Imperial College London

Module Outline

Statistical Mechanics

Module Code	PHYS96039	FHEQ Level	Level 6
Pre-requisites	None	Co-requisites	None
Primary Department	Physics		
Module Leader	Dr Tim Evans and Prof Kim Christensen		
Additional Teaching Departments	None		
Teaching Staff	Module leaders + Rapid Feedback Demonstrator + Associate		

Programmes on which the Module is delivered	Core/Elective
All UG Physics programmes (F300, F303, F309, F325, F390, F3W3)	Elective

Learning Outcomes

On completing the Statistical Mechanics course, students will be able to:

Phase Transitions and the Scaling Hypothesis

- describe the notion of an order parameter in a phase transition
- understand the relationship between the divergence of the characteristic length scale and the onset of scale invariance as a critical point is approached
- use the scaling hypothesis to obtain scaling forms for critical quantities and derive scaling relations between critical exponents
- write down and explain the form of the mean-field Landau free energy for a system with a scalar order parameter near a phase transition; deduce the critical behaviour predicted by Landau theory
- use a simple decimation-based real-space renormalization group (RSRG) procedure to estimate critical points and exponents
- discuss qualitatively Wilson's Renormalization Group Theory

Percolation

- derive exact solutions in one dimension and on the Bethe lattice for the mean cluster size, cluster size distribution, and strength of the percolating cluster
- describe near-threshold percolation in terms of a divergent cluster length scale; write down the scaling hypothesis and derive the scaling relations for percolation
- use the RSRG procedure to obtain estimates for the percolation threshold and the power-law divergence of the cluster length scale

Ising model

- define magnetisation, magnetic susceptibility, the spin correlation function and the spin correlation length for the Ising model
- understand the role of magnetisation as an order parameter
- use transfer matrices to solve the 1D Ising model analytically
- use the Landau mean-field theory of the Ising model to find its behaviour near the critical point, including critical exponents
- derive the Widom scaling form for the singular part of the free energy and derive the scaling relations
- apply the RSRG procedure to the Ising model in zero field in 1D and 2D; identify and interpret the fixed points

Description of Content	Atomic physics and quantum mechanics determine the physics of matter on the microscopic scale, but how do the short-ranged interactions at the nanoscale give rise to macroscopic phenomena such as the freezing of liquids, the magnetization of a ferromagnet, and the onset of superfluidity and superconductivity? Is the interaction of 10^{23} degrees of freedom too complicated to describe? It is certainly beyond any brute-force computer simulation. The aim of this course is to study how the co-operative behaviour of many simple constituents can lead to the emergence of new physics that could not have been predicted directly from the microscopic laws of motion. More specifically, we investigate continuous phase transitions, where the features that characterise a system suddenly change.
	
	Surprisingly, it turns out that systems near phase transitions have universal properties that do not depend on microscopic details. Instead, the key is to understand how phase transitions are related to spontaneously broken symmetries. The development of a conceptual framework to understand these phenomena was one of the most significant scientific advances of the late 20th century and culminated in Kenneth Wilson's renormalization group, for which he won the 1982 Nobel Prize in Physics.
	
	This course focuses on two simple models displaying phase transitions: the percolation model and the Ising model. We show how simple effective theories of percolation and magnetic ordering break down near critical points and how renormalization group ideas are then required. The mathematical framework developed is applicable in many other areas, including networks and finance.
	
	Course aims
	• To use two simple models, **percolation theory** and the **Ising model**, as an introduction to the **scaling hypothesis** and the **renormalization group**
	• To introduce and illustrate the following concepts in the modern theory of **phase transitions and critical phenomena**:
	  ○ spatial correlations controlled by a single length scale
	  ○ scale invariance at the critical point
	  ○ critical exponents
	  ○ divergent fluctuations and susceptibility at critical points
	  ○ universal classes for critical exponents
	  ○ dimensionality and symmetry
	  ○ real-space renormalisation group

Assessment	Assessment Type	Weighting
Written exam	Exam	100%

Learning & Teaching Hours	Independent Study Hours	Placement Hours	Total Hours
57	93	0	150
ECTS Credit	6	CATS Credit	12
Date of introduction	October 2016	Date of Last Revision	22/04/20