets and the trading therein.

### Learning Outcome

On completion of this module, you will be able to display mastery of a complex and specialised area of knowledge and skills in Derivative Pricing. In particular you will be able to:

- Apply the principles of derivatives pricing rigorously and apply them to newly issued options in financial markets;
- Identify the mechanisms of prevalent structured products offered by investment banks;
- Utilise tools from stochastic analysis to compute prices of exotic products such as quanto, barrier, lookback and Asian options;
- Define and compute implied volatilities;
- Outline the precise interaction between pricing and hedging of options, and apply these ideas to volatility arbitrage;
- Price derivatives (e.g. variance swap) under a model-independent framework

### Module Content

Derivatives pricing is at the core of trading and model validation, in so far as traders and quantitative analysts rely on stochastic models to build their trades and monitor their risks. The goal of this module is to introduce the technical tools needed to understand the specificities of these models and their inherent risks.

In order to do so, the module will constantly strike a fair balance between the mathematical framework and specific tools (stochastic analysis, Fourier methods, fractional calculus), the numerical aspects (actual implementation of the models, optimization routines) and the data (calibration on real data, backward testing of hedging strategies)

Literature:

Gatheral: The Volatility Surface

Guyon, Henry-Labordere - Nonlinear option pricing

Pascucci - PDE and martingale methods in option pricing

Joshi - CPP design patterns and derivatives pricing

## MATH70122: Optimisation in Machine Learning

**Module Leader:** Dr Y. Jiang

**Assessment:** 100% Exam

## Description

The module covers both the theoretical underpinnings of convex optimisation and its applications to important problems in mathematical finance especially in the context of machine learning.

## Learning Outcome

On completion of this module you will be able to display mastery of a complex and specialised area of knowledge and skills in optimisation. In particular you will be able to:

- Recognise and formulate optimisation problems that are (or can be recast as) convex and not convex, and identify particularly important subclasses of such problems; in particular as it pertains to mathematical finance;
- Apply duality theory to solve the particular class of convex optimisation problems;
- Apply specific numerical algorithms (gradient descent, particle swarm, simulated annealing) to solve non-convex optimisation problems in the framework of machine learning.

## Module Content

The course covers both the theoretical underpinnings of optimisation and its applications to important problems in machine learning and mathematical finance. A brief outline of the course reads as follows:

1. Fundamental properties of convex sets and convex functions
2. The basics of convex optimisation with special emphasis on duality theory
3. Markowitz portfolio theory and the CAPM model
4. Numerical algorithms for non-convex problems: particle swarm, (stochastic) gradient descent, simulated annealing
5. Importance of different loss functions for optimisation problems
6. Applications to classification problems, PDE solvers

The modules will strike a fair balance between the theoretical analysis of (convex and non-convex) optimisation problems and the related numerical solvers and the applications (and tailor-made solutions) to machine learning and mathematical finance.

The course material will consist of a full set of self-contained lecture notes and will draw strong inspirations from the reference books 'Optimization Methods in Finance' (2nd ed) by Cornuéjols, Peña, and Tütüncü and 'Algorithms for Optimization', by Kochenderfer and Wheeler.

## MATH70125: Market Microstructure

**Module Leader:** Prof. M. Rosenbaum

**Assessment:** 100% CW

## Description

The goal of the module is to develop thorough understanding of how trades occur in financial markets.

## Learning Outcome

On completion of this module you will be able to display mastery of a complex and specialised area of knowledge and skills in market microstructure. In particular you will be able to:

- Analyse market types and explain why traders trade;
- Recognise the main order types and their purpose in trading;
- Derive measures of market liquidity and models of price impact;
- Explain the recent trends of market fragmentation and dark liquidity
- Analyse the impact of tick size on market behaviour;

- Apply statistical models of order flow data;

### **Module Content**

The goal of the module is to develop thorough understanding of how form, information is aggregated, and trades occur in financial markets. The main market types will be described as well as traders' main motives for why they trade. Market manipulation and high-frequency trading strategies have received a lot of attention in the press recently, so the module will illustrate them and examine recent developments in regulations that aim to limit them. Liquidity is a key theme in market microstructure, and the students will learn how to measure it and to recognise the recent increase in liquidity fragmentation and hidden, "dark" liquidity. The Flash Crash of 6 May 2010 will be analysed as a case study of sudden loss of liquidity.

### **MATH70126: Stochastic Control in Finance**

**Module Leader:** Prof. H. Zheng

**Assessment:** 100% Exam

### **Description**

The goal of this module is to bring the main concepts and techniques from dynamic stochastic optimisation and stochastic control theory to the realm of quantitative finance.

