A Hierarchical Model for the Age of Halo White Dwarfs

Shijing Si*, David van Dyk*, Ted von Hippel**
*Department of Mathematics, Imperial College London, U.K.
**Department of Physical Science, Embry Riddle Aeronautical University, U.S.

Imperial College London

Background and Motivation

Stellar evolution is a branch of astronomy which focuses on how stars are formed and how they evolve over time. Astrophysicists characterise a star using parameters such as age, distance from the Earth, mass, composition. Computer models predict stellar photometry from these parameters. Photometry is a commonly used measure of an astronomical object's brightness. Photometric data are observed via telescopes with certain measurement errors.

To make inference about stellar parameters, van Dyk et al. (2009) has developed a principled Bayesian statistical technique that embeds computer models for stellar evolution into a multilevel model that accounts for data contemination, measurement errors and multi-star systems. Markov chain Monte Carlo(MCMC) is employed to explore the joint posterior distribution of parameters of this model. Figure 1 is a typical serpent-shaped joint posterior density of logAge (short for $\log_{10}(\mathrm{Age})$), distance modulus, and mass of a white dwarf star.

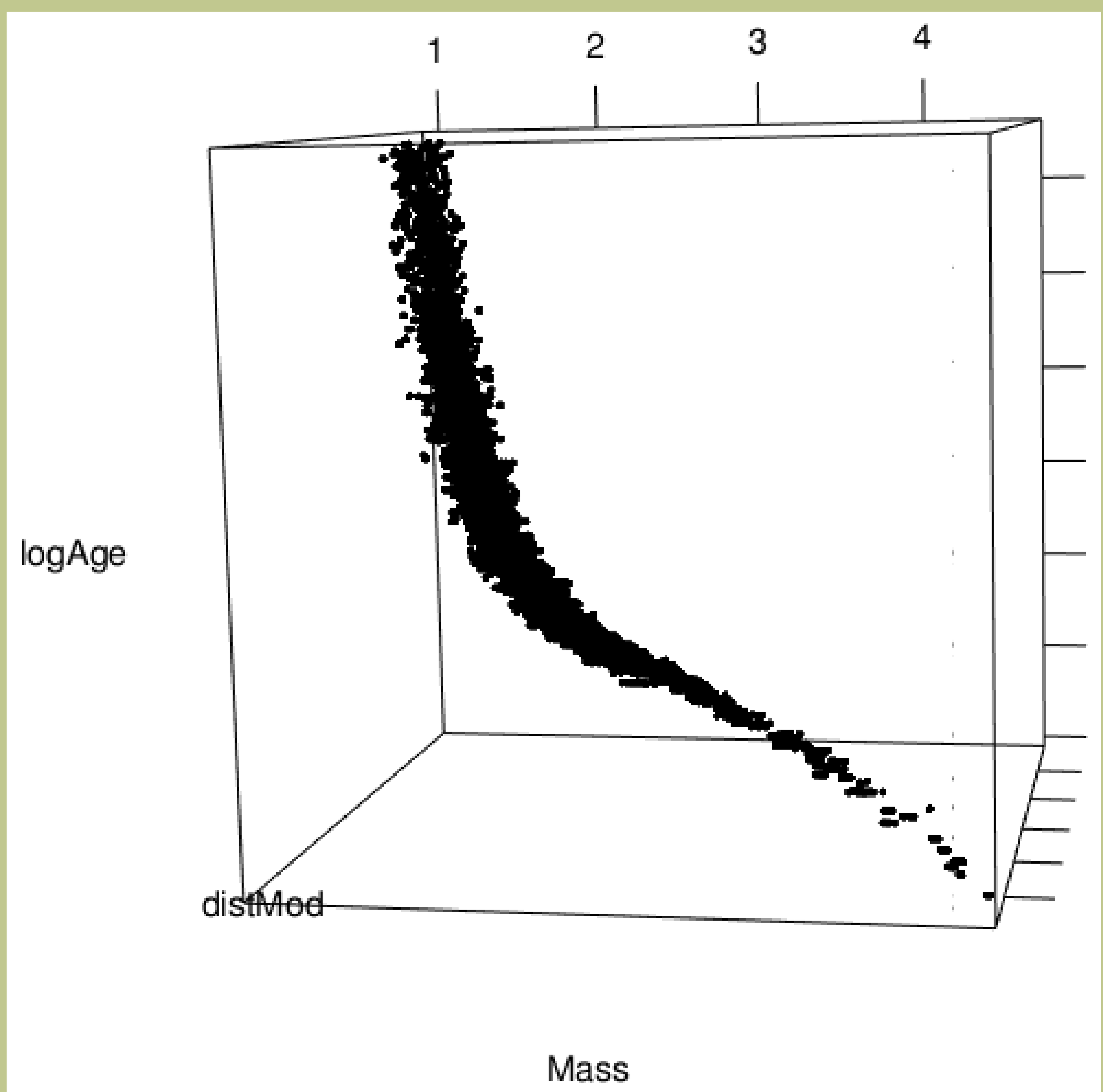


Figure 1: Marginal posterior density of age, distance modulus and mass.

