Mathematical Methods

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<th>Module Code</th>
<th>PHYS95011</th>
<th>FHEQ Level</th>
<th>Level 5</th>
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Pre-requisites     | None      | Co-requisites | None    |

Primary Department | Physics   |             |         |
Module Leader       | Professor Dimitry Turaev |             |         |
Additional Teaching Departments | Maths |             |         |

Teaching Staff      | Professor Dimitry Turaev |             |         |

Programmes on which the Module is delivered | Core/Elective |
BSc Physics with Theoretical Physics (F325) & MSci Physics with Theoretical Physics (F390) | Core |
All other UG Physics Programmes (F300, F303, F309, F3W3) | Elective |

Learning Outcomes
On completing the Mathematical Methods course, students will:
- Be able to identify and use mathematical methods useful in Physics.
- Be able to perform analytic continuation and use the residue method for evaluating integrals.
- Be able to use Fourier transform for the solution of linear equations.
- Get familiar with the least action principle and Lagrangian/Hamiltonian formalism.
- Get familiar with tensor calculus.
- Be able to apply basic numerical methods.

Description of Content
The module is split into five parts:

1. CALCULUS OF VARIATIONS
Shortest curve joining two points and Brachistochrone problems, Euler-Lagrange equation as a stationarity condition on

$$ S = \int_a^b L(y(x), y'(x), x)dx, $$

with $y(a)$ and $y(b)$ fixed. If $L$ does not depend explicitly on $x$ then

$$ H = y'^1 \frac{\partial L}{\partial y'^1} - L $$

is constant.

Hamilton's Principle (Lagrangian mechanics), $L = T - V$ for conservative forces, generalised coordinates and momenta, cyclic coordinates, Lagrangian for a charged particle using scalar and vector potentials.

Hanging rope problem, Lagrange multipliers, isoperimetric problems.

Geodesics and the metric tensor $g_{ij}$; transformation properties of $g_{ij}$, Einstein's summation convention, inverse metric $g^{ij}$, Kronecker delta $\delta^i_j$. 

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2. COMPLEX VARIABLES
Complex differentiation, analytic functions, Cauchy-Riemann equations, entire functions, harmonic property of analytic functions.

Complex integration, Fundamental Theorem of Calculus, Cauchy's theorem (anti derivative and standard form), Deformation theorem, Cauchy's integral formula and applications (proof of Liouville's theorem and proof that analytic functions are infinitely differentiable), Taylor’s theorem, isolated singularities and Laurent series, poles and essential singularities, meromorphic functions, Laurent’s theorem.
Residue theorem and application to computing real integrals, Principal value of real and complex integrals, Residue theorem with simple poles on the contour.

3. FOURIER TRANSFORMS
Review of Fourier transforms and Fourier integrals, computation of Fourier transforms using contour integration.
Heaviside and sign function, delta function as a derivative, delta function as a limit of smooth functions, definition through the integral formula
\[ \int_{-\infty}^{\infty} f(x) \delta(x - a) \, dx = f(a). \]
Properties of the delta function, Fourier transform (and Fourier integral representation) of the delta function. Higher dimensional delta functions and Green’s functions.
Application of Fourier transforms to solving linear ODEs and PDEs (eg. driven oscillator ODEs and Laplace’s equation in an infinite strip and half-plane).

4. TENSORS
Definition of vectors via their transformation properties, cartesian tensors, tensor algebra, contraction of tensor indices.
Vectors and pseudo-vectors (or polar and axial vectors), cartesian tensors, Levi-Civita symbol, cross product, grad, div, curl and Laplacian. Physical examples of Cartesian tensors.

5 NUMERICAL METHODS
Numerical integration (trapezium rule and Simpson’s rule), Newton-Raphson method, Runge-Kutta algorithm.

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