### **Learning Outcome**

On completion of this module you will be able to display mastery of a complex and specialised area of knowledge and skills in stochastic control. In particular you will be able to:

- Explain the main concepts underlying the Dynamic Programming Principle;
- Define the notion of viscosity solutions to Partial Differential Equations;
- Derive Hamilton-Jacobi-Bellman equations and prove their properties;
- Apply dynamic programming methods to solve various problems in mathematical finance (portfolio allocation, investment problems, super-replication, utility maximisation) and develop a critical understanding of the benefits and limitations of these tools for practical purposes.

### **Module Content**

Many problems in mathematical finance (and in other areas) are essentially optimisation problems subject to random perturbations, where some controls play the role of a performance criterion. The goal of this module is to bring the main concepts and techniques from dynamic stochastic optimisation and stochastic control theory to the realm of quantitative finance. It will therefore naturally start with a theoretical part focussing on required elements of stochastic analysis, and with a motivation through several examples of control problems in Finance. We will then turn to the classical PDE approach of dynamic programming, including controlled diffusion processes, dynamic programming principle, the Hamilton-Jacobi-Bellman equation and its verification theorem. We will finally see how to derive and solve dynamic programming equations for various financial problems such as the Merton portfolio problem, pricing under transaction costs, super-replication with portfolio constraints, and target reachability problems.

### **MATH70127: Quantitative Trading and Price Impact**

**Module Leader:** Prof. J. Muhle-Karbe

**Assessment:** 100% CW

### **Description**

In this module, we will study specificities of automated trading. In particular, we shall develop the mathematical tools required to construct trading algorithms in a high-frequency framework.

### **Learning Outcome**

On completion of this module you will be able to display mastery of a complex and specialised area of knowledge and skills in algorithmic trading. In particular you will be able to:

- Apply probabilistic, variational and dynamic programming methods to solve problems that arise from algorithmic and high frequency trading;
- Prove the key results of the module;
- Apply the ideas in the module to analyse and review related trading models and interpret the results.

### **Module Content**

The increase in computer power over the last decades has given rise to prices being quoted and stocks being traded at an ever-increasing pace. Since humans are not able to place orders at this speed, algorithms have replaced classical traders to optimise portfolios and investments. In this module, we will study specificities of this market, and in particular, we shall develop the mathematical tools required to develop such algorithms in this high-frequency framework. The module will start with a short review of stochastic optimal control, which forms the mathematical background. We shall then move on to study optimal execution, namely how and when to place buy/sell orders in this market, both assuming continuous trading and in the context of limit and market orders. The last part of the module will be dedicated to the concept of market making and statistical arbitrage in high-frequency settings.

### **MATH70128: Volatility Modelling**

**Module Leader:** Dr V. Lucic and Prof J. Jacquier

**Assessment:** 100% CW

### **Description**

The goal of this module is to complement the other option-flavoured modules, specifically focusing on modelling the volatility of Equity markets. **Learning Outcome**

On completion of this module you will be able to display mastery of a complex and specialised area of knowledge and skills in quantitative finance. In particular you will be able to:

- Recognise the (always evolving) difference between vanilla and exotic options;
- Define properly the notion of model risk;
- Characterise the properties and pitfalls of stochastic volatility models;
- Analyse dynamics of interest rates and the notion of the volatility cube;
- Analyse and pre-process option price data on several trading markets;
- Evaluate path-dependent options and monitor their risks.

### **Module Content**

Option markets are extremely diverse, spanning several different asset classes and many pricing and hedging strategies. The goal of this module is to complement the other option-flavoured modules, focusing on the specificities of Foreign Exchange and Fixed Income markets. For each of these markets, the module will study their specific characteristics and evolutions, develop the technical tools needed to understand the pricing of derivatives, and explain how to set up trading and hedging strategies therein. A strong emphasis will be given on the actual implementation of the models and their calibration to real data.

### **MATH70129: Portfolio Management**

**Module Leader:** Dr C. Rossato

**Assessment:** 40% CW, 60% Exam

### **Description**

This module provides a foundation for quantitative portfolio management and for understanding market price determination.

## **Learning Outcome**

On completion of this module you will be able to display mastery of a complex and specialised area of knowledge and skills in portfolio management. In particular you will be able to:

- Explain the basic principles of quantitative portfolio management;
- Derive (equilibrium) implications for market prices;
- Apply and implement widely used approaches such as factor models and momentum strategies;
- Characterise the important empirical and institutional properties of different markets.
- Develop a critical understanding of current practices in portfolio management in the financial industry.

## **Module Content**

This module gives students a foundation for quantitative portfolio management and for understanding market price determination. Key concepts include risk measurement, risk-reward trade-offs, portfolio optimisation, benchmarking, equilibrium asset pricing, market efficiency, and pricing anomalies. Specific portfolio management tools include mean-variance optimisation, CAPM and APT asset pricing, factor models (e.g., Fama-French), momentum strategies, and performance evaluation. The course will present essential theories and formulas and will also review important institutional and empirical facts about equity, bond, and commodity markets.