Hierarchical Modelling a Cluster of Halo Stars

The aim of this project is to learn the population distribution of white dwarf stars in the Galactic halo. Galactic halo is the outer region of a galaxy. Figure 2 shows the plain view of a Galaxy.

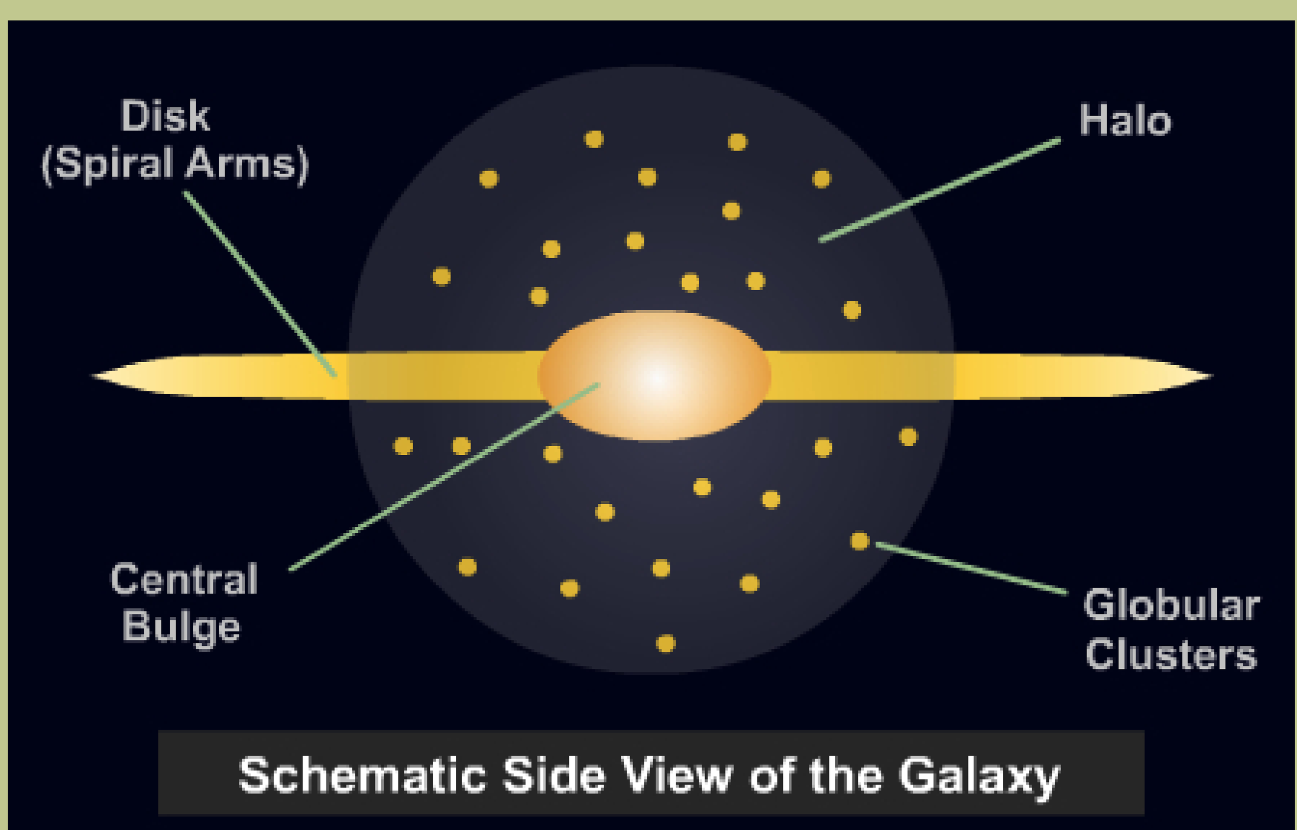


Figure 2: The plain figure of a Galaxy.

For brevity, let $A_j = \log_{10}(\mathrm{Age})$ for star j and θ_j be the mass, distance modulus and metallicity of star j. We consider the hierarchical model:

$$(X_j|\theta_j, A_j) \sim N(G(\theta_j, A_j), \Sigma_j);$$
$$\theta_j \sim p(\theta_j);$$
$$(A_j|\gamma, \tau) \sim N(\gamma, \tau^2), \quad j = 1, \cdots, J,$$

where $G(\cdot)$ is the stellar evolution model. Based on astronomers' understanding, we can set an objective and proper joint prior on γ and τ, denoted by $\pi(\gamma, \tau) \propto 1$.

References

[1] O'Malley, E., von Hippel, T., van Dyk, D. (2013) A Bayesian approach to deriving ages of individual field white dwarfs. *Astrophysical Journal*, **775**.

[2] Stein, N., van Dyk, D., von Hippel, T., DeGennaro, S., Jeffery, E. and Jefferys, W. (2013) Combining computer models to account for mass loss in stellar evolution. *Statistical Analysis and Data Mining*, **6**, 34-52.

[3] van Dyk, D., Degennaro, S., Stein, N., Jefferys, W., von Hippel, T. (2009) Statistical analysis of stellar evolution . *The Annals of Applied Statistics*, **3**, 117-143.

[4] von Hippel, T., Jefferys, W., Scott, J., Stein, N., Winget D. (2006) Inverting color-magnitude diagrams to access precise cluster parameters: A Bayesian approach. *The Astrophysical Journal*, **645**, 1436-1447.

Methodology

In our method, we take empirical Bayes to fit the hierarchical model. It proceeds in two steps. Step 1: find the MAP (Maximum a Posterior) estimate of hyperparameters, γ and τ. Step 2: plug this estimate in the population level model leads to a prior for all individual stars. To circumvent the case that τ peaks at 0, we take the logarithm transformation by taking $\xi = \log(\tau)$.

Here is the description of our algorithm for obtaining the MAP estimates for γ and τ.

Phase 1: MCEM

1. Initialise $\gamma = \gamma^{(1)}, \xi = \xi^{(1)}$ and $\tau = \exp(\xi^{(1)})$;
2. Given the t-th iteration $\gamma^{(t)}$ and $\tau^{(t)}$, for star $i = 1, \cdots, I$, we draw a sample $A_i^{[s,t]}, D_i^{[s,t]}, M_i^{[s,t]}, T_i^{[s,t]}, s = 1, \cdots, S$ of size S from its joint posterior $p(A_i, D_i, M_i, T_i|X_i, \gamma^{(t)}, \tau^{(t)})$ via BASE9 program;
3. $$\gamma^{(t+1)} = \frac{1}{S \cdot I} \sum_{i=1}^{I} \sum_{s=1}^{S} A_i^{[s,t]},$$
$$\xi^{(t+1)} = \log\left(\frac{1}{S \cdot (I-1)} \sum_{i=1}^{I} \sum_{s=1}^{S} (A_i^{[s,t]} - \gamma^{(t+1)})^2\right)/2;$$
$$\tau^{(t+1)} = \exp(\xi^{(t+1)});$$
4. Replicate step 2 and 3 until distance between two consecutive iterations is small enough;

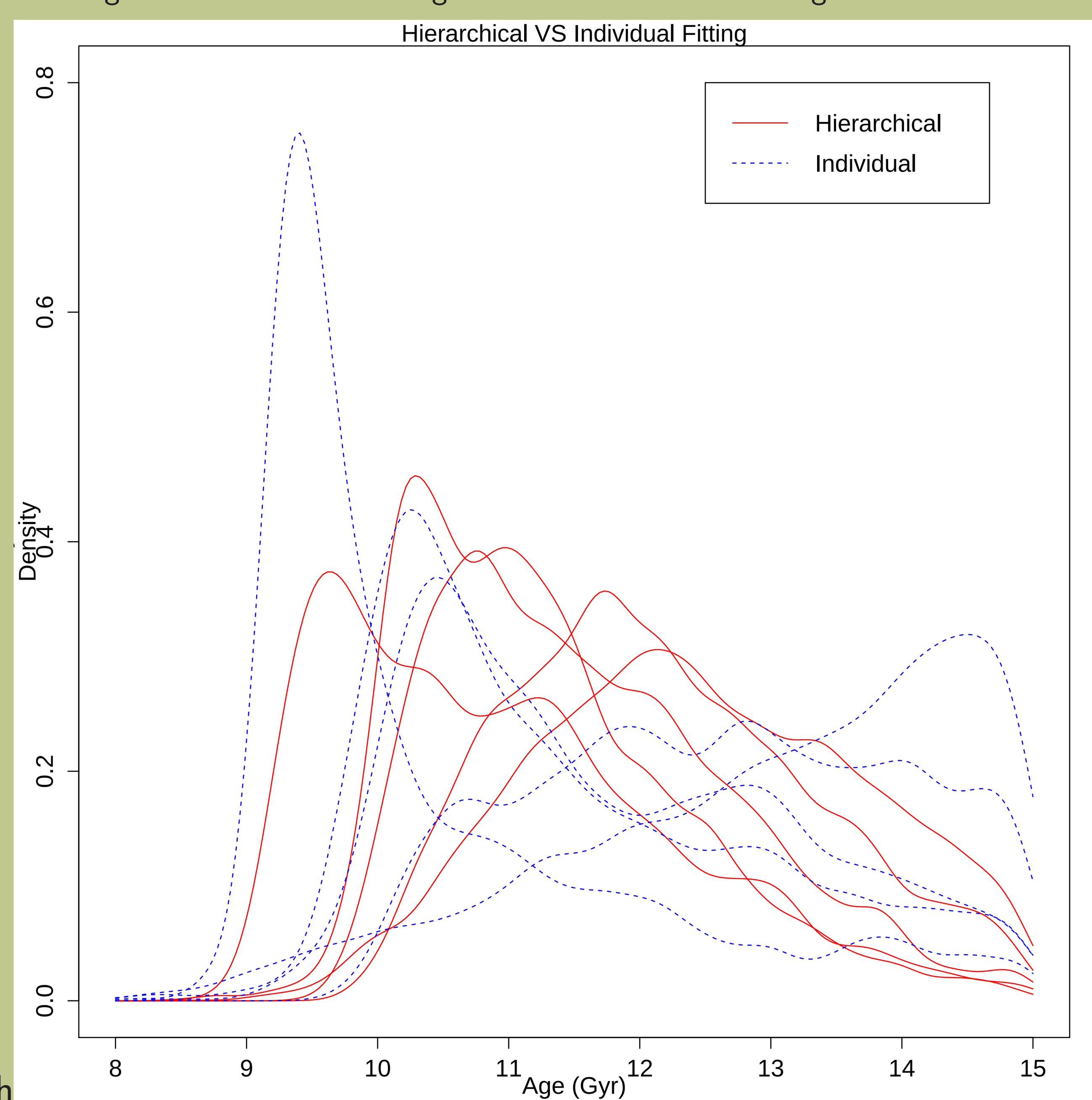
Phase 2: EM with importace sampling

1. Record the last a few samples for all stars in phase 1. For breivity, we take just one sample here, $(A_i^{[*,s]}, M_i^{[*,s]}, D_i^{[*,s]}, T_i^{[*,s]}), i = 1, \cdots, I, s = 1, \cdots, S$ obtained when given $\gamma = \gamma^*, \ \tau = \exp(xi^*)$.
2. Suppose current iteration $\gamma^{(t)}, \tau^{(t)} = \exp(\xi^{(t)})$, then
$$w_i^{[t,s]} = \frac{\frac{\phi(A_i^{[*,s]}|\gamma^{(t)},\tau^{(t)})}{\phi(A_i^{[*,s]}|\gamma^*,\tau^*)}}{\sum_{s=1}^{S} \frac{\phi(A_i^{[*,s]}|\gamma^{(t)},\tau^{(t)})}{\phi(A_i^{[*,s]}|\gamma^*,\tau^*)}};$$
$$\gamma^{(t+1)} = \frac{1}{I} \sum_{i=1}^{I} \sum_{s=1}^{S} A_i^{[*,s]} w_i^{[t,s]};$$
$$\xi^{(t+1)} = \log\left(\frac{1}{(I-1)} \sum_{i=1}^{I} \sum_{s=1}^{S} [A_i^{[*,s]} - \gamma^{(t+1)}]^2\right)/2;$$
$$\tau^{(t+1)} = \exp(\xi^{(t+1)})$$
in which $\phi(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(x-\mu)^2}{2\sigma^2})$;
3. Repeat step 2 until some stopping criteria is satisfied.

Real Data Analysis

Our method is used to fit those five stars. Figure 3 is the comparison between non-hierarchical fitting and its hierarchical countpart. We can see the shrinkage effect of hierarhical modelling, which compromises all available age information from different stars.

Figure 3: Hierarchical fitting versus nonhierarchical fitting of 5 stars



chch

Acknowledgements

The authors are sincerely thankful to Elliot Robinson for his R interface with BASE9, which makes this work go smoothly. Also, thanks department of mathematics for the Roth studentship to support Shijing Si.

 s.si13@imperial.ac.